CE 371 Surveying
Circular Curves

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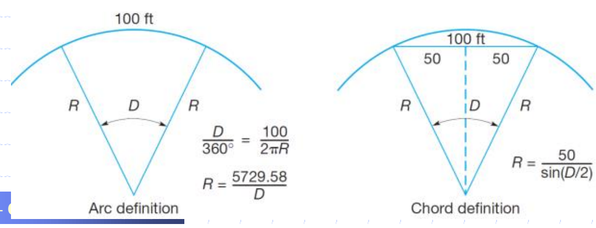
Overview

- Introduction
- Definition of Terms
- Parameters of Circular Curves (Arc Definition)
- Curve Stationing
- Design Criteria

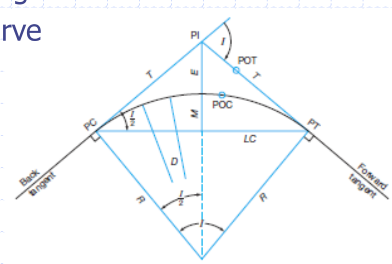
Introduction

- Horizontal curves are used in route surveying for construction of highways, railroads, and pipelines.
- Horizontal curves can be circular or spiral, however, we will be explaining circular curves only.
- A circular curve is either defined by its radius (ex: a 500-m curve), or by its degree (ex: 2-degree curve).
- Degree of curve (D_c) is the angle at the center of a circular arc subtended by a 100-m chord C (Chord definition)
- Degree of curve (D_a)is the angle at the center subtended by a 100-m circular arc S (Arc definition)



Definition of Terms

- PI Point of Intersection of the tangents
- PC Point of Curvature, or beginning of the curve
- PT Point of Tangent, or end of curve
- POT Point On the Tangent
- POC Point On the Curve
- I Deflection angle
- R Radius of curve
- T Tangent length
- LC Long Chord, or the distance from PC to PT
- E External distance, or the radial distance from PI to the curve
- M Middle ordinate, or the radial distance from midpoint of the long chord to midpoint of the curve



Parameters of Circular Curves
(Arc Definition)

For 100-m stationing, S = 100 m. Circular curve parameters are:

Length of curve = $L = S(I/D) = 2\pi R\ (I/360)$

Radius of Curve = $R = (S/D)\text{x}(180/\pi)$

Degree of Curve = $D = (S/R)\text{x}(180/\pi)$

Tangent distance = $T = R\tan(I/2)$

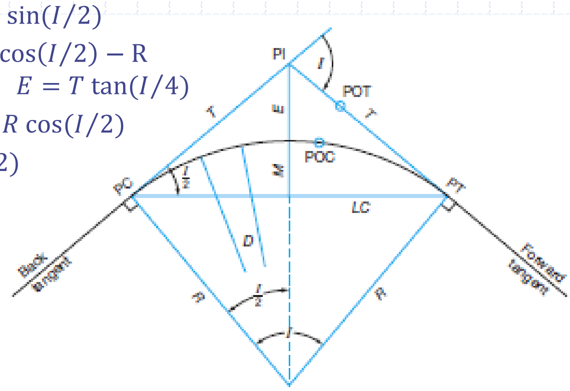
Long chord= $LC = 2R\sin(I/2)$

External distance = $E = R/\cos(I/2) - R$

$E = T\tan(I/4)$

Middle ordinate = $M = R - R\cos(I/2)$

$= E\cos(I/2)$



Example

A circular curve has 300 m radius and 60° deflection angle. What is its degree by (a) arc definition and (b) chord definition of standard length 30 m. Also calculate (i) length of curve, (ii) tangent length, (iii) length of long chord, (iv) mid-ordinate and (v) apex (ext.) distance.

Solution

Arc definition S= 30 $D = (S/R)\text{x}(180/\pi)$ = 30/300 x(180/π) = 5.73°

Chord definition Rx sin (D_c/2) = S/2 sin(D_c/2) = 30/ (2x 300)

D_c = 5.732°

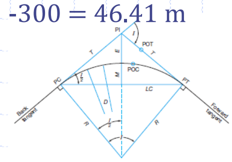
Length of curve = $L = 2\pi R(I/360)$ = 2xπx300(60/360) = 314.16 m

Tangent distance = $T = R\tan(I/2)$ = 300 x tan(60/2) = 173.21 m

Long chord= $LC = 2R\sin(I/2)$ = 2x 300 x sin (60/2) = 300 m

Middle ordinate = $M = R - R\cos(I/2)$ = 300x (1- cos(60/2)) = 40.19 m

External distance = $E = R/\cos(I/2) - R = 300/\cos(60/2) - 300 = 46.41$ m



Curve Stationing

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- In route surveys, lines and deflection angles along the traverse are measured first and PI_s are established.
- Each traverse station (PI) has a station value indicating its distance from starting point. Curves are then designed and staked out.
- To compute the PC station, tangent distance T is subtracted from the PI station, and to calculate the PT station, curve length L is added to the PC station, as in the following example

Example 1

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- Let deflection angle I=8° 24′, station of PI = 64+27.46 (100-m stationing),D=2° 0′ (arc definition). Calculate stationing of PC and PT along the curve, external distance (E), and middle ordinate distance (M) of this curve.
- **Solution:**

From Equation (1): Length of curve L = 100(8° 24′/2°) = 420.00 m

From Equation (2): Radius of Curve R = 100x 360/(2π x2°)= 2864.79 m

From Equation (4): Tangent distance T= 2864.79 tan((8° 24')/2) =210.38 m

Stationing: PI station= 64 + 27.46
-T= 02 + 10.38
PC station= 62 + 17.08
+ L= 04 + 20.00
PT station= 66 + 37.08 m

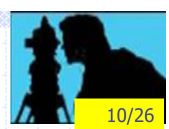
From Equation (6): External distance E = 210.38 tan((8° 24')/4) = 7.71 m

From Equation (7): Middle ordinate M = 7.71 cos((8° 24')/2) = 7.69 m

Note

- Note that the stationing of the PT cannot be obtained by adding the tangent distance to the station of the PI, although the location of the PT on the ground is determined by measuring the tangent distance from the PI.
- Points representing the PC and PT must be carefully marked and placed exactly on the tangent lines at the correct distance from the PI so other computed values will fit their fixed positions on the ground.

Design Criteria



- A curve is selected to fit ground conditions. Specification limitations, such as maximum **D** or minimum **R,** are also used as design criteria. The purpose of the route, such as highways or railroads, is also a criteria.
- Ground conditions such as presence of lakes, mountains, and high slopes determine the curve parameters to be fixed. Example is when fixing external distance **E** or middle ordinate **M** to miss a stream or a slope.
- Tangent distance **T** is sometimes used as a criteria for curve design. Example is when a proposed location of a railroad station is to be built on the tangent instead of a super-elevated curve.
- Length of curve **L** is rarely used to design the curve.

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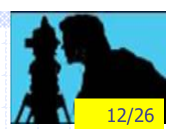
Overview



- Curve Layout
- Curve Layout by Deflection Angles
- Computing Deflection Angles and Chords
- General Curve Layout Procedures

Curve Layout

- Except for unusual cases, the radii of curves on route surveys are too large to permit swinging an arc from the curve center.
- Circular curves are therefore laid out by more practical methods, including

(1) deflection angles, (2) coordinates, (3) tangent offsets (4) chord offsets, (5) middle ordinates, (6) ordinates from the long chord

- Layout by deflection angles has been the standard approach, although with the advent of total station instruments, the coordinate method is becoming increasingly popular.
- In all methods, the positions of full stations (such as every 100 m) along the curve are first computed then staked out. These full stations are the ones at which cross-sections are normally taken.

Curve Layout by Deflection Angles

- Layout of a curve by deflection angles can be done by either
- *incremental chord method* or
- *total chord method.*
- In the past, the incremental chord method was almost always used as it could be readily accomplished with a theodolite and tape.
- The method can still be used with total station, although then the distances are observed by taping rather than electronically.
- The total chord method was not practical until the advent of total stations, but with these instruments it is now conveniently employed even though longer distance measurements are involved.

Incremental chord method

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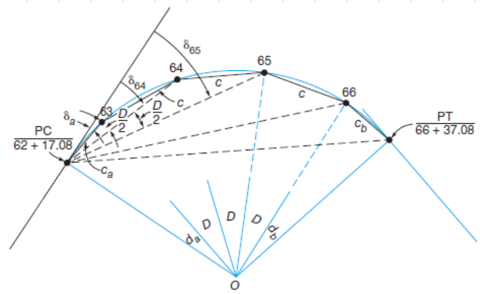
- The incremental chord method is illustrated in Figure. Assume that the instrument is set up over the PC (station 62 + 17.08 in Example). For this illustration, assume that each full station is to be marked along the curve,
- The first station to be set in this example is 63 + 00.
- To mark that point from the PC, a backsight is taken on the PI with zero set on the instrument's horizontal circle.
- Deflection angle δ_a to station 63 + 00 is then turned and two tape_persons measure chord C_a from the PC and set 63 + 00 at the end of the chord on the instrument's line of sight.



Incremental chord method

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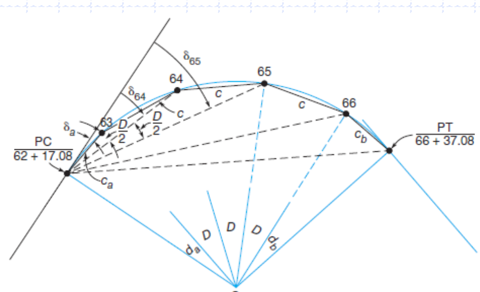
- With station 63 + 00 set, the tape_persons next measure the chord length *c* from it and stake station where the line of sight of the instrument, now set to δ_{64} , intersects the end of that chord.
- This process is repeated until the entire curve is laid out.
- In this procedure it is seen that the accuracy in the placement of each succeeding station depends on the accuracies of all those stations previously set.



Total chord method

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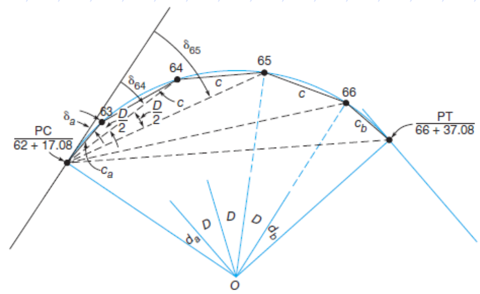
- The total chord method can also be described with reference to the Figure
- In this procedure, a total station instrument is set up at the PC, a backsight taken on the PI, and zero indexed on the horizontal circle.
- To set station 63 + 00, deflection angle δ_a is turned with the instrument, the reflector placed on line and adjusted until its distance from the instrument is C_a and the stake set.
  - To set station 64 + 00 deflection angle δ_{64} is turned, the reflector placed on this line of sight and adjusted in position until the total chord from the PC to station 64 + 00 is obtained and the stake set.



Total chord method

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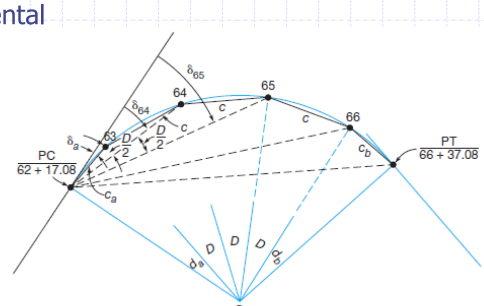
- This procedure is repeated, with each station being set independently of the others, until the entire curve is staked.
- This method of staking a curve has some drawbacks.
- One is that in some areas vegetation or other obstructions can block sight lines along the chords.
- Another is that each station is set independently and thus there is no check at the end of the curve.
- For these reasons, the incremental

chord approach is often preferred



Computing Deflection Angles and Chords

- Using 100-m stationing, the arc length **S** between any two full stations along the curve is equal to 100 m, this arc subtends a central angle equal to the degree of curve **D**.
- The chord length **C** between any two full stations along the curve can be computed from Equation (5) by substituting **C** for LC and D for I to get:

$C = 2R\sin(D/2)$ 8

- Usually the first arc $\mathbf{S_a}$ from the PC to the first full station (63+00 in our example) is not equal to **S**. The same is true for the arc length $\mathbf{S_b}$ between the last full station (66+00 in our example) and the PT.

$S_a = \text{Stationing of the first full station} - \text{Stationing of PC}$ 9

$S_b = \text{Stationing of PT} - \text{Stationing of the last full station}$ 10

Computing Deflection Angles and Chords

- Arcs **Sa** and **Sb** subtend central angles denoted by $\mathbf{d_a}$ and $\mathbf{d_b}$ respectively. The two angles ($\mathbf{d_a}$ and $\mathbf{d_b}$) are calculated by proportion from:

$d_a = S_a(D/S)$ 11

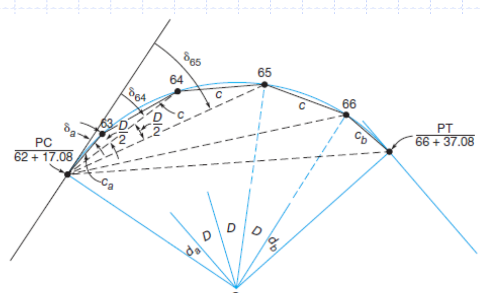
$d_b = S_b(D/S)$ 12

- The chord lengths $\mathbf{C_a}$ and $\mathbf{C_b}$ corresponding to arcs $\mathbf{S_a}$ and $\mathbf{S_b}$ are calculated from Equation (5) in a way similar to that of chord **C**.

$C_a = 2R\sin(d_a/2)$ 13

$C_b = 2R\sin(d_b/2)$ 14

- Deflection angle increment for any full station = half the central angle.



Example 2

- If PC=62+17.08, PT= 66+37.08, D=2°, R=2864.79 m. Compute:

(a) arcs S, S_a, and S_b.

(b) central angles d_a and d_b subtended by S_a and S_b respectively.

(c) chords C, C_a, and C_b. Prepare a table of deflection angles and chords.

- **Solution:**
- By definition, arc S = 100.00 m
- S_a = (63+00) - PC = 6300 - 6217.08 = 82.92m
- S_b = PT - (66+00) = 6637.08 - 6600 = 37.08 m
- $d_a = S_a(D/S)$ = 82.92(2°/100) = 1° 39′ 30.2″
- $d_b = S_b(D/S)$ = 37.08(2°/100) = 0° 44′ 29.8″
- C = 2R sin(D/2) = 99.99 m
- C_a = 2R sin(d_a/2) = 82.92 m
- C_b = 2R sin(d_b/2) = 37.08 m

Example

- To complete the table:
- Deflection angle increment between any two full stations=D/2= 1° 00′ 00″
- Deflection angle increment for the first full station =d_a/2=0° 49′ 45.1″
- Deflection angle increment from the last full station =d_b/2=0° 22′ 14.9″
- The following is a complete table of deflection angles and increment chords method.

Station	Stationing Value (m)	Arc length (m)	Subtended angle at the center (deg,min,sec)	Chord length (m)	deflection angle Increment (deg,min,sec)	Deflection angle (deg,min,sec)
PT	66+37.08					
		037.08	0° 44' 29.8"	37.08	0° 22' 15"	4° 12' 00"
66	66+00.00					
		100.00	2° 00' 00.0"	99.99	1° 00' 00"	3° 49' 45"
65	65+00.00					
		100.00	2° 00' 00.0"	99.99	1° 00' 00"	2° 49' 45"
64	64+00.00					
		100.00	2° 00' 00.0"	99.99	1° 00' 00"	1° 49' 45"
63	63+00.00					
		082.92	1° 39' 30.2"	82.92	0° 49' 45"	0° 49' 45"
PC	62+17.08					
		420	8° 24' 00.0"		4° 12' 00"	

General Curve Layout Procedures

- Use the data of examples 1 and 2, and use a tape and theodolite.

1. With the theodolite at PI, back sight the previous traverse station. Mark point PC along this direction using a tape.
2. With the theodolite still at PI, back sight the next traverse station. Mark point PT along this direction using a tape.
3. From this direction, turn theodolite an angle equal to (180-I)/2. Mark a point at a distance E from PI. This point is the top of the curve.
4. Set up the theodolite over point PC. Initialize the horizontal circle to zero, then back sight point PI.
5. Turn the theodolite until horizontal circle reading is $d_a/2$= 0º 49' 45.1''.
6. With the tape zero end at PC extend the tape a distance C_a= 82.92 m. Swing the tape until the 82.92 mark falls on the theodolite line of sight. Mark the point which is the first full station (63) along the curve.

General Curve Layout Procedures

7. Turn the theodolite until the horizontal circle reads an angle = $(d_a+D)/2$.
8. With the tape zero end on station (63), extend the tape a distance C = 99.99 m. Swing the tape until the 99.99-m mark falls on the theodolite line of sight. Mark this point which is full station (64).
9. Turn the theodolite until horizontal circle reading is $(d_a+2D)/2$.
10. With the tape zero end on station (64), extend the tape a distance C = 99.99 m. Swing the tape until the 99.99-m mark falls on the theodolite line of sight. Mark this point which is full station (65).
11. Turn the theodolite until horizontal circle reading is $(d_a+3D)/2$.
12. With the tape zero end on station (65), extend the tape a distance C = 99.99 m. Swing the tape until the 99.99-m mark falls on the theodolite line of sight. Mark this point which is full station (66).
13. Turn the theodolite until horizontal circle reading is $(d_a+d_b+3D)/2$

General Curve Layout Procedures

14. With the tape zero end on station (66), extend the tape a distance C_b =37.08 m. Swing the tape until the 37.08-m mark falls on the theodolite line of sight. Mark this point which should be point PT.
15. If the two marked points PT of steps 2 and 14 do not coincide, Measure the distance between them. This distance **e** is called **arc misclosure.**
16. Compute **Curve precision** using curve length **L** and tangent distance **T**:

$$Curve\ precision = \frac{1}{(L + 2T)/e}$$

Suitable chord length

- The following chord lengths are commonly used for the degrees of curve shown:
- 100 m —0 to 3 degrees of curve
- 50 m —3 to 8 degrees of curve
- 25 m —8 to 16 degrees of curve
- 10 m -over 16 degrees of curve

- The above chord lengths are the maximum distances in which the discrepancy between the arc length and chord length will fall within the allowable error for taping.
- The allowable error is 0.02 m per 100 m on most construction surveys

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