Dr. Saeed A. Dobbah Alghamdi

Department of Statistics
Faculty of Sciences
Building 90, Office 26F41
King Abdulaziz University
http://saalghamdy.kau.edu.sa

Dr. Saeed's Website

Statistics for Business & Economics

David R. Anderson
Dennis J. Sweeney
Thomas A. Williams

Hypothesis Tests

Chapter 9

Learning Objectives

LO1 Define a hypothesis and hypothesis testing.

LO2 Explain the five-step hypothesis-testing procedure.

LO3 Describe Type I and Type II errors.

LO4 Distinguish between a one-tailed and two-tailed hypothesis

LO5 Conduct a test of hypothesis about a population mean.

LO6 Conduct a test of hypothesis about a population proportion.

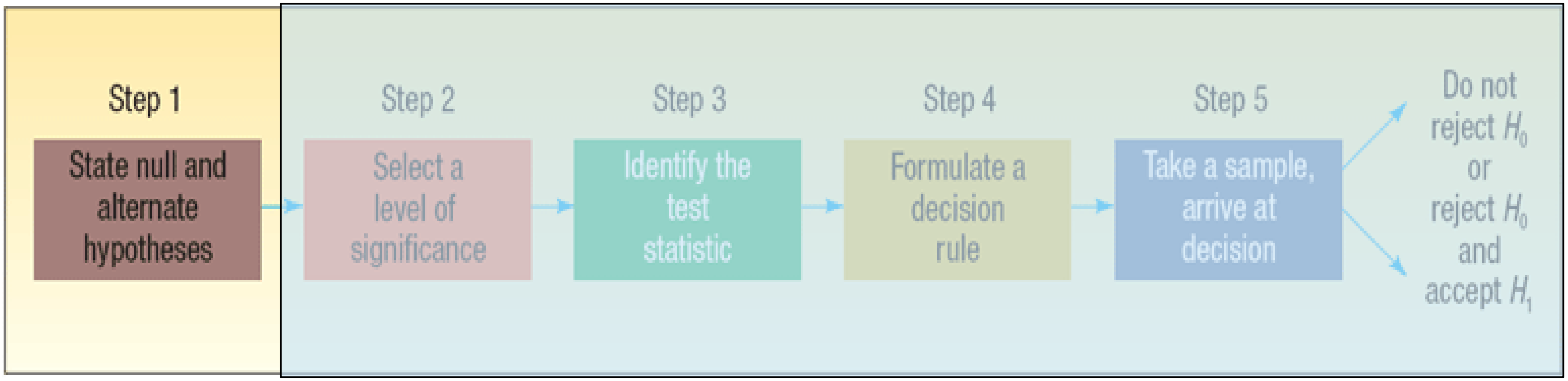
Hypothesis and Hypothesis Testing

A hypothesis is a statement about a population. Data are then used to check the reasonableness of the statement. To begin we need to define the word hypothesis.

HYPOTHESIS A statement about the value of a population parameter developed for the purpose of testing.

HYPOTHESIS TESTING A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.

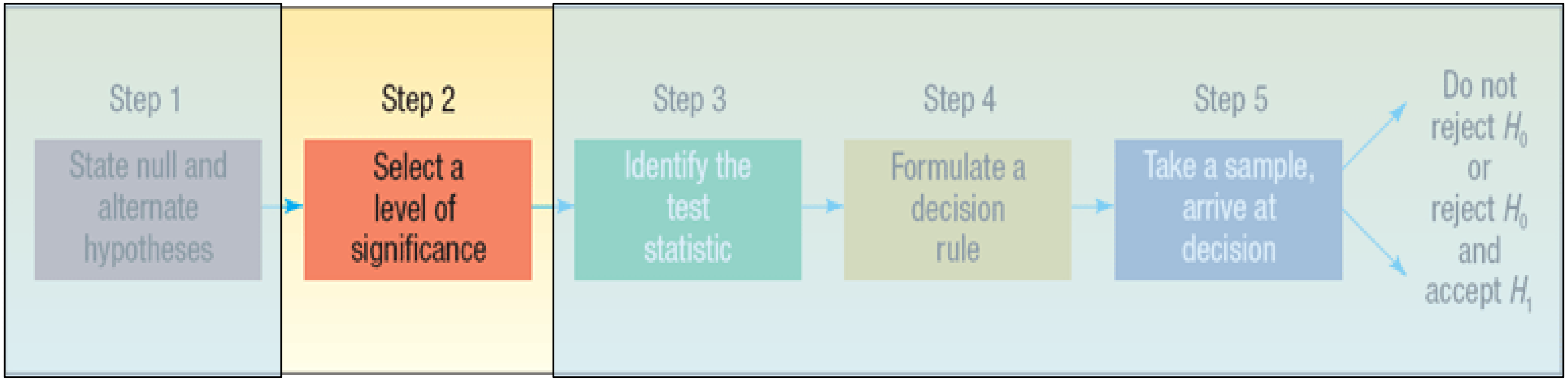
Five-Step Procedure for Testing a Hypothesis



NULL HYPOTHESIS A statement about the value of a population parameter developed for the purpose of testing numerical evidence, designated H_0

ALTERNATE HYPOTHESIS A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false, designated H_1

Five-Step Procedure for Testing a Hypothesis

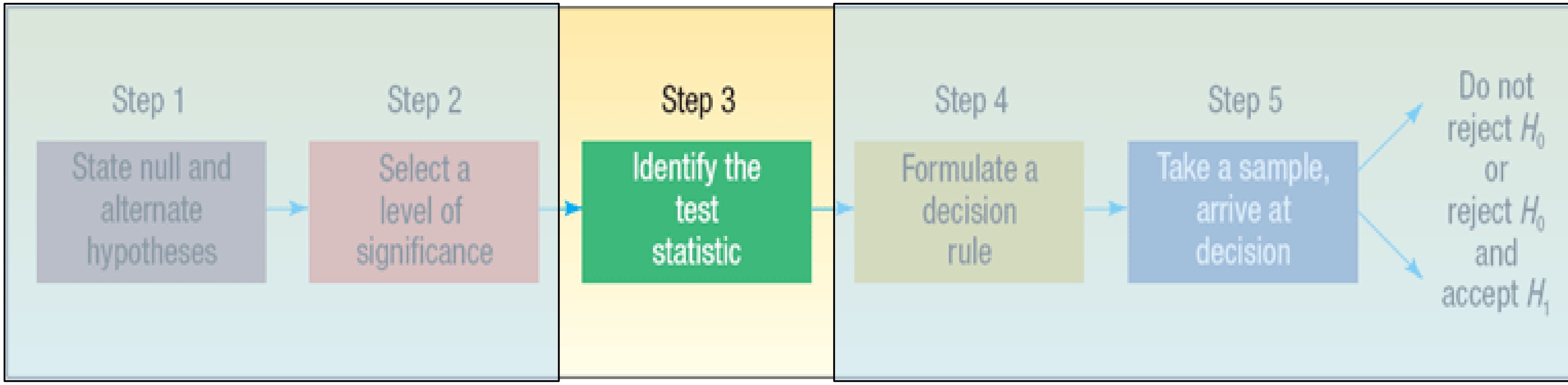


LEVEL OF SIGNIFICANCE The probability of rejecting the null hypothesis when it is true, designated α.

TYPE I ERROR Rejecting the null hypothesis, H_0 , when it is true.

TYPE II ERROR Accepting the null hypothesis, H_0 , when it is false.

Five-Step Procedure for Testing a Hypothesis

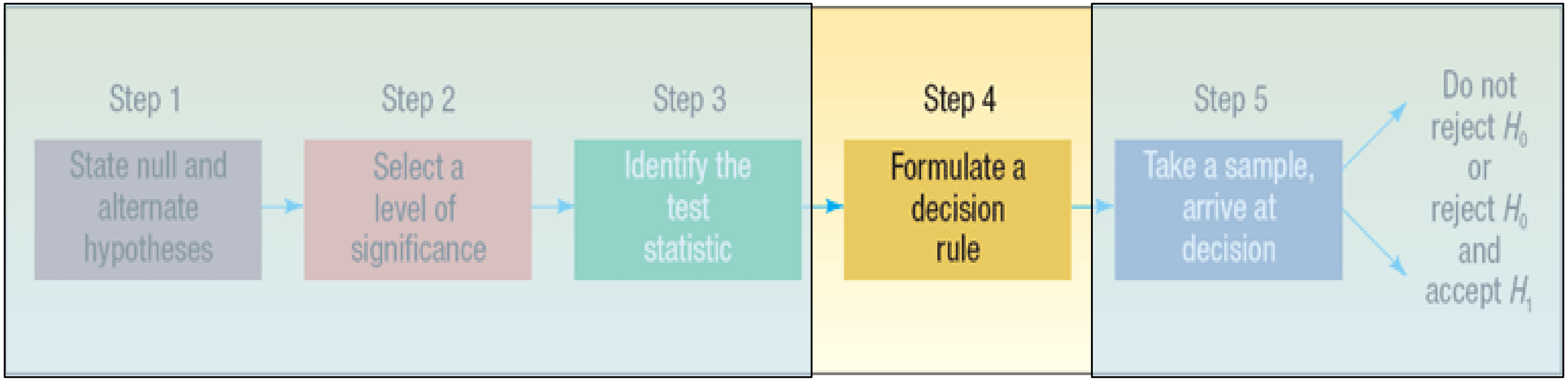


There are many test statistics. In this chapter we use both z and t as the test statistic.

TEST STATISTIC A value, determined from sample information, used to determine whether to reject the null hypothesis.

$$\text{Test value} = \frac{(\text{observed value}) - (\text{expected value})}{\text{standard error}}$$

Five-Step Procedure for Testing a Hypothesis



A decision rule is a statement of the specific conditions under which the null hypothesis is rejected and the condition under which it is not rejected.

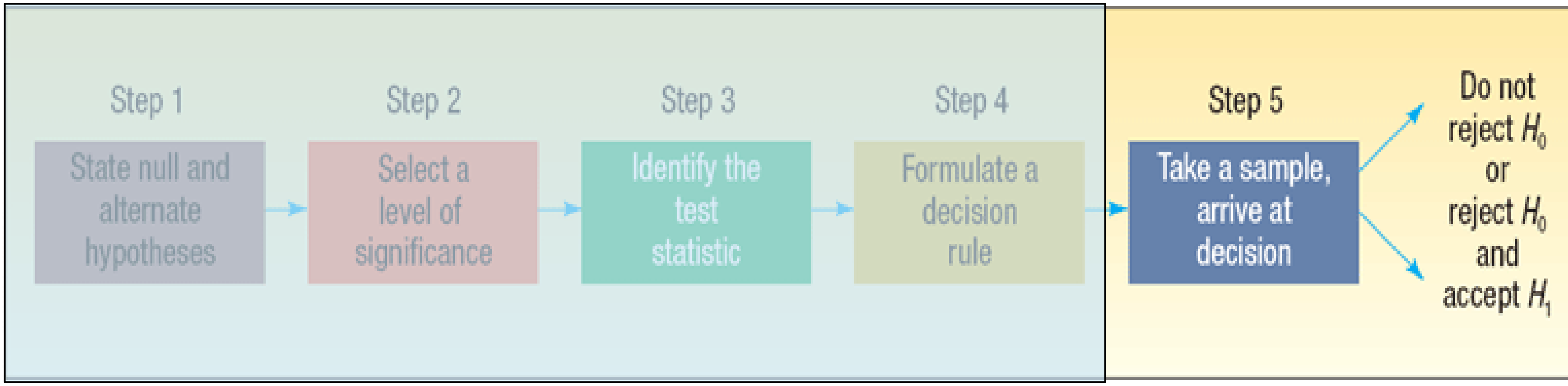
P-VALUE The probability, computed using the test statistic, that measure the degree to which the sample supports the null hypothesis.

Decision Rule When Using a *P*-Value

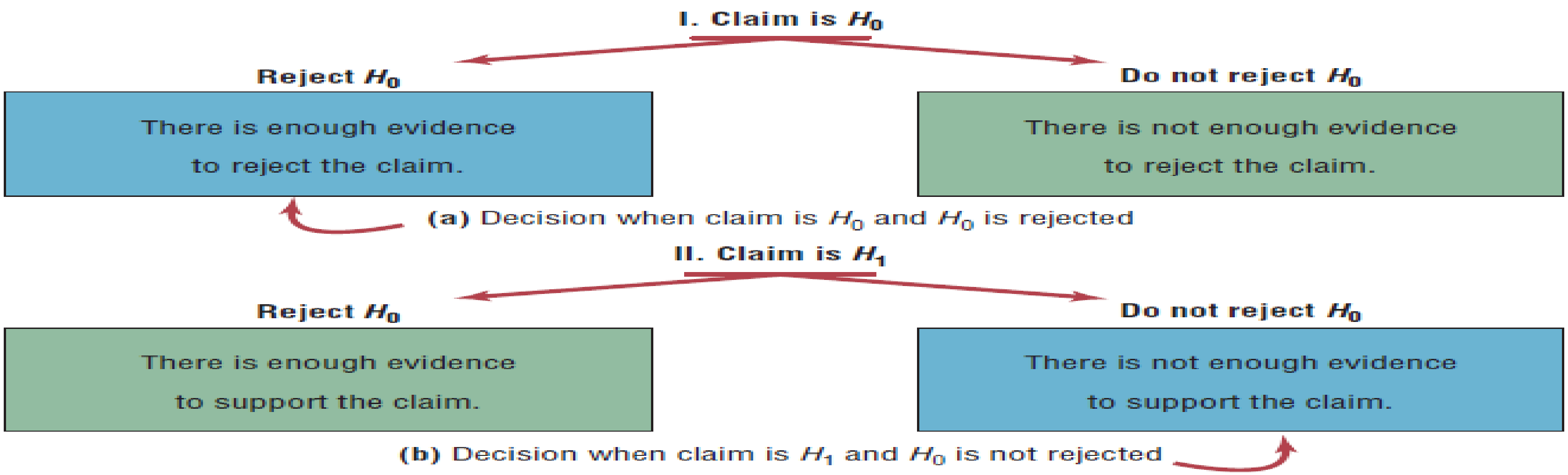
If P-value $\leq \alpha$, reject the null hypothesis.

If P-value $> \alpha$, do not reject the null hypothesis.

Five-Step Procedure for Testing a Hypothesis



The fifth and final step in hypothesis testing is computing the test statistic, comparing it to the critical value, and making a decision to reject or not to reject the null hypothesis.



Possible Outcomes of a Hypothesis Test

The following table summarizes the decisions the researcher could make and the possible consequences.

Null Hypothesis	Researcher: Does Not Reject H_0	Researcher: Rejects H_0
H_0 is true	Correct decision	Type I error
H_0 is false	Type II error	Correct decision

Important Things to Remember about H_0 and H_1

- H_0: null hypothesis and H_1: alternate hypothesis
- H_0 and H_1 are mutually exclusive and collectively exhaustive
- H_0 is always presumed to be true
- H_1 has the burden of proof
- A random sample (n) is used to "*reject* H_0"
- If we conclude 'do not reject H_0', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not enough evidence to reject H_0; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of H_0 ("$=$" , "$\geq$" , "$\leq$").
- "$\neq$" "$<$" and "$>$" always part of H_1
- In problem solving, look for **key words** and convert them into symbols. Some key words include: "***improved, better than, as effective as, different from, has changed***, etc."

Keywords	Inequality Symbol	Part of:
Larger (or more) than	$>$	H_1
Smaller (or less)	$<$	H_1
No more than	$\leq$	H_0
At least	$\geq$	H_0
Has increased	$>$	H_1
Is there difference?	$\neq$	H_1
Has not changed	$=$	H_0
Has "improved", "is better than". "is more effective"	See left text	H_1

Hypothesis for Testing a Mean (μ) or Proportion (π)

MEAN

$H_0: \mu = \text{value}$ $H_1: \mu \neq \text{value}$	$H_0: \mu \geq \text{value}$ $H_1: \mu < \text{value}$	$H_0: \mu \leq \text{value}$ $H_1: \mu > \text{value}$

<u>Use *Z*-distribution</u>

If the population standard deviation is known or the sample is greater than 30.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

<u>Use *t*-distribution</u>

If the population standard deviation is unknown and the sample is less than 30.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

PROPORTION

$H_0: \pi = \text{value}$ $H_1: \pi \neq \text{value}$	$H_0: \pi \geq \text{value}$ $H_1: \pi < \text{value}$	$H_0: \pi \leq \text{value}$ $H_1: \pi > \text{value}$

- The sample proportion is denoted by *p* and is found by ***x/n***
- It is assumed that the binomial assumptions discussed in Chapter 6 are met
- The normal distribution can be used as an approximation to the binomial distribution
- The test statistic is computed as shown:

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}}$$

If P-value $\leq \alpha$, reject the null hypothesis.

Testing for a Mean with a Known σ

Example

Jamestown Steel Company manufactures and assembles desks and other office equipment. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of **200** and a standard deviation of **16**. Recently, new production methods have been introduced and new employees hired. The VP would like to investigate whether there has been a **change** in the weekly production of the Model A325 desk. A sample of 50 weeks shows a mean number of desks produced to be 203.5. Test the claim and interpret the result at 1% significance value.

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu = 200 \qquad H_1 : \mu \neq 200$

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution since σ is known

$$z = \frac{\bar{X}-\mu}{\sigma} = \frac{203.5-200}{16/\sqrt{50}} = 1.55$$

Step 4: Formulate the decision rule.

If P-value ≤ 0.01, reject H_0

Step 5: Make a decision and interpret the result.

Since the p-value is 0.1219 is greater than 0.01, H_0 is not rejected. We conclude that there is not enough evidence to support the claim that the production rate at the plant has changed from 200 per week.

Testing for a Mean with a Known σ

Example

Jamestown Steel Company manufactures and assembles desks and other office equipment. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of **200** and a standard deviation of **16**. Recently, new production methods have been introduced and new employees hired. The VP would like to investigate whether there has been an **increase** in the weekly production of the Model A325 desk. A sample of 50 weeks shows a mean number of desks produced to be 203.5. Test the claim and interpret the result at 1% significance value.

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu \leq 200 \qquad H_1 : \mu > 200$

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution since σ is known

$$z = \frac{\bar{X} - \mu}{\sigma} = \frac{203.5 - 200}{16/\sqrt{50}} = 1.55$$

Step 4: Formulate the decision rule.

If P-value ≤ 0.01, reject H_0

Step 5: Make a decision and interpret the result.

Since the p-value is 0.0610 is greater than 0.01, H_0 is not rejected. We conclude that there is not enough evidence to support the claim that the production rate at the plant is more than 200 per week.

Testing for a Mean with an Unknown σ

Example

The McFarland Insurance Company Claims Department reports the **mean** cost to process a claim is **$60**. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of **26** claims processed last month. The sample information is reported as shown.

45	49	62	40	43	61	48	53	67
63	78	64	48	54	51	56	63	69
58	51	58	59	56	57	38	76	

At the **.01** significance level is it reasonable a claim is ***now less than $60***?

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0: \mu \geq \$60 \qquad H_1: \mu < \$60$

Step 2: Select the level of significance.

$\alpha = 0.01$ as stated in the problem

Step 3: Select the test statistic.

Use t-distribution since σ is unknown

Step 4: Formulate the decision rule.

If P-value ≤ 0.01, reject H_0

Step 5: Make a decision and interpret the result.

Since the p-value is 0.0407 is greater than 0.01, H_0 is not rejected. We conclude that there is not enough evidence to support the claim that the mean cost to process a claim is now less than $60.

Testing for a Population Proportion

Example

Suppose prior elections in a certain state indicated it is necessary for a candidate for governor to receive at least 80 percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office and plans to conduct a survey of 2,000 registered voters in the northern section of the state. The sample revealed that 1550 planned to vote for the incumbent governor. Using the hypothesis-testing procedure at 5% significance level, assess the governor's chances of reelection.

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0: \pi \geq .80$ $H_1: \pi < .80$

Step 2: Select the level of significance.

$\alpha = 0.05$ as stated in the problem

Step 3: Select the test statistic.

Use z-distribution

Step 4: Formulate the decision rule.

If P-value ≤ 0.05, reject H_0

Step 5: Make a decision and interpret the result.

Since the p-value is 0.0026 is less than 0.05, H_0 is rejected. We conclude that there is enough evidence to reject the claim that the governor will be reelected.

Summary

Null hypothesis The hypothesis tentatively assumed true in the hypothesis testing procedure.

Alternative hypothesis The hypothesis concluded to be true if the null hypothesis is rejected.

Type I error The error of rejecting $H0$ when it is true.

Type II error The error of accepting $H0$ when it is false.

Level of significance The probability of making a Type I error when the null hypothesis is true as an equality.

Test statistic A statistic whose value helps determine whether a null hypothesis should be rejected.

***p*-value** A probability that provides a measure of the evidence against the null hypothesis given by the sample. Smaller *p*-values indicate more evidence against $H0$.