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CE 371 Surveying

TRAVERSING _2, 3

Traverse Computations

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Overview



- Traverse Computations
 1. Balancing Measured Angles
 2. Computing Azimuth Angles
 3. Computing Departures and Latitudes
 4. Computing Rectangular Coordinates
 5. Traverse Precision
- Length and Azimuth From Coordinates or Latitude and Departure
- Geometrically open traverse

Traverse Computations

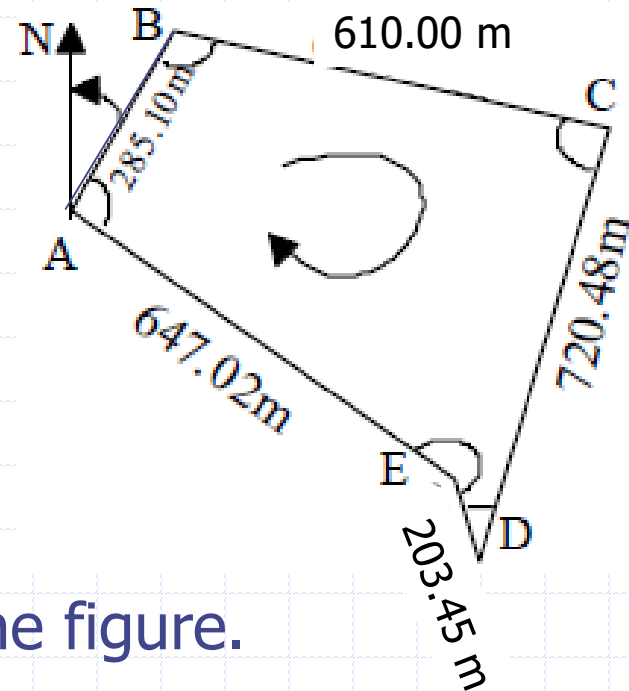


- The usual procedures followed in traverse computations are:
 1. Adjusting (balancing) measured angles to fixed geometric conditions.
 2. Computing azimuths (or bearings) for each traverse side.
 3. Computing departure and latitude then adjusting them for misclosure.
 4. Computing rectangular coordinates of traverse stations (corners).



Example

- A clockwise polygon traverse ABCDEA is made to determine the interior angles, lengths, and directions of all traverse sides.
- Interior angles are:
 - $A=100^{\circ} 44.5'$,
 - $B=101^{\circ} 35.0'$,
 - $C=089^{\circ} 05.5'$,
 - $D=017^{\circ} 12.0'$, and
 - $E=231^{\circ} 24.5'$.
- Azimuth of AB= $26^{\circ} 10.0'$
- Lengths of all sides are given in the figure.
- $A(1000.00,1000.00)$.
- Do complete traverse computations.



Balancing Measured Angles



5/27

- (Interior angles) =
 $A+B+C+D+E= 540^{\circ} 1.5'$
- Angle misclosure
- $EC = 540^{\circ} 1.5' - 180^{\circ}(5-2)$
- $EC = 540^{\circ} 1.5' - 540^{\circ} = +1.5'$
- Correction per angle = $-1.5'/5 = -0.3'$
- Add $-0.3'$ to each interior angle.
- Check: (Adjusted interior angles) = $540^{\circ} 0.0'$.

Table (1) Balancing interior angles

Interior angle	Measured angles	Balanced angles
A	100° 44.5'	100° 44.2'
B	101° 35.0'	101° 34.7'
C	089° 05.5'	089° 05.2'
D	017° 12.0'	017° 11.7'
E	231° 24.5'	231° 24.2'
Σ	540° 01.5'	540° 00.0'



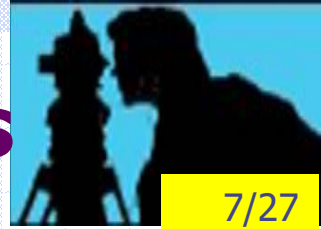
Azimuth Computations

For a clockwise polygon traverse:

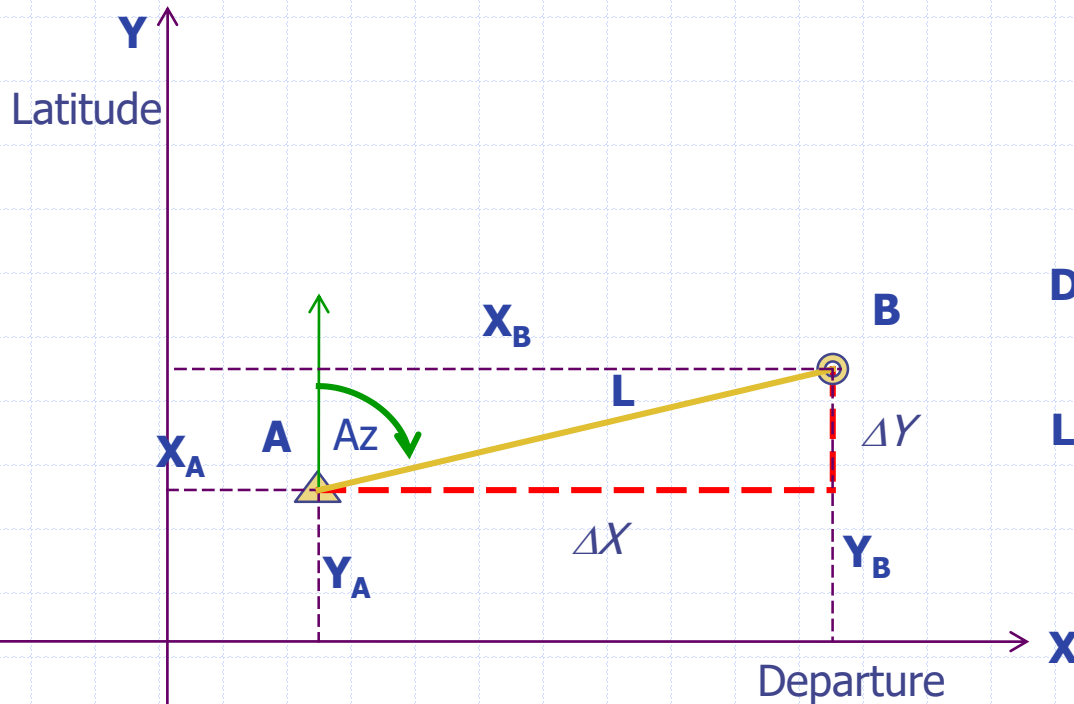
- Azimuth (Az) of a side = back azimuth (BAz) of the previous side - the interior angle between the two sides
- Add 360 for negative result

Line/ station	Forward Azimuth	Adjusted Left angle	Back Azimuth
AB	026° 10.0'	+ 180	206° 10.0'
B		101° 34.7'	
BC	104° 35.3'	+ 180	284° 35.3'
C		089° 05.2'	
CD	195° 30.1'		15° 30.1'
D		017° 11.7'	
DE	358° 18.4'		178° 18.4'
E		231° 24.2'	
EA	306° 54.2'		126° 54.2'
A		100° 44.2'	
AB (for check)	26° 10.0'		

Computing Departures and Latitudes



- Departure (ΔX) of a traverse side is its projection on the east-west axis.
- Latitude (ΔY) of a traverse side is its projection on the north-south axis.



$$\text{Departure } (\Delta X) = L \sin (Az)$$

$$\text{Latitude } (\Delta Y) = L \cos (Az)$$



Computing Departures and Latitudes

Side	Length (m)	Azimuth	ΔX (m)	ΔY (m)	$C_{\Delta X}$	$C_{\Delta Y}$	$\Delta X'$	$\Delta Y'$
AB	285.10	026° 10.0'	125.72	255.88	-0.010	0.018	125.710	255.898
BC	610.00	104° 35.3'	590.33	-153.64	-0.023	0.040	590.307	-153.600
CD	720.48	195° 30.1'	-192.56	-694.27	-0.026	0.047	-192.586	-694.223
DE	203.45	358° 18.4'	-6.01	203.36	-0.007	0.013	-6.017	203.373
EA	647.02	306° 54.2'	-517.39	388.51	-0.024	0.042	-517.414	388.552
Σ	2466.05		0.09	-0.16	-0.09	0.16	0.00	0.00

$$\text{Departure misclosure} = \sum_{i=1}^n \Delta X_i$$

$$\text{Latitude misclosure} = \sum_{i=1}^n \Delta Y_i$$

$$\text{Traverse Perimeter } P = \sum_{i=1}^n L_i$$

Bowditch method

$$\text{Correction to departure } C_{\Delta X} = -L \left(\frac{\sum_{i=1}^n \Delta X_i}{P} \right)$$

$$\text{Correction to latitude } C_{\Delta Y} = -L \left(\frac{\sum_{i=1}^n \Delta Y_i}{P} \right)$$

$$\text{Adjusted departure} = \widehat{\Delta X} = \Delta X + C_{\Delta X}$$

$$\text{Adjusted latitude} = \widehat{\Delta Y} = \Delta Y + C_{\Delta Y}$$

Compute Rectangular Coordinates



- Rectangular coordinates of traverse stations can be calculated from the adjusted departure and latitude of traverse sides provided the (X,Y) coordinates of at least one station is known.

$$X_B = X_A + \Delta X'_{AB}$$

$$Y_B = Y_A + \Delta Y'_{AB}$$

Station/ Line	$\Delta X'$	$\Delta Y'$	X	Y
A			1000.00	1000.00
AB	125.710	255.898		
B			1125.710	1255.898
BC	590.307	-153.600		
C			1716.017	1102.298
CD	-192.586	-694.223		
D			1523.431	408.075
DE	-6.017	203.373		
E			1517.414	611.448
EA	-517.414	388.552		
A (for check)			1000.00	1000.00

Traverse Precision



- Traverse precision is the linear misclosure ΔL of the traverse divided by traverse perimeter P . These are defined by the following two equations:

$$\text{Linear misclosure } \Delta L = \sqrt{(\sum \Delta X)^2 + (\sum \Delta Y)^2}$$

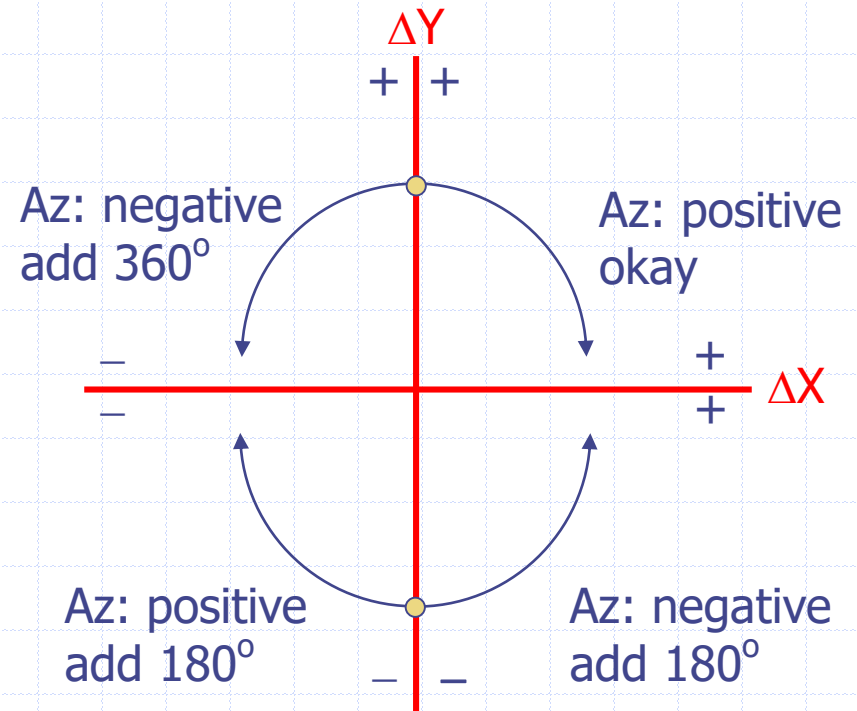
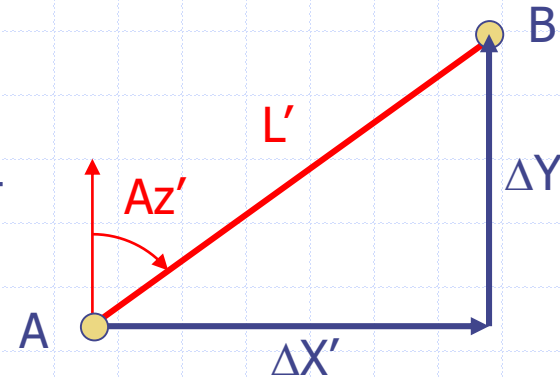
$$\text{Traverse precision} = \Delta L/P$$

- $\Delta L = \sqrt{(\Delta X)^2 + (\Delta Y)^2} = \sqrt{(0.09)^2 + (-0.16)^2} = 0.184$
- Traverse precision = $0.184/2466.05 = 7.444\text{E-}5 = 1/13433$
- Accepted traverse precisions
 - construction surveys: 1/3000 to 1/10000
 - Control survey: 1/30000



Length and Azimuth From ΔX & ΔY

- $L' = \sqrt{(\Delta X')^2 + (\Delta Y')^2}$
- $L' = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$
- $Az' = \tan^{-1} \left(\frac{\Delta X'}{\Delta Y'} \right)$



Length and Azimuth From ΔX & ΔY



- $$L' = \sqrt{(\Delta X')^2 + (\Delta Y')^2}$$

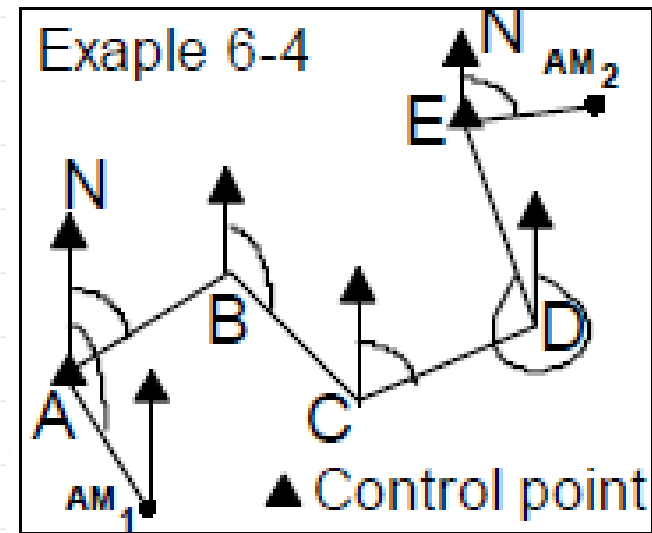
$$Az' = \tan^{-1} \left(\frac{\Delta X'}{\Delta Y'} \right)$$

Station/ Line	$\Delta X'$	$\Delta Y'$	X	Y	Adjusted Az'	Adjusted L'
A			1000.00	1000.00		
AB	125.710	255.898			026° 09.75'	285.11
B			1125.710	1255.898		
BC	590.307	-153.600			104° 35.44'	610.73
C			1716.017	1102.298		
CD	-192.586	-694.223			195° 30.28'	720.44
D			1523.431	408.075		
DE	-6.017	203.373			358° 18.32'	203.46
E			1517.414	611.448		
EA	-517.414	388.552			306° 54.28'	647.06
A (for check)			1000.00	1000.00		



Example 2

- Traverse ABCDE in the figure starts at control point A and closes at control point E.
- A (2765.48, 3280.21)
- E (4797.12, 4384.41)
- $AZ_{AM_1-A} = 319^\circ 5' 45''$ and
- $AZ_{E-AM_2} = 86^\circ 20' 47''$
- The measured values:



side	length	angle	measured
AB	1,045.50	A	$283^\circ 50' 10''$
BC	1,007.38	B	$256^\circ 17' 18''$
CD	897.81	C	$098^\circ 12' 41''$
DE	960.36	D	$103^\circ 30' 34''$
		E	$285^\circ 24' 59''$

Complete traverse computations



Azimuth Computations

For a counterclockwise polygon traverse:

- Azimuth (Az) of a side = back azimuth (BAz) of the previous side + the interior angle between the two sides
- Add 360 for negative result

Line/ station	Forward Azimuth	Measured angle	Back Azimuth
AM ₁ -A	319° 05' 45"	- 180	139° 05' 45"
A		283° 50' 10"	
AB	062° 55' 55"	+ 180	242° 55' 55"
B		256° 17' 18"	
BC	139° 13' 13"		319° 13' 13"
C		098° 12' 41"	
CD	057° 25' 54"		237° 25' 54"
D		103° 30' 34"	
DE	340° 56' 28"		160° 56' 28"
E		285° 24' 59"	
E-AM ₂	086° 21' 27"		

solution



Using unadjusted azimuth

Station/ Line	Length (m)	Azimuth	ΔX (m)	ΔY (m)	Prelim. X	Prelim. Y	$C_{\Delta X}$	$C_{\Delta Y}$	X	Y
A					2765.48	3280.21			2765.48	3280.21
AB	1,045.50	062° 55' 55"	930.98	475.75			-0.093	0.067		
B					3696.463	3755.963	-0.093	0.067	3696.369	3756.030
BC	1,007.38	139° 13' 13"	657.97	-762.81			-0.090	0.064		
C					4354.436	2993.149	-0.183	0.131	4354.252	2993.279
CD	897.81	057° 25' 54"	756.63	483.30			-0.080	0.057		
D					5111.065	3476.444	-0.264	0.188	5110.802	3476.632
DE	960.36	340° 56' 28"	-313.60	907.72			-0.086	0.061		
E					4797.469	4384.161	-0.349	0.249	4797.120	4384.410
E (correct)					4797.12	4384.41				
Σ	3911.05				0.349	-0.249	-0.349	+0.249		

Azimuth Adjustment



Angle misclosure

$$EC = 086^\circ 21' 27'' - 086^\circ 20' 47'' = 40''$$

$$\text{Correction per Azimuth} = -40''/5 = -8''$$

Add $-8''$ to each Azimuth in an Accumulative manner.

Check: (the last Azimuth = its corrected value)

A (2765.48, 3280.21)

E (4797.12, 4384.41)

$$\Delta X = -2031.64$$

$$\Delta Y = -1104.2$$

$$EA = 2312.32$$

$$Az_{EA} = 241^\circ 28' 33''$$

Line/ station	Forward Azimuth	Back Azimuth
AM ₁ -A	319° 05' 45"	319° 05' 45"
A		
AB	062° 55' 55"	062° 55' 47"
B		+1*(-8)
BC	139° 13' 13"	139° 12' 57"
C		+2*(-8)
CD	057° 25' 54"	057° 25' 30"
D		+3*(-8)
DE	340° 56' 28"	340° 55' 56"
E		+4*(-8)
E-AM ₂ (for check)	086° 21' 27"	086° 20' 47"
		+5*(-8)

Another solution



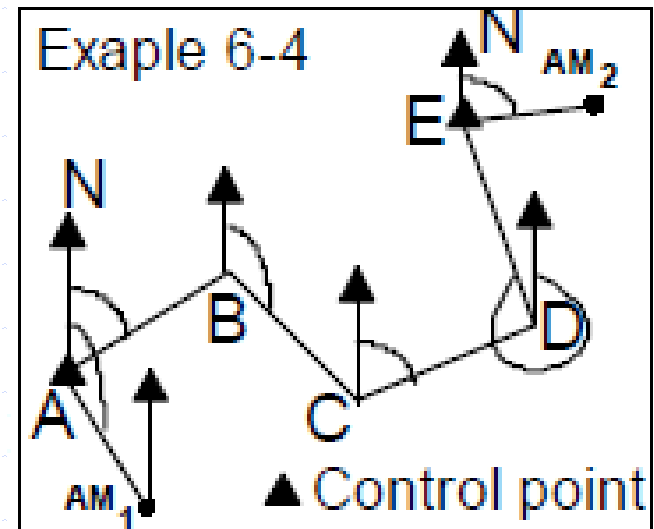
Using adjusted azimuth

Station/ Line	Length (m)	Azimuth	ΔX (m)	ΔY (m)	Prelim. X	Prelim. Y	$C_{\Delta X}$	$C_{\Delta Y}$	X	Y
A					2765.48	3280.21			2765.48	3280.21
AB	1,045.50	062° 55' 47"	930.96	475.79			-0.052	0.033		
B					3696.444	3755.999	-0.052	0.033	3696.393	3756.032
BC	1,007.38	139° 12' 57"	658.03	-762.76			-0.050	0.032		
C					4354.477	2993.236	-0.101	0.064	4354.375	2993.300
CD	897.81	057° 25' 30"	756.57	483.38			-0.044	0.028		
D					5111.050	3476.620	-0.146	0.093	5110.904	3476.712
DE	960.36	340° 55' 56"	-313.74	907.67			-0.047	0.030		
E					4797.313	4384.287	-0.193	0.123	4797.12	4384.41
E (correct)					4797.12	4384.41				
Σ	3911.05				0.193	-0.123	-0.193	+0.123		

Geometrically open traverse



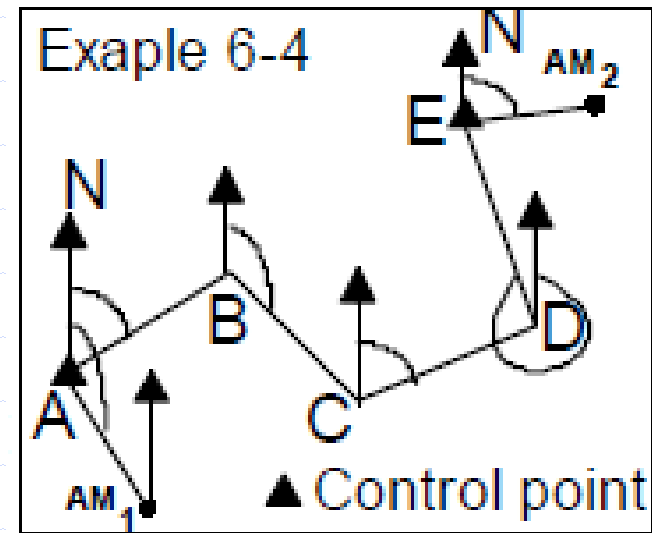
- it starts at control point A and ends at another control point E
- If point A can see point E, we deal with it exactly as the geometrically closed traverse.
- If not, The azimuth of a line from control point A to an azimuth mark, such as AM_1 in the figure, must be known along with the azimuth of another line from control point E to another azimuth mark AM_2 .
- The computational procedures start with azimuth computations





Example

- Traverse ABCDE in the figure starts at control point A and closes at control point E.
- A (2765.48, 3280.21)
- E (4797.12, 4384.41)
- $AZ_{AM_1-A} = 319^\circ 5' 45''$ and
- $AZ_{E-AM_2} = 86^\circ 20' 47''$
- The measured values:



side	length	angle	measured
AB	1,045.50	A	$283^\circ 50' 10''$
BC	1,007.38	B	$256^\circ 17' 18''$
CD	897.81	C	$098^\circ 12' 41''$
DE	960.36	D	$103^\circ 30' 34''$
		E	$285^\circ 24' 59''$

Complete traverse computations



Azimuth Computations

For a counterclockwise polygon traverse:

- Azimuth (Az) of a side = back azimuth (BAz) of the previous side + the interior angle between the two sides
- Add 360 for negative result

Line/ station	Forward Azimuth	Adjusted Left angle	Back Azimuth
AM ₁ -A	319° 05' 45"	- 180	139° 05' 45"
A		283° 50' 10"	
AB	062° 55' 55"	+ 180	242° 55' 55"
B		256° 17' 18"	
BC	139° 13' 13"		319° 13' 13"
C		098° 12' 41"	
CD	057° 25' 54"		237° 25' 54"
D		103° 30' 34"	
DE	340° 56' 28"		160° 56' 28"
E		285° 24' 59"	
E-AM ₂	086° 21' 27"		

Azimuth Computations



Angle misclosure

$$EC = 086^\circ 21' 27'' - 086^\circ 20' 47'' = 40''$$

$$\text{Correction per Azimuth} = -40''/5 = -8''$$

Add $-8''$ to each Azimuth in an Accumulative manner.

Check: (the last Azimuth = its corrected value)

A (2765.48, 3280.21)

E (4797.12, 4384.41)

$$\Delta X = -2031.64$$

$$\Delta Y = -1104.2$$

$$EA = 2312.32$$

$$Az_{EA} = 241^\circ 28' 33''$$

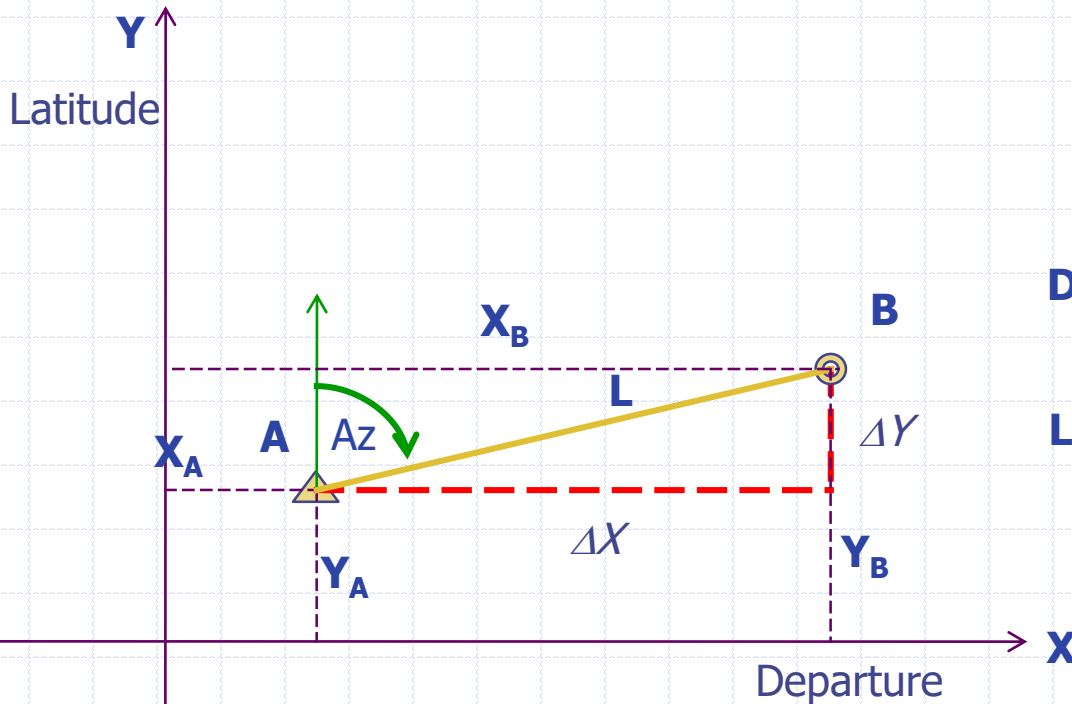
Line/ station	Forward Azimuth	Back Azimuth
AM ₁ -A	319° 05' 45"	319° 05' 45"
A		
AB	062° 55' 55"	062° 55' 47"
B		+1*(-8)
BC	139° 13' 13"	139° 12' 57"
C		+2*(-8)
CD	057° 25' 54"	057° 25' 30"
D		+3*(-8)
DE	340° 56' 28"	340° 55' 56"
E		+4*(-8)
E-AM ₂ (for check)	086° 21' 27"	086° 20' 47"
		+5*(-8)

Computing Departures and Latitudes



22/27

- Departure (ΔX) of a traverse side is its projection on the east-west axis.
- Latitude (ΔY) of a traverse side is its projection on the north-south axis.



$$\alpha_{AC} = \alpha + \theta$$

$$\text{Departure } (\Delta X) = L \sin (Az)$$

$$\text{Latitude } (\Delta Y) = L \cos (Az)$$

Computing Departures and Latitudes



Side	Length (m)	Azimuth	ΔX (m)	ΔY (m)	$C_{\Delta X}$	$C_{\Delta Y}$	$\Delta X'$	$\Delta Y'$
AB	1,045.50	062° 55' 47"	930.96	475.79	-0.048	0.032	930.91	475.82
BC	1,007.38	139° 12' 57"	658.03	-762.76	-0.046	0.031	657.98	-762.73
CD	897.81	057° 25' 30"	756.57	483.38	-0.042	0.028	756.53	483.41
DE	960.36	340° 55' 56"	-313.74	907.67	-0.044	0.029	-313.78	907.70
EA (Correct)	2312.32	241° 28' 33"	-2031.64	-1104.2			-2031.64	-1104.2
Σ	3911.05		0.18	-0.12	-0.18	+0.12	0.00	0.00

$$\text{Departure misclosure} = \sum_{i=1}^n \Delta X_i$$

$$\text{Latitude misclosure} = \sum_{i=1}^n \Delta Y_i$$

$$\text{Traverse Perimeter } P = \sum_{i=1}^n L_i$$

Bowditch method

$$\text{Correction to departure } C_{\Delta X} = -L \left(\frac{\sum_{i=1}^n \Delta X_i}{P} \right)$$

$$\text{Correction to latitude } C_{\Delta Y} = -L \left(\frac{\sum_{i=1}^n \Delta Y_i}{P} \right)$$

$$\text{Adjusted departure} = \overline{\Delta X} = \Delta X + C_{\Delta X}$$

$$\text{Adjusted latitude} = \overline{\Delta Y} = \Delta Y + C_{\Delta Y}$$

Compute Rectangular Coordinates



- Rectangular coordinates of traverse stations can be calculated from the adjusted departure and latitude of traverse sides provided the (X,Y) coordinates of at least one station is known.

$$X_B = X_A + \Delta X'_{AB}$$

$$Y_B = Y_A + \Delta Y'_{AB}$$

Station/ Line	$\Delta X'$	$\Delta Y'$	X	Y
A			2765.48	3280.21
AB	930.91	475.82		
B			3696.39	3756.03
BC	657.98	-762.73		
C			4354.37	2993.30
CD	756.53	483.41		
D			5110.90	3476.71
DE	-313.78	907.70		
E (for check)			4797.12	4384.41

Traverse Precision



- Traverse precision is the linear misclosure ΔL of the traverse divided by traverse perimeter P . These are defined by the following two equations:

$$\text{Linear misclosure } \Delta L = \sqrt{(\sum \Delta X)^2 + (\sum \Delta Y)^2}$$

$$\text{Traverse precision} = \Delta L/P$$

- $\Delta L = \sqrt{(\Delta X)^2 + (\Delta Y)^2} = \sqrt{(0.18)^2 + (-0.12)^2} = 0.216$
- Traverse precision = $0.216 / 3911.05 = 5.531E-5 = 1/18078$
- Accepted traverse precisions
 - construction surveys: 1/3000 to 1/10000
 - Control survey: 1/30000

Length and Azimuth From ΔX & ΔY



- $L' = \sqrt{(\Delta X')^2 + (\Delta Y')^2}$

$$Az' = \tan^{-1} \left(\frac{\Delta X'}{\Delta Y'} \right)$$

Station/ Line	$\Delta X'$	$\Delta Y'$	X	Y	Adjusted Az'	Adjusted L'
A			2765.48	3280.21		
AB	930.91	475.82			062° 55' 37"	1045.47
B			3696.39	3756.03		
BC	657.98	-762.73			139° 13' 01"	1,007.32
C			4354.37	2993.30		
CD	756.53	483.41			057° 25' 20"	897.79
D			5110.90	3476.71		
DE	-313.78	907.70			340° 55' 49"	960.40
E			4797.12	4384.41		

Summary



27/27

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