Lec. 3

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CE 371 Surveying
Error analysis

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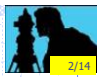
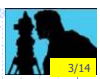
Overview



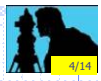
- Mean and Standard Error
- Properties of Random Errors
- The Probability Intervals
- Weighted Measurements

Mean and residual



- The most probable value of the true value is simply the arithmetic mean, or
- $\overline{M} = \frac{\sum M}{n}$
- Since the true value of a measurement is never known, the exact error e is unknown too.
- The residual of a measurement v_i is the best estimate of the error e_i. The residual v_i is defined as:
- $v_i = M_i - \overline{M}$

Standard Error & Variance



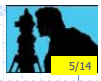
- The standard error of each measurement σ can be calculated as:
- $\sigma = \pm\sqrt{\frac{\sum v^2}{n-1}}$
- Standard error describes precision of the measurements, or how close the measurements are to their mean value.

High precision

Low precision

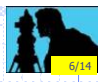
- Variance σ^2 is another way of defining precision.

standard error of the mean



- Since the mean is computed from individual observed values, each of which contains an error = σ , the error of the mean is found by the relation:
- $\bar{\sigma} = \frac{\sigma}{\sqrt{n}}$

Example



Line AB was measured 6 times as: 81.240; 81.245; 81.243; 81.242; 81.241; and 81.246 m.

Calculate the mean , residual of each measurement, standard error of each measurement , and standard error of the mean.

- Solution

length	Residual (V)	v²
81.240	-0.003	9E-6
81.245	0.002	4E-6
81.243	0.000	0.000
81.242	-0.001	1E-6
81.241	-0.002	4E-6
81.246	0.003	9E-6
487.458		27E-6

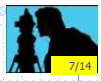
$\bar{M} = \frac{\sum M}{n} = \frac{487.458}{6} = 81.243$

$\sigma = \pm\sqrt{\frac{\sum v^2}{n-1}} = \sqrt{\frac{27E-6}{6-1}} = 2.32E\text{-}3$

$= \pm 0.002$ m

$\bar{\sigma} = \frac{\sigma}{\sqrt{n}} = \frac{0.002}{\sqrt{6}} = \pm 0.001$ m

Example



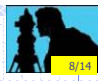
- Two teams measure the height of a flagpole.

Height in cm

Team A	Team B
183	183.0
182	183.5
185	182.7
181	182.5
183	183.1
184	183.3

- Which team did the better job?
- Why do you think so?

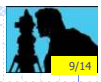
Random Errors



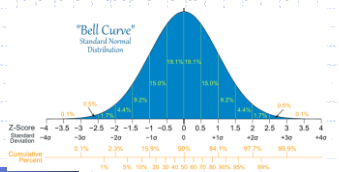
Random errors have the following properties:

1. Their values have equal chances of being negative or positive.
2. Small errors occur more frequently than large errors.
3. Random errors have a bell-shaped Normal Distribution Curve.
4. Standard Error gives an indication of the spread of random errors.
5. From probability theory, %68.27 of the error values will fall within the interval from $\bar{M} - \sigma \ to \ \bar{M} + \sigma$.

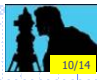
The Probability Intervals



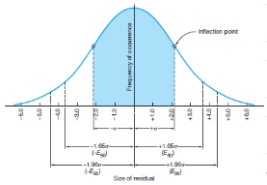
- If the Normal Distribution Function is integrated between the two limits $\bar{M} - \sigma \ and \ \bar{M} + \sigma$ the area under the curve will be equal to 68.27% of the total area.
- The two limits define the standard error probability interval.
- It means that the true value of the measurement has a 68.27% probability of falling within the standard error limit.
- It also means that 68.27% of the measurements will fall within the interval $\bar{M} \pm \sigma$



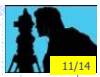
The Probability Intervals



- The 50% error E_{50} is the limit within which 50% of the measurements fall.
- $E_{50} = 0.6745\ \sigma$
- The 90% error $\mathbf{E_{90}}$ is the limit within which 90% of the measurements fall.
- $E_{90} = 1.6449\ \sigma$
- The 95% error $\mathbf{E_{95}}$ is the limit within which 95% of the measurements fall.
- $E_{95} = 1.9599\ \sigma$

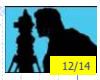


Example



- In the former Example compute the 90% probability interval.
- **Solution:**
- The two limits of the 90% probability interval are:
- $\bar{M} - 1.6449\sigma$ =81.243−1.6449 x 2.32E-3=81.239 m
- $\bar{M} + 1.6449\sigma$ =81.243+1.6449 x 2.32E-3=81.247 m

Weighted Measurements

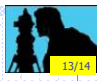


- Measurements of the same quantity can have different precisions due to using different instruments, different measuring procedures and so on.
- We should give more weight to measurements with smaller σ values when calculating the mean and the standard error.
- If w_i is the weight of M_i, then w_i is proportional to $1/\sigma_i^2$.
- If the proportionality constant is assumed equal to a reference variance σ_0^2, then the weight w_i is equal to:

- $w_i = \sigma_0^2 / \sigma_i^2$

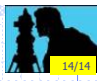
- The reference variance σ_0^2 is usually assumed equal to the largest variance in the measurements.

The weighted mean



- The weighted mean
- $\overline{M_w} = \frac{\sum wM}{\sum w}$
- The weighted standard deviation
- $\sigma_w = \pm\sqrt{\frac{\sum wv^2}{(n-1)\sum w}}$

Example 2



- Distance AB was measured 3 times as: 1.652±0.005 m, 1.659±0.008 m, and 1.655±0.001 m. Calculate the weighted mean and its standard error
- Solution
- Let σ_0 = 0.008, therefore

Meas.	M	σ	w	wM	v	wv²
1	1.652	0.005	2.56	4.229	-0.003	2.3E-5
2	1.659	0.008	1	1.659	0.004	1.6E-5
3	1.655	0.001	64.0	105.92	0.000	0.000
Σ	4.966		67.56	111.808		3.9E-5

$\overline{M_w} = \frac{\sum wM}{\sum w} = \frac{111.808}{67.56}$ = 1.655 m

$\sigma_w = \pm\sqrt{\frac{\sum wv^2}{(n-1)\sum w}}$ $= \sqrt{\frac{3.9E-5}{(3-1)x67.56}}$ = 5.38E-4 = ±0.0005 = ±0.001 m

Summary

- Mean and Standard Error
- Random Errors
- The Probability Intervals
- Weighted Measurements