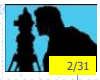
Lec 13 &
Lec 14

CE 371 Surveying

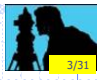
ANGLE MEASURING_2_3

Dr. Ragab Khalil
Department of Landscape Architecture
Faculty of Environmental Design
King AbdulAziz University
Room LIE15



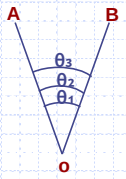
Overview

- Repetition method
- Directional method
- Relationship of Angles And Distances
- Measurement Precision With Repeating Theodolites
- Deflection Angle Measurement
- Azimuth Measurement
- Theodolite Applications
- Sources of Errors in Theodolites



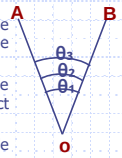
Repetition method

- This method is used for very accurate work. In this method ,the same angle is added several times mechanically and the correct value of the angle is obtained by dividing the accumulated reading by the no. of repetitions.
- The No. of repetitions made usually in this method is six, three with the face left and three with the face right .



Repetition method Procedure

- Setup the theodolite at station O.
- Bisect the point A with vertical hair of theodolite and move telescope in clockwise and direction to bisect at point B.
- Note this circle reading in the book and fix this circle reading, then again bisect the point A by keeping the circle reading fixed.
- Now, release the circle reading and rotate the telescope again in clockwise direction till it bisect again point B.
- Similarly get 3rd and 4th repetition and note the circle reading after 4th repetition in the book.
- Change the face of telescope and repeat the above steps, an example and method of booking observations have given below,



Repetition method

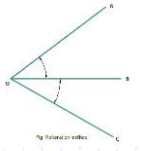
- Angle AOB is measured 8 times with a repeating theodolite. Compute the average value of angle AOB.

Station occupied	Station sighted	Direct reading	Reversed reading
O	A	000° 00.0′	180° 00.0′
	B	035° 10.0′	215° 10.1′
	B	070° 19.9′	250° 20.3′
	B	105° 29.8′	285° 30.5′
	B	140° 39.6′	320° 40.5′
Average values		035° 9.9′	035° 10.1′
Final Angle		035° 10.0′	

- Average direct=
- (140° 39.6′)/4 =35° 9.9′
- Average indirect =
- [(320° 40.5′)-180°]/4=35° 10.1′
- Average of angle AOB=
- (35° 9.9′ + 35° 10.1′)/2 = 35° 10.0′

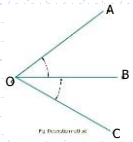
Directional method

- This method is another precise and comparatively less tedious method of measuring the horizontal angles.
- It is generally preferred when several angles with multiple repetitions are to be measured at a particular station.
- The needed angles are measured on face left (from left to right), then measured on face right (from right to left)



Directional method Procedure

- Setup the theodolite at station O, bisect the point A with a certain circle reading with face left.
- Rotate the instrument in clockwise direction and bisect B, note the circle reading.
- Then rotate and the telescope till it bisect the point C, note this circle reading also. All these reading will book into face left position.
- Transit the telescope and rotate the instrument through 180°, this time bisect the point C firstly and then rotate telescope in anti clockwise direction towards B and then ultimately towards A. Put these readings in face right position.
- You can do more than one sets of measurements for the accurate results, three sets are done and booking as follows,

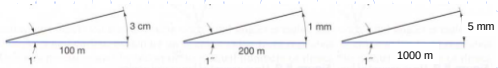


Directional method

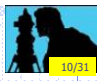
Station occupied	Station sighted	Face Left	Face Right	Mean	Reduced Angle	Final Angle
O	A	000 20 05	180 20 07	000 20 06	000 00 00	
	B	047 30 10	227 30 12	047 30 11	047 10 05	AOB = 047 10 06
	C	088 40 20	268 40 30	088 40 25	088 20 19	
O	A	045 20 16	225 20 18	045 20 17	000 00 00	
	B	092 30 21	272 30 23	092 30 22	047 10 05	BOC = 088 20 19
	C	133 40 34	313 40 42	133 40 38	088 20 21	
O	A	090 10 04	270 10 06	090 10 05	000 00 00	
	B	137 20 12	317 20 10	137 20 12	047 10 07	
	C	178 30 18	358 30 26	178 30 22	088 20 17	
	AOB = (047 10 05 + 047 10 05 + 047 10 07)/3 = 047 10 06					
	BOC = (088 20 19 + 088 20 21 + 088 20 17)/3 = 088 20 19					

Relationship of Angles And Distances

- The best quality surveys result when there is compatibility between the accuracies of distance and angle observations.
- In other words, keep the offsets in line, caused by errors in angular measurements, approximately equal to the errors in linear measurements.
- The formula for relating distance to angle is
- $S=R\theta$
- S is the arc length subtended at a distance R by an arc of θ in radians



Measurement Precision With Repeating Theodolites



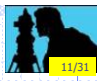
Precision of an angle increases as we repeat the measurements. This is, however, true up to a certain limit. Some types of errors increase as we repeat measurements, such as pointing error E_p, reading error E_r of the least division (interpolated digit), and setting (initializing) error E_s

$$E_a = \frac{1}{n}\sqrt{E_s^2 + 2nE_p^2 + E_r^2}$$

optimum number of measurements (n) =6 or 8

E_a= The maximum random error of an angle measured by repetition
E_p=Pointing error usually ranges from 3" to 5" with an average observer
E_r =Reading error is less or equal to half the least division of the angle read
E_s = initializing error = E_r is applicable if the observer sets the circle to zero or any value at the back station

Measurement Precision With Repeating Theodolites



Example

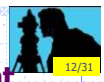
Compute the maximum random error of an angle measured 8 times with a repeating theodolite with 20″ accuracy. Assume E_p=5″

Solution

The angle is measured 8 times, with a 20″ theodolite.
E_r=10″ , E_s= 10″

$$E_a = \frac{1}{8}\sqrt{10^2 + 2x8x5^2 + 10^2} = 3.06''$$

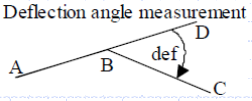
Deflection Angle Measurement



To measure deflection angle **def** of line BC from line AB with a repeating theodolite, do the following:

1. Initialize the horizontal circle to 0° while theodolite on B (face-left mode).
2. With lower motion, backsight point A. Invert the telescope (the telescope now is pointing at point D along line AB).
3. With upper motion, turn theodolite to C. Record the reading (def_r).
4. With lower motion, backsight point A. Invert the telescope again.

Deflection angle measurement



Deflection Angle Measurement

5. With upper motion, turn theodolite to C. Record this reading (def_i).
6. Divide the sum by 2 to get the average of the deflection angle of line BC.
7. If the deflection angle is to be measured **n** times, then repeat steps 2 to 5 an n/2 number of times, then read the total angle and divide by **n** to get the average.
8. If the final mean does not agree with the first recorded value, repeat the measurements

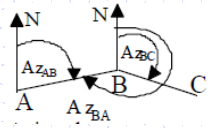
DEFLECTION ANGLES

Sta	BS/FS Sta	No. Reps.	Circle Rdg ° ′ ″	Mean Angle ° ′ ″	Right/ Left
	A	1	12 15 12		
B		2	24 30 20		
	C	4	49 00 40	12 15 10	R

Azimuth Measurement

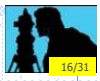
In order to measure azimuth of a line, such as Az_{BC} of line BC in the figure, one must have the azimuth of a reference line, such as the azimuth Az_{AB} of reference line AB. With a repeating theodolite, measuring steps are:

1. Initialize the horizontal circle to the back azimuth of AB (i.e. Az_{BA}), while theodolite on B (face-left mode). With lower motion, backsight point A.
2. With upper motion, turn theodolite clockwise to point C. Record the reading.



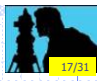
Azimuth Measurement

3. Invert the telescope (face-right mode), and initialize the horizontal circle to the back azimuth of AB, or Az_{BA}. With lower motion, backsight point A.
4. With upper motion, turn theodolite clockwise to point C. Record the reading.
5. Take the average of the direct and reversed readings of steps 2 and 4.



Theodolite Applications

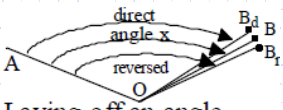
1. Laying off an Angle
2. Prolonging a Straight Line
3. Prolonging a Straight Line Past an Obstacle
4. Balancing in
5. Random Line



Laying off an Angle

To lay off angle **x** at O from point A to point B with a repeating theodolite, do the following:

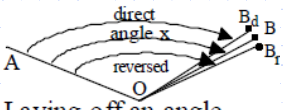
1. Initialize the horizontal circle to 0° while theodolite on O (face-left mode).
2. With lower motion, backsight point A.
3. With upper motion, turn theodolite until the horizontal circle reads **x**.
4. Mark point $\mathbf{B_d}$ on the ground along this direction.



Laying off an angle

Laying off an Angle

5. Invert the telescope (face right). Initialize the horizontal circle to 180°.
6. With lower motion, backsight point A.
7. With upper motion, turn theodolite until the circle reads (180°+x), mark point $\mathbf{B_r}$ on the ground along this direction (if it is different from B_d).
8. Bisect distance B_d B_r to locate point B on the ground.

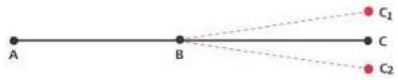


Laying off an angle

Prolonging a Straight Line

To extend line AB to a point such as C do the following:

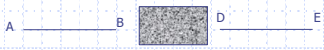
1. With the theodolite at B (face left), backsight point A, then invert telescope (face right). Mark point C_r on the ground along this direction.
2. Backsight point A again with the telescope still inverted, then invert it again (face left). If C_r is not along this direction, mark a new point C_l.
3. Bisect the distance C_r C_l to get point C, which is the extension of AB.



Prolonging a Straight Line Past an Obstacle

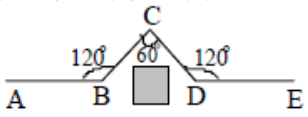
There are series of methods for prolonging line AB past an obstacle (like a building) such that line DE is along AB:

1. Equilateral-triangle method
2. Angle Offset method
3. Right-angle-offset method
4. One random line method
5. Two random lines method



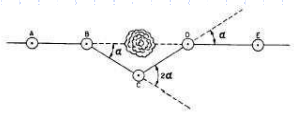
Equilateral-triangle method

1. With the theodolite at B, turn an angle equal to 120° from A and mark point C along this direction, then measure a suitable distance such as BC.
2. Set up the theodolite on C and turn an angle equal to 60° from B and mark point D such that CD=CB.
3. Set up the theodolite on D and turn an angle equal to 120° from C then mark a point such as E along this direction. Line DE is collinear with AB.



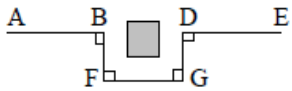
Angle Offset method

1. With the theodolite at B, lay off a deflection angle α and mark point C along this direction, then measure a suitable distance such as BC.
2. Set up the theodolite on C and lay off a deflection angle equal to 2 α and mark point D such that CD=CB.
3. Set up the theodolite on D and lay off a deflection angle equal to α then mark a point such as E along this direction. Line DE is collinear with AB.



Right Angle Offset method

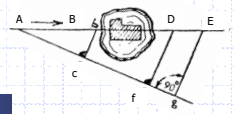
1. With theodolite at B, turn 90° from A and mark a point such as F. Measure distance BF.
2. With theodolite at F, turn 90° from B. Mark point G beyond the obstacle.
3. With theodolite on G, turn 90° from F. Mark point D where DG=BF.
4. With theodolite on D, turn 90° from G. Mark point E. Line DE is collinear with AB.

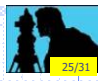


One random line method

1. With theodolite at A, run a random line Ag. mark points c, f and g. Measure distance Ac, Af and Ag.
2. With theodolite at c, turn 90° from A. Mark point b on the line AB and measure the distance cb.
3. With theodolite on f, turn 90° from A. Mark point D where fD=cb. Af/Ac
4. With theodolite on g, turn 90° from A. Mark point E where Eg=Bc. Ag/Ac

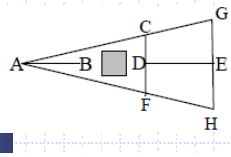
Line DE is collinear with AB.

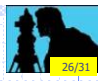




Two random line method

1. With theodolite on A, a suitable angle is turned and points such as C, F, G, and H are marked (as shown in the figure) where AC=AF and AG=AH.
2. Bisect distances CF and GH to get D and E. DE is collinear with AB.

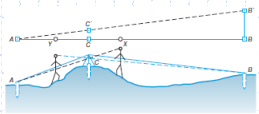


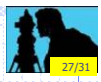


Balancing in (*wiggling-in*)

Balancing in is to establish a point such as C on line AB where A and B are not intervisible. The steps are:

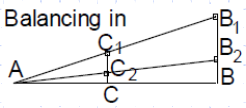
1) Set up theodolite (face left) on a trial point between A and B, such as C1.
2) Backsight A, then invert the telescope. If point B is not along the telescope direction, mark point B1, and measure BB1.

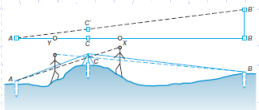




Balancing in (*wiggling-in*)

3) Move the theodolite a distance equals to . *BB*1 x *AC*1/*AB*
4) Repeat steps 2 and 3 until point B is exactly along telescope direction. The point under the theodolite is then point C which is along line AB.
5) If the required shift is less than 2 cm, the theodolite can be shifted over the tripod head.





Random Line

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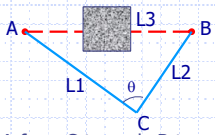
To find the direction of a line joining 2 non-intervisible points, such as line AB in the figure, do the following:

1) Establish a third point C from which A and B are visible.
2) Measure angle C, and measure distances AC and CB.
3) Compute length of AB and angle A from the equations:

$$L_3^2 = L_1^2 + L_2^2 - 2L_1L_2(\cos\theta)$$

$$AB = \sqrt{AC^2 + CB^2 - 2AC \times CB \times \cos C}$$

$$\text{Angle A} = \arcsin[(BC/AB) \times \sin(C)]$$



4) With theodolite at A, turn angle A from C towards B to get the required direction of point B.

Sources of Errors in Theodolites

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Instrumental errors:

1. plate bubble out of adjustment,
2. axis of sight not perpendicular to the horizontal axis,
3. horizontal axis not perpendicular to the vertical axis,
4. horizontal and vertical circles not perfectly graduated.

Most of the instrumental errors are systematic errors that can be eliminated by double centering and angle repetition.

Sources of Errors in Theodolites

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Natural errors:

1. wind effect,
2. temperature effect,
3. Refraction effect,
4. settling of the tripod.

Personal errors:

1. instrument not set up exactly over the point,
2. plate bubbles not leveled perfectly,
3. poor focusing and targeting.

Summary

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