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STAT 110 Slides

موقع المكتب

Main Reference

Elementary Statistics

A Step by Step Approach

By

Allan Bluman

Chapter 8

Hypothesis Testing

Objectives

- Understand the terms used in hypothesis testing.
- Explain the five-step hypothesis-testing procedure.
- Describe Type I and Type II errors.
- Test hypothesis about a population mean using z test.

Introduction

Researchers are interested in answering many types of questions. For example,

- A physician might want to know whether a new medication will lower a person's blood pressure.
- An educator might wish to see whether a new teaching technique is better than a traditional one.
- A retail merchant might want to know whether the public prefers a certain color in a new line of fashion.
- Automobile manufacturers are interested in determining whether a new type of seat belt will reduce the severity of injuries caused by accidents.

These types of questions can be addressed through statistical **hypothesis testing,** which is a decision-making process for evaluating claims about a population.

In hypothesis testing, the researcher must define the population under study, state the particular hypotheses that will be investigated, give the significance level, select a sample from the population, collect the data, perform the calculations required for the statistical test, and reach a conclusion.

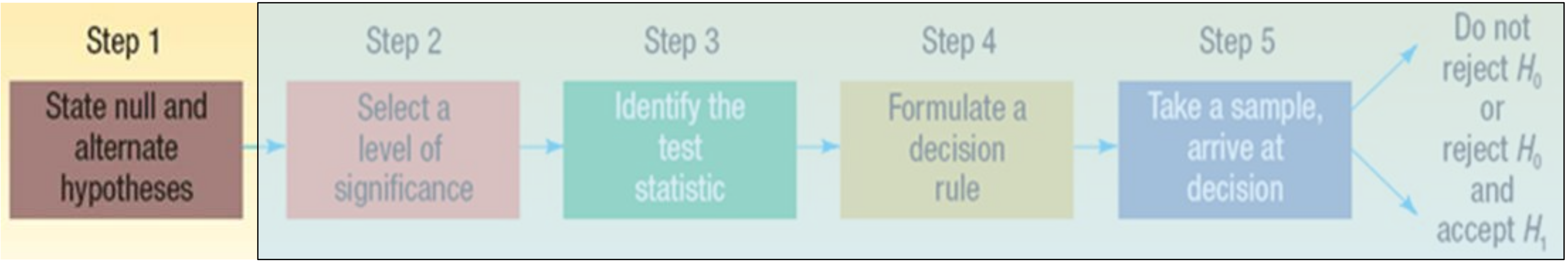
Hypothesis and Hypothesis Testing

A hypothesis is a statement about a population. Data are then used to check the reasonableness of the statement. To begin we need to define the word hypothesis.

HYPOTHESIS is a statement about the value of a population parameter developed for the purpose of testing.

HYPOTHESIS TESTING is a decision-making process for evaluating claims about a population based on sample evidence and probability theory.

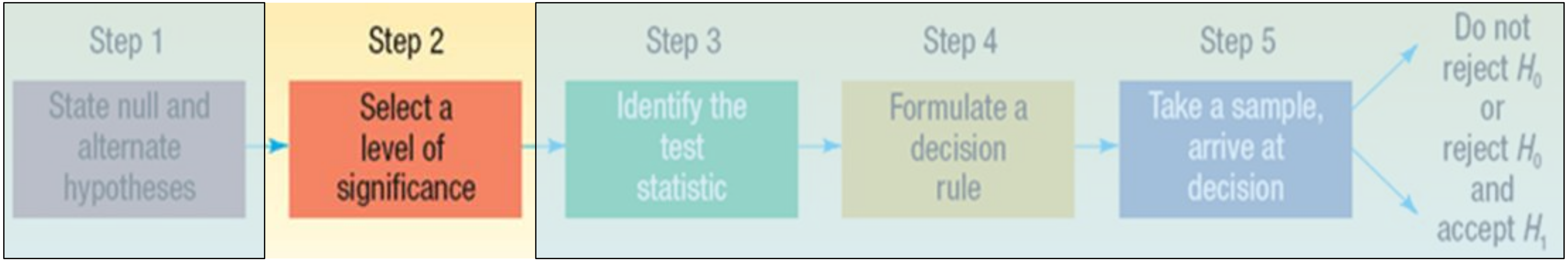
Five-Step Procedure for Testing a Hypothesis



NULL HYPOTHESIS is a statement about the value of a population parameter developed for the purpose of testing numerical evidence, designated H_0

ALTERNATE HYPOTHESIS is a statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false, designated H_1

Five-Step Procedure for Testing a Hypothesis

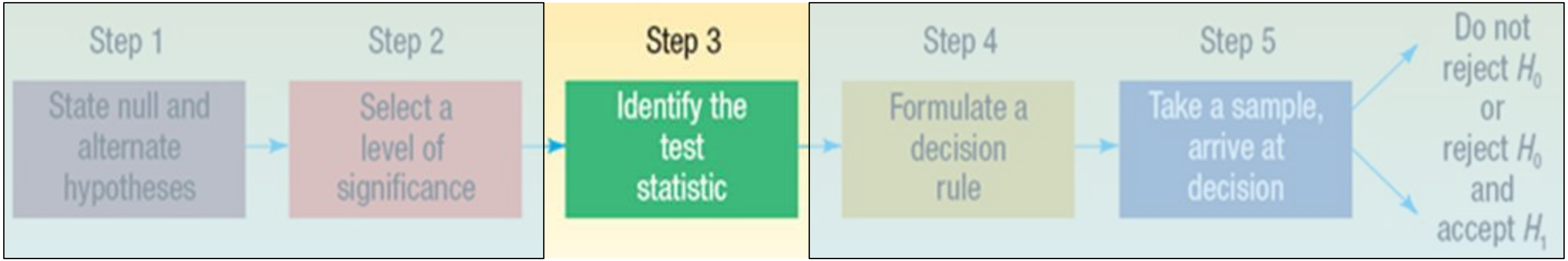


LEVEL OF SIGNIFICANCE is the probability of rejecting the null hypothesis when it is true, designated α.

TYPE I ERROR is rejecting the null hypothesis, H_0 , when it is true.

TYPE II ERROR is accepting the null hypothesis, H_0 , when it is false.

Five-Step Procedure for Testing a Hypothesis



There are many test statistics. In this chapter we use only z test statistic

TEST STATISTIC is a value, determined from sample information, used to determine whether to reject the null hypothesis.

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

Five-Step Procedure for Testing a Hypothesis

A decision rule is a statement of the specific conditions under which the null hypothesis is rejected or not rejected.

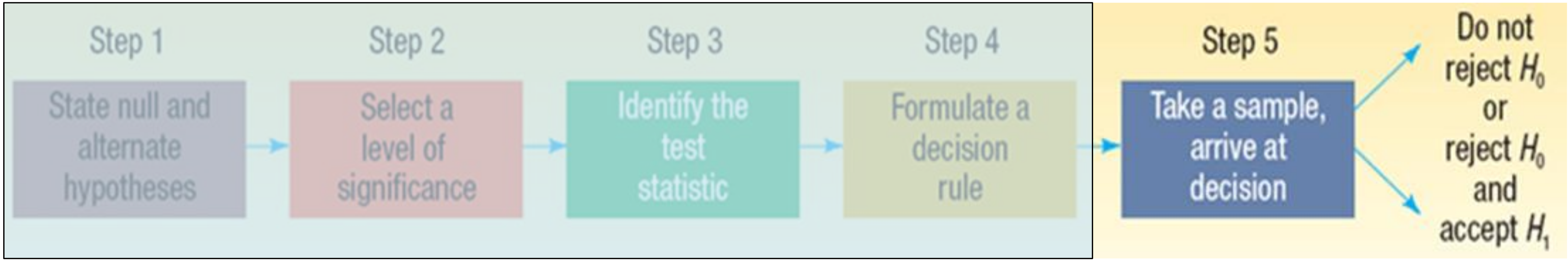
P-VALUE is the probability, computed using the test statistic, that measure the degree to which the sample will support the null hypothesis.

Decision Rule When Using a *P*-Value

If P-value $\leq \alpha$, reject the null hypothesis.

If P-value $> \alpha$, do not reject the null hypothesis.

Five-Step Procedure for Testing a Hypothesis



The final step in hypothesis testing is using the sample data to compute the test statistic and p-value, comparing the p-value to the significance level, and making a decision to reject or not to reject the null hypothesis.

	Claim	
Decision	**Claim is H_0**	**Claim is H_1**
Reject H_0	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Do not reject H_0	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Possible Outcomes of a Hypothesis Test

The following table summarizes the decisions the researcher could make and the possible consequences.

Null Hypothesis	Researcher: Does Not Reject H_0	Researcher: Rejects H_0
H_0 is true	Correct Decision	Type I Error
H_0 is false	Type II Error	Correct Decision

Important Things to Remember about H_0 and H_1

- H_0: null hypothesis vs H_1: alternate hypothesis
- H_0 and H_1 are mutually exclusive and collectively exhaustive
- H_0 is always presumed to be true, H_1 has the burden of proof
- A random sample (n) is used to "*reject* H_0"
- If we conclude 'do not reject H_0', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not enough evidence to reject H_0; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of H_0 ($=$, $\geq$, $\leq$).
- $\neq$, $<$, and $>$ always part of H_1
- In problem solving, look for **key words** and convert them into symbols. Some key words include: "***improved, better than, as effective as, different from, has changed***, etc."

Keywords	Symbol	Part of:
larger (more) than *greater than, above* *higher (bigger) than* *Increased*	$>$	H_1
smaller (less) than lower (shorter) than below decreased (reduced)	$<$	H_1
not more than *less than or equal* *at most*	$\leq$	H_0
not less than *greater than or equal* *at least*	$\geq$	H_0
different from *not equal to* *changed from* *not the same as*	$\neq$	H_1
exactly the same as *equal to* *not changed from* the same as	$=$	H_0

Hypothesis Setups for Testing a Mean (μ)

$H_0: \mu = \text{value}$	$H_0: \mu \geq \text{value}$	$H_0: \mu \leq \text{value}$
$H_1: \mu \neq \text{value}$	$H_1: \mu < \text{value}$	$H_1: \mu > \text{value}$

<u>Use *Z*-distribution</u>

If the population standard deviation is known

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

If the population standard deviation is not known and the sample size is at least 30.

$$z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Hypotheses

EXAMPLES

State the null and alternative hypotheses for each conjecture.

- The average number of monthly visits/sessions on the Internet by a person at home has increased from 36 in 2009.

$$H_0:\ \mu \le 36 \qquad \text{vs.} \qquad H_1:\ \mu > 36$$

- The average age of first-year medical school students is at least 27 years.

$$H_0:\ \mu \ge 27 \qquad \text{vs.} \qquad H_1:\ \mu < 27$$

- The average weight loss for a sample of people who exercise 30 minutes per day for 6 weeks is 8.2 pounds.

$$H_0:\ \mu = 8.2 \qquad \text{vs.} \qquad H_1:\ \mu \ne 8.2$$

z Test for a Mean

EXAMPLE

A researcher believes that the mean age of medical doctors in a large hospital system is older than the average age of doctors in the United States, which is 46. Assume the population standard deviation is 4.2 years. A random sample of 28 doctors from the system is selected, and the mean age of the sample is 48.6. Test the claim at $\alpha = 0.05$.

Step 1: State the null and alternate hypotheses.

$H_0 : \mu \leq 46 \qquad H_1 : \mu > 46$

Step 2: Select the level of significance.

$\alpha = 0.05$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution since σ is known

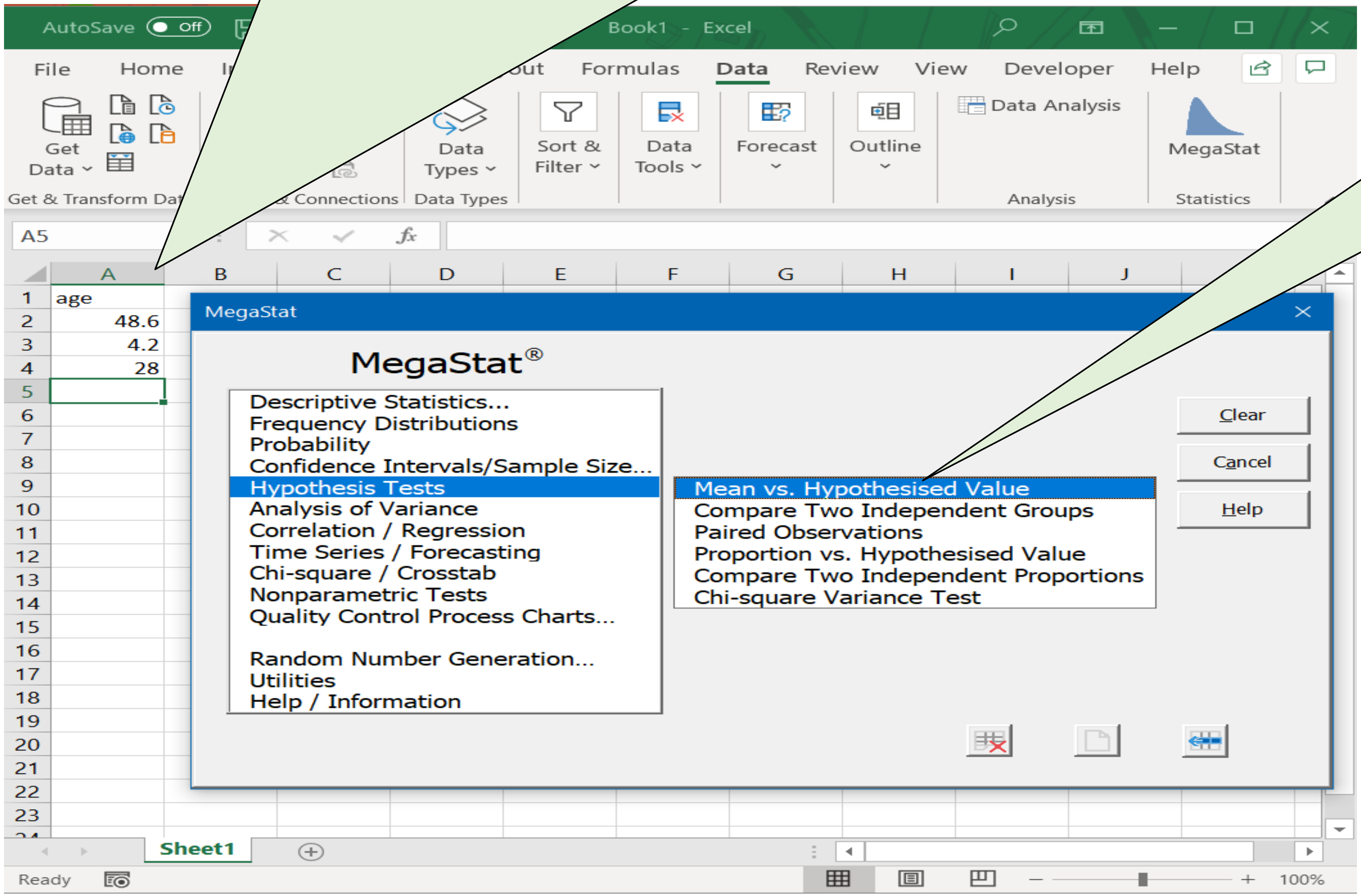
Step 4: Decision rule.

If P-value ≤ 0.05, reject H_0

Step 5: Analyze the data and interpret the result.

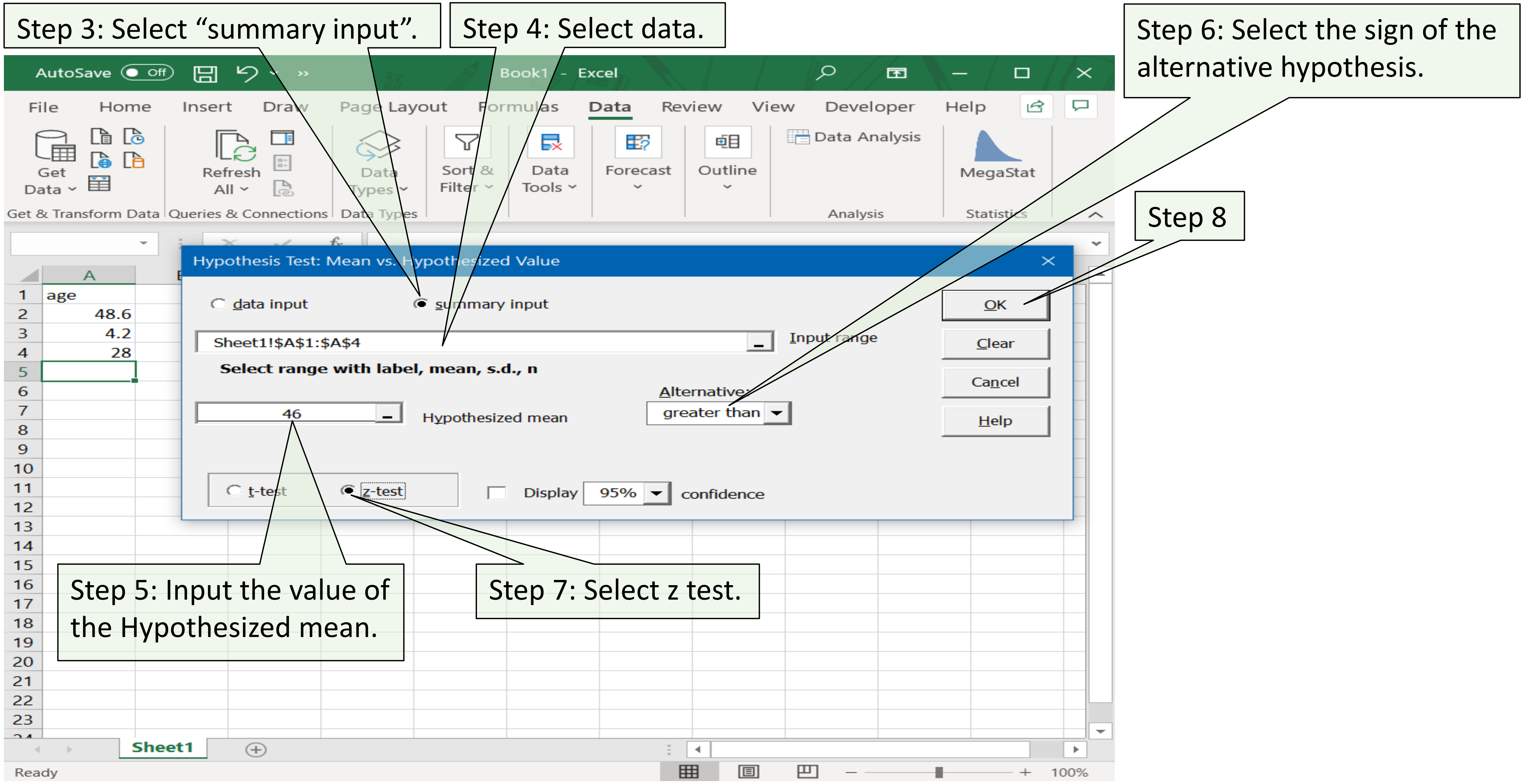
z Test for a Mean

Step 1: Enter the data summary as **Label, mean, Standard deviation and sample size** respectively.

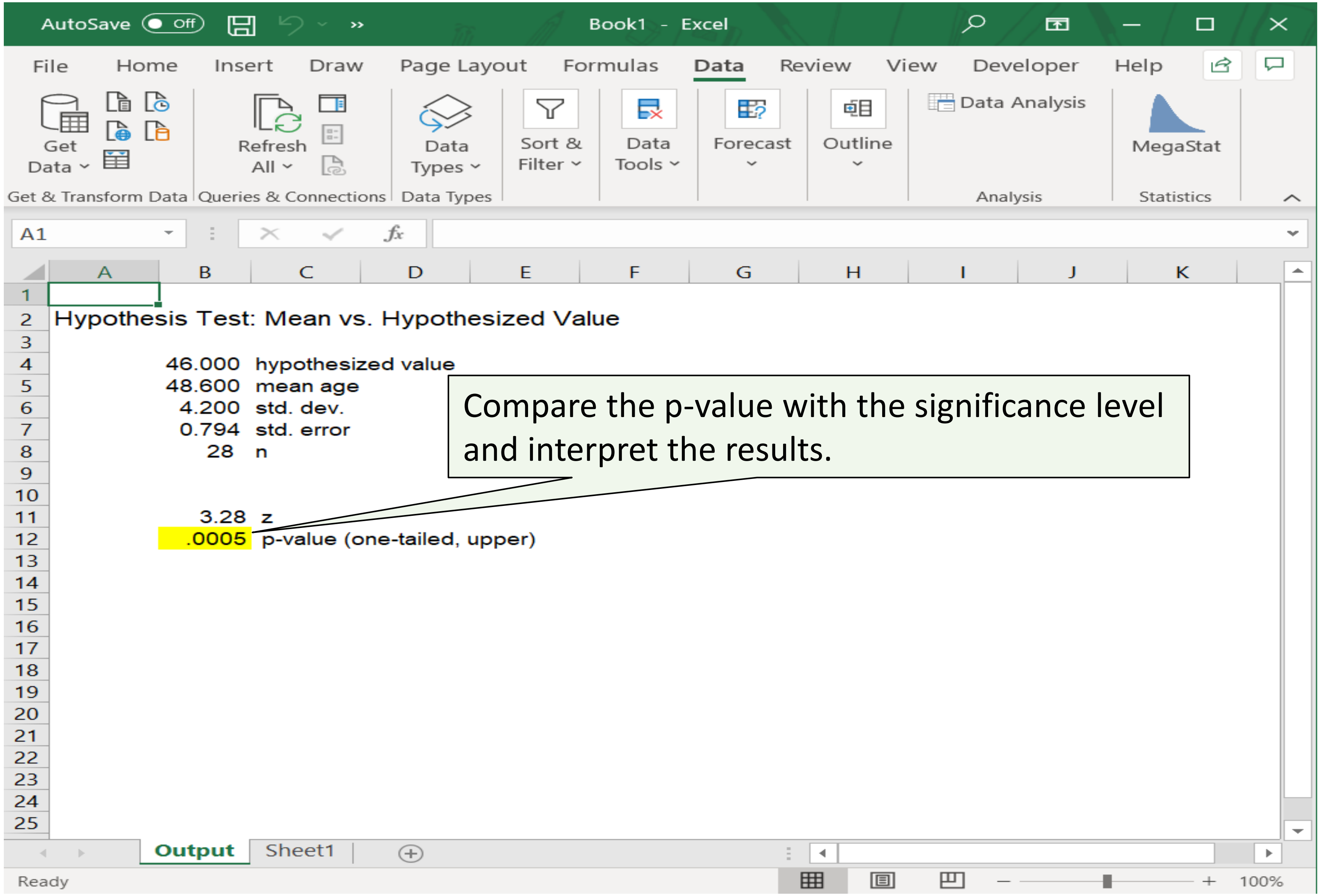


Step 2:
Select “DATA”
then “MegaStat”
then “Hypothesis Tests”
then “Mean vs. Hypothesized Value”

z Test for a Mean



z Test for a Mean



Since the p-value 0.0005 is less than 0.05, H_0 is rejected. We conclude that there is enough evidence to support the claim that the mean age of medical doctors in a large hospital system is older than the average age of doctors in the United States, which is 46.

z Test for a Mean

EXAMPLE

A researcher believes that the mean age of medical doctors in a large hospital system is **not less** than the average age of doctors in the United States, which is 46. Assume the population standard deviation is 4.2 years. A random sample of 28 doctors from the system is selected, and the mean age of the sample is 48.6. Test the claim at $\alpha = 0.05$.

Step 1: State the null and alternate hypotheses.

$H_0 : \mu \geq 46$ $\qquad$ $H_1 : \mu < 46$

Step 2: Select the level of significance.

$\alpha = 0.05$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution since σ is known

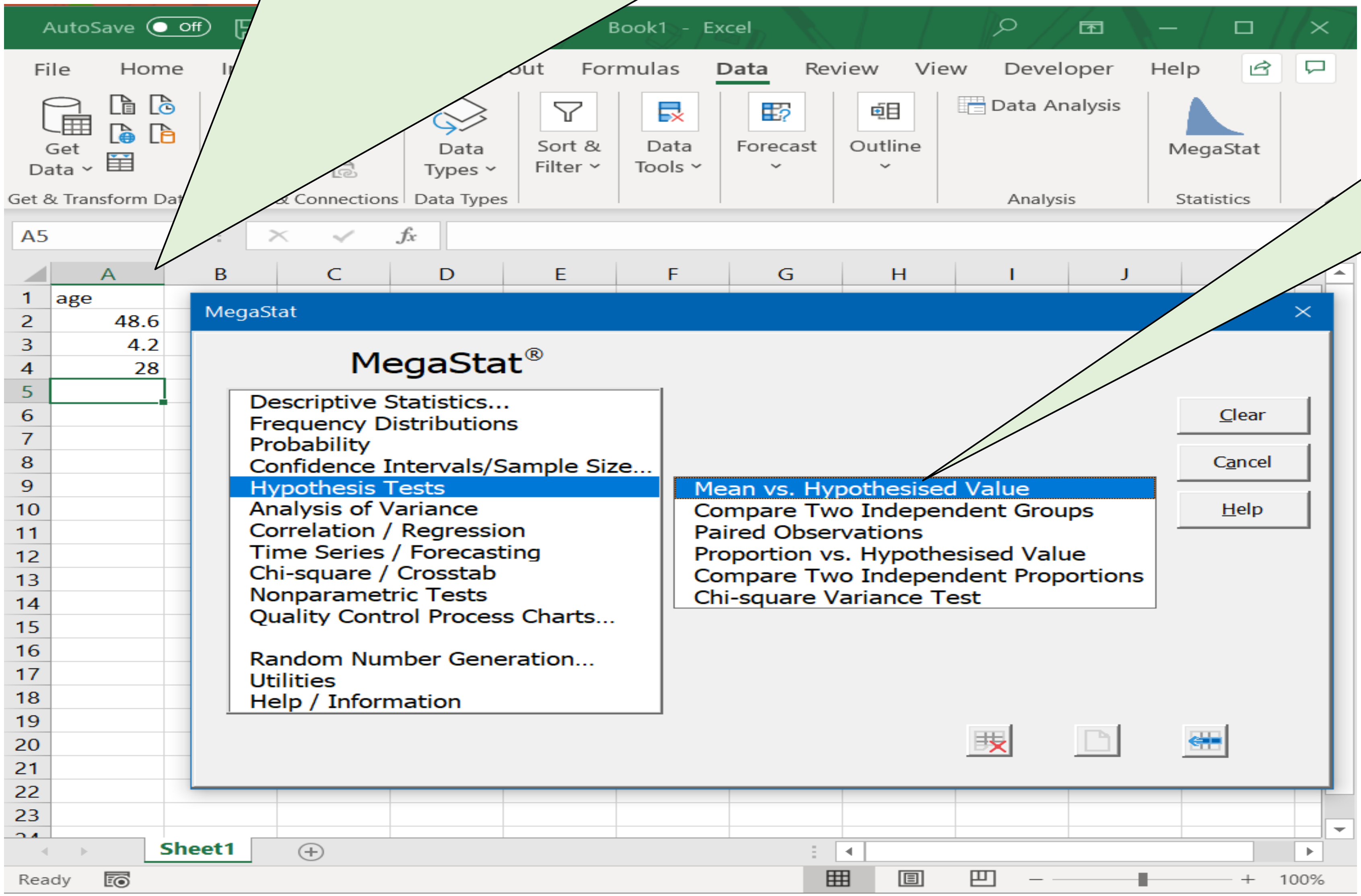
Step 4: Decision rule.

If P-value ≤ 0.05, reject H_0

Step 5: Analyze the data and interpret the result.

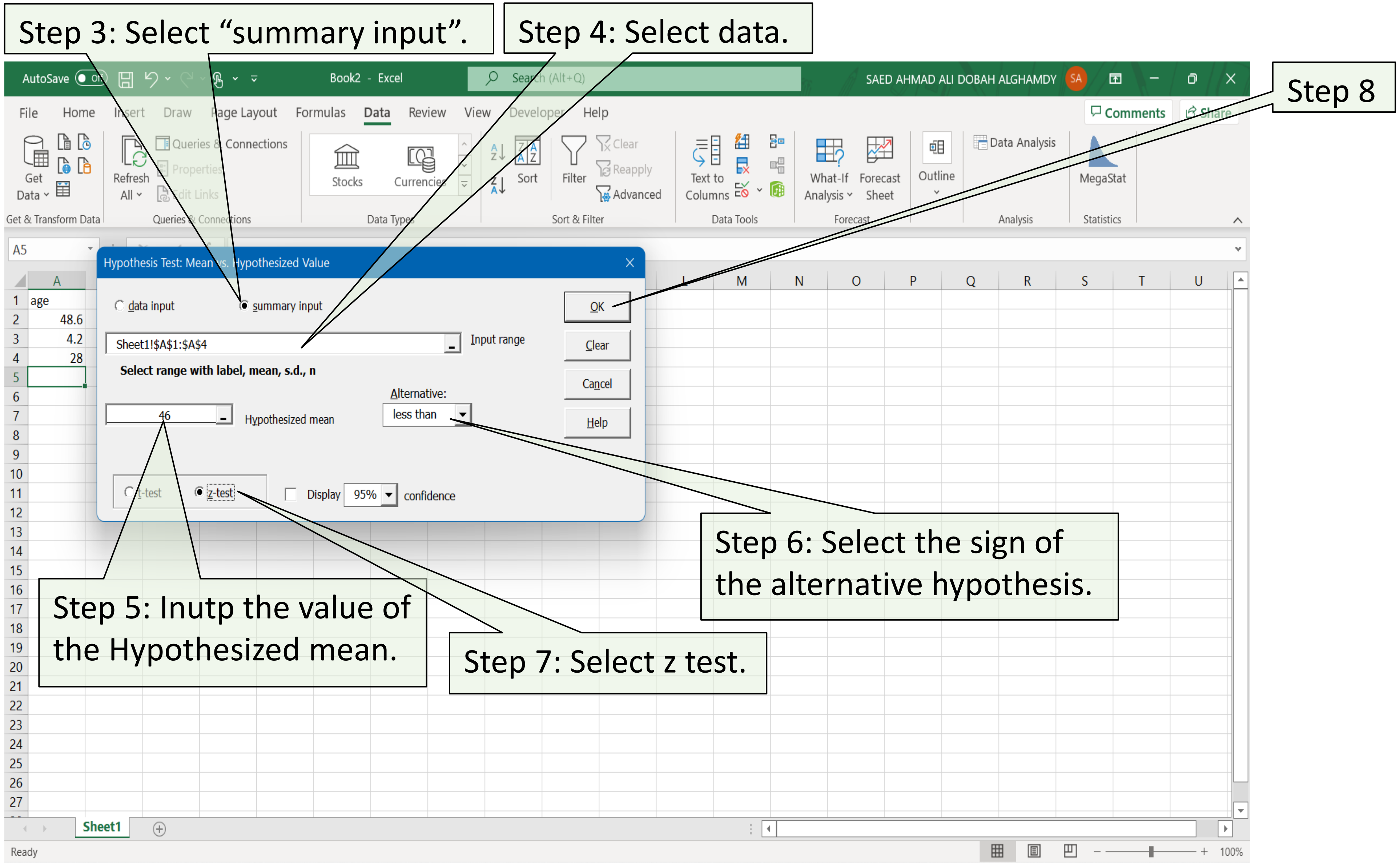
z Test for a Mean

Step 1: Enter the data summary as **Label, mean, Standard deviation and sample size** respectively.

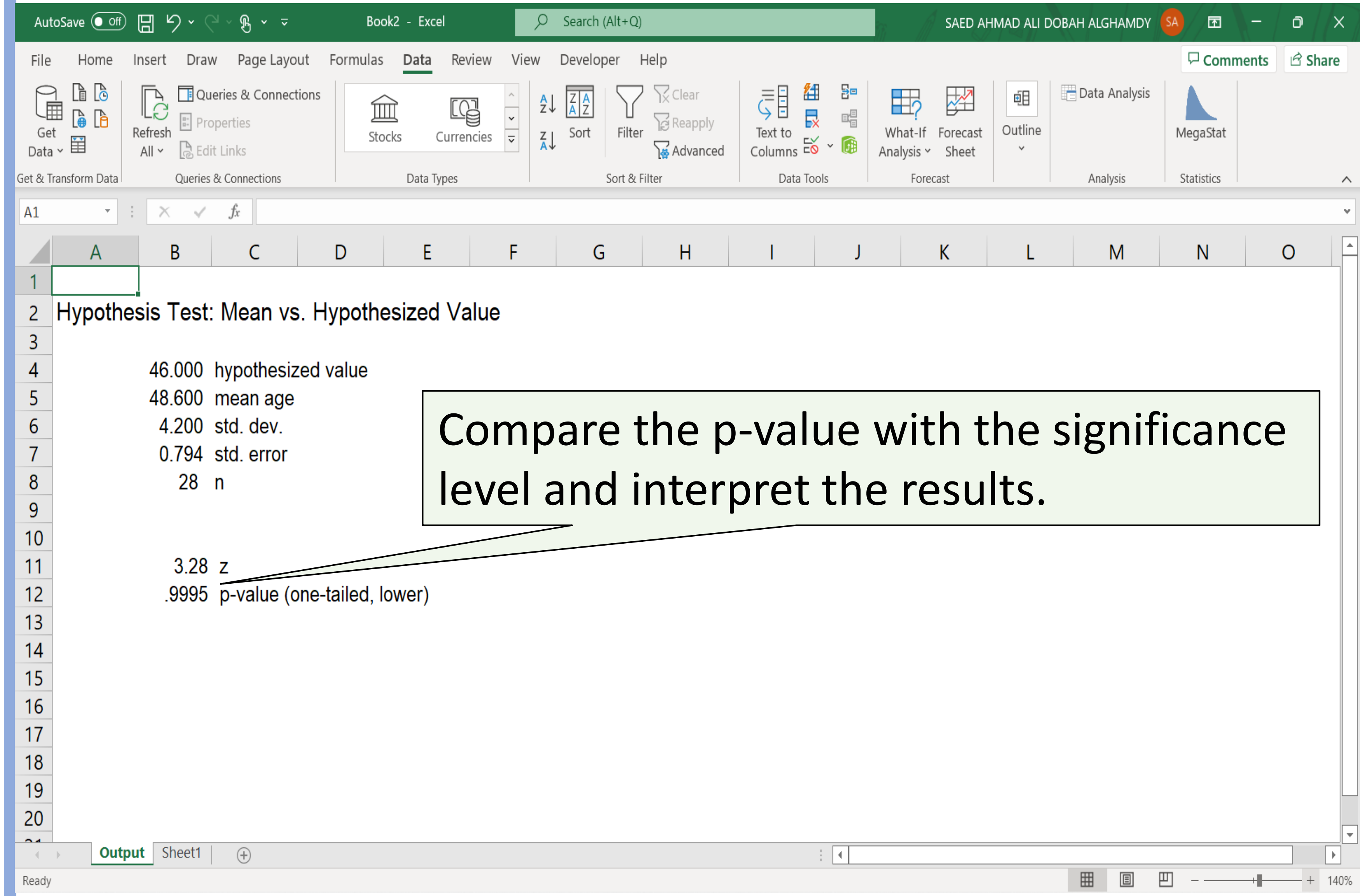


Step 2:
Select "DATA"
then "MegaStat"
then "Hypothesis Tests"
then "Mean vs. Hypothesized Value"

z Test for a Mean



z Test for a Mean



Since the p-value 0.9995 is greater than 0.05, H_0 is not rejected. We conclude that there is not enough evidence to reject the claim that the mean age of medical doctors in a large hospital system is not less than the average age of doctors in the United States, which is 46.

z Test for a Mean

EXAMPLE

A motorist claims that the city police issue an average of 60 speeding tickets per day. The following data show the number of speeding tickets issued each day for a randomly selected period of 32 days. Is there enough evidence to reject the claim at 0.10?

72	45	36	68	69	71	57	60
83	26	60	72	58	87	48	59
60	56	64	68	42	57	57	66
58	63	49	73	75	42	63	59

Step 1: State the null and alternate hypotheses.

$H_0: \mu = 60$ $\qquad$ $H_1: \mu \neq 60$

Step 2: Select the level of significance.

$\alpha = 0.10$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution since n is more than 30

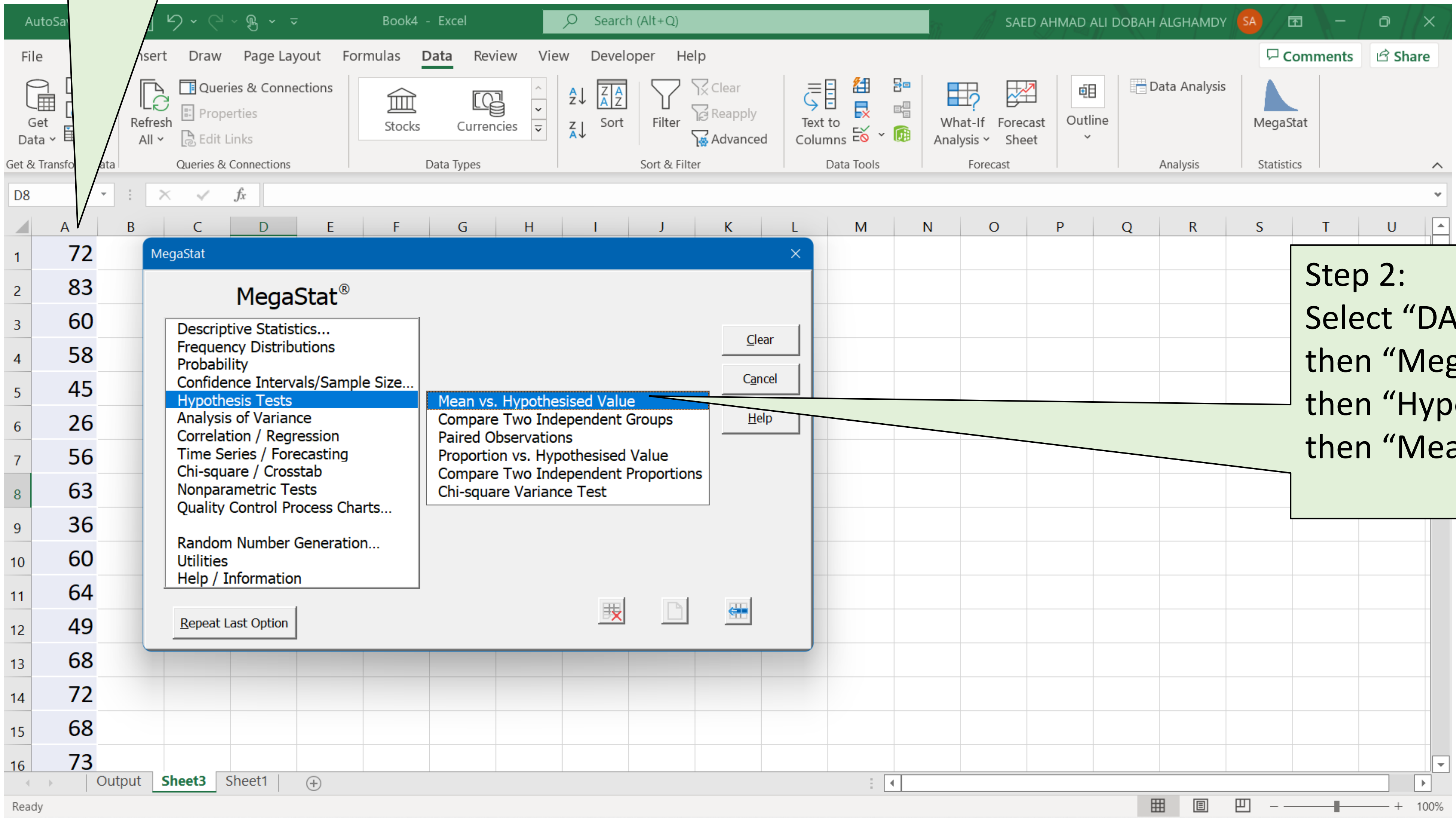
Step 4: Decision rule.

If P-value ≤ 0.1, reject H_0

Step 5: Analyze the data and interpret the result.

z Test for a Mean

Step 1: Enter the data



Step 2:
Select "DATA"
then "MegaStat"
then "Hypothesis Tests"
then "Mean vs. Hypothesized Value"

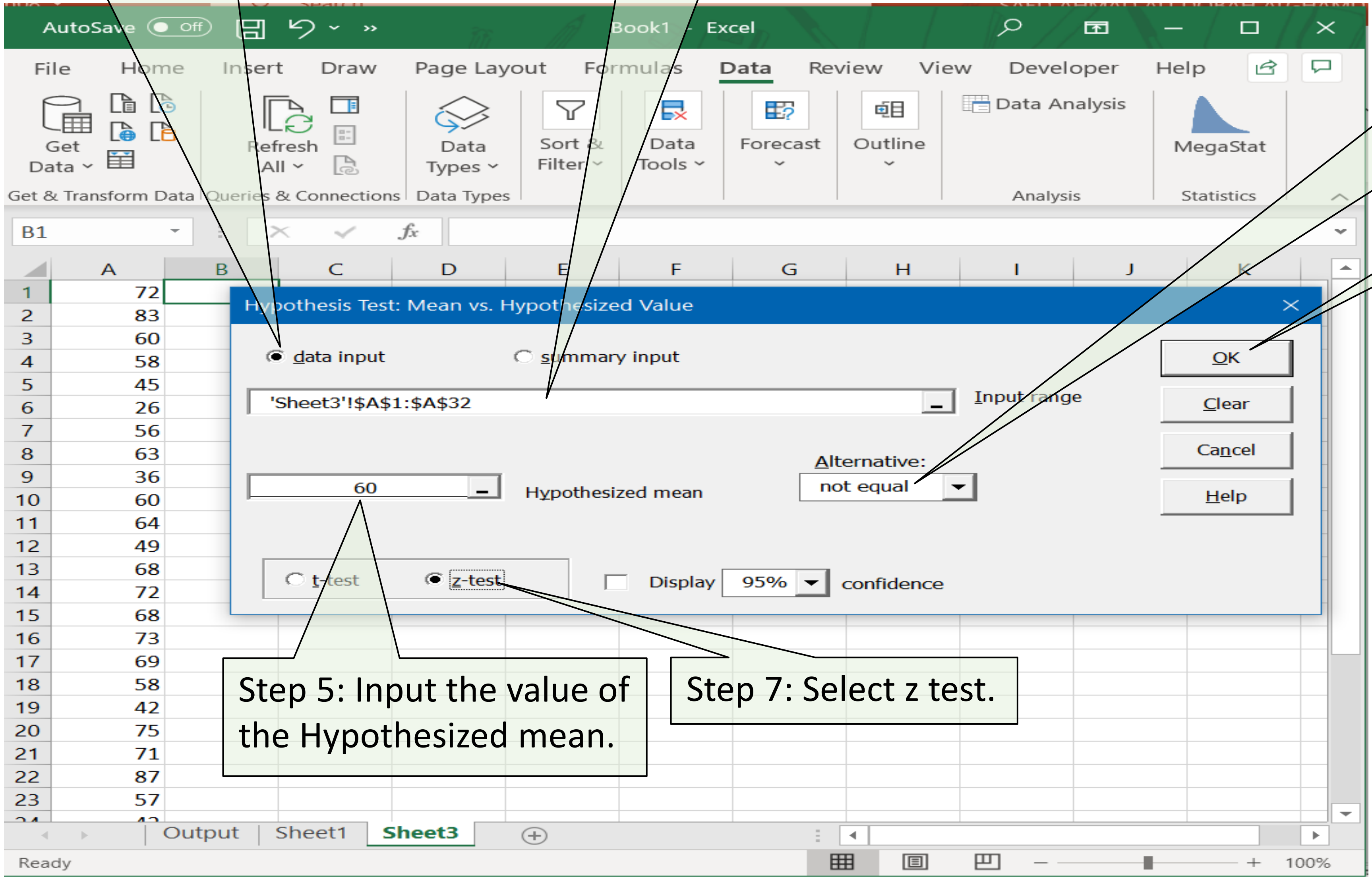
z Test for a Mean

Step 3: Select “Data input”.

Step 4: Select data.

Step 6: Select the sign of the alternative hypothesis.

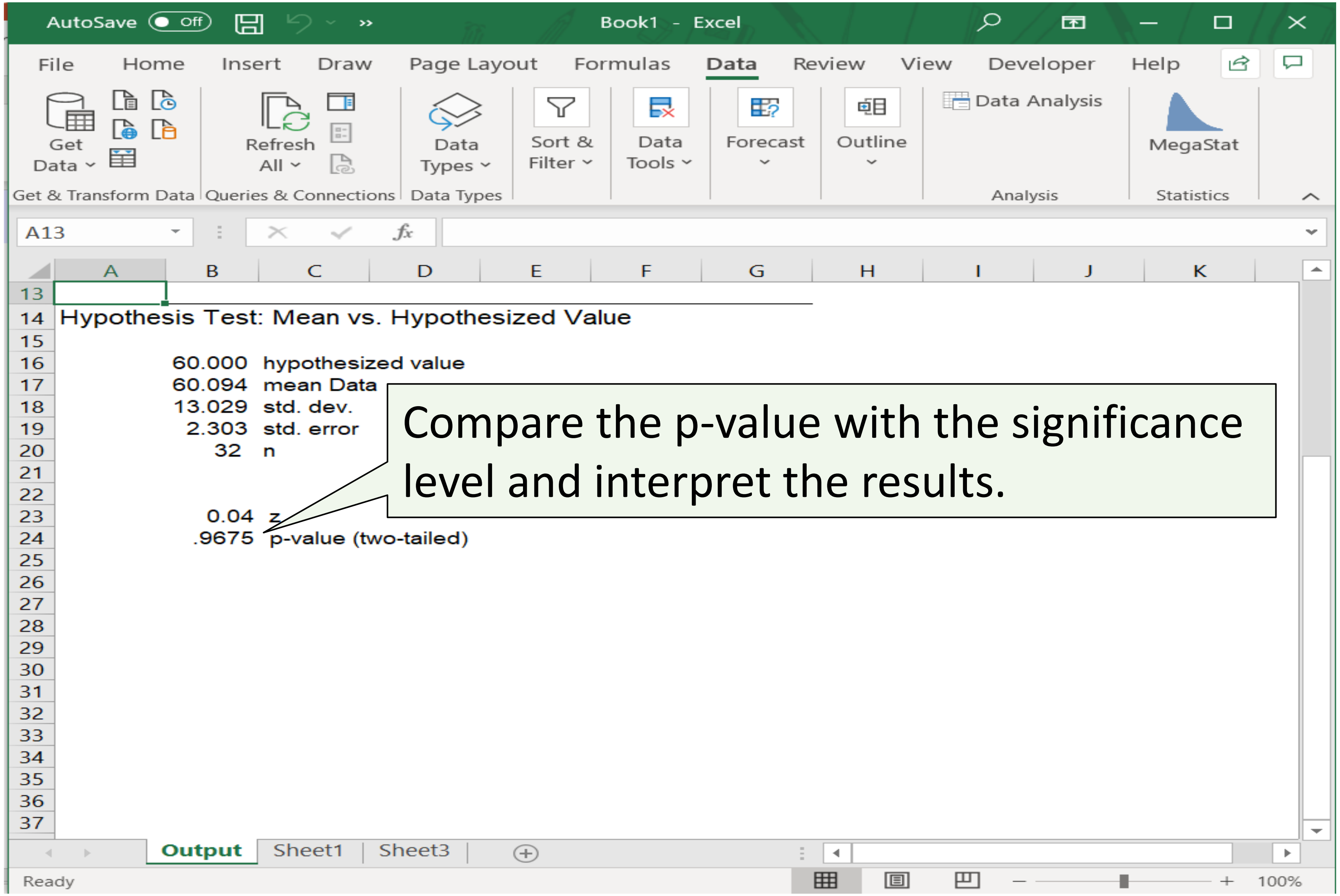
Step 8



Step 5: Input the value of the Hypothesized mean.

Step 7: Select z test.

z Test for a Mean



Since the p-value 0.9675 is greater than 0.1, H_0 is not rejected. We conclude that there is not enough evidence to reject the claim that the city police issue an average of 60 speeding tickets per day.

EXAMPLE

A motorist claims that the city police issue an average of at least 65 speeding tickets per day. The following data show the number of speeding tickets issued each day for a randomly selected period of 32 days. Is there enough evidence to reject the claim at 0.01?

72	45	36	68	69	71	57	60
83	26	60	72	58	87	48	59
60	56	64	68	42	57	57	66
58	63	49	73	75	42	63	59

Step 1: State the null and alternate hypotheses.

$H_0 : \mu \geq 65$ $\quad\quad$ $H_1 : \mu < 65$

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution since n is greater than 30

Step 4: Decision rule.

If P-value ≤ 0.01, reject H_0

Step 5: Analyze the data and interpret the result.

?

Summary

- ✓ A statistical hypothesis is a conjecture about a population.
- ✓ There are two of statistical hypotheses: the null and the alternative hypotheses. The equality is always part of the null hypothesis.
- ✓ Researchers compute a test value from the sample data to decide whether the null hypothesis should be rejected.
- ✓ The null hypothesis is rejected when the difference between the population parameter and the sample statistic is said to be significant.
- ✓ The level of significance, α , is the probability of committing a type I error which is rejecting the null hypothesis when it is true.
- ✓ A second kind of error, the type II error, can occur when the null hypothesis is not rejected when it is false.

Summary

- ✓ All hypothesis-testing situations using the *P*-value method should include the following steps:
  1. State the hypotheses and identify the claim.
  2. Select the significance level, α.
  3. Select the test statistics.
  4. Formulate the decision rule.
  5. Analyze the data and interpret the results.
- ✓ The z test is used to test a mean when the population standard deviation is known or when the sample size is at least 30.