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CE 371 Surveying
Error analysis

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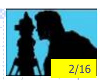
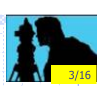
Overview



- Definition of Error
- Sources of Errors
- Types of Errors
- Elimination of Errors
- Probability
- Surveying Measurements and Probability
- Normal Distribution Function

Definition of Error



- An error e is the difference between the measured value of a quantity M and the true value M' .
- error = measured value – true value.
- In mathematical form the error e is given by:

$$e = M - M'$$

Sources of Errors

1. **Natural**:
   - These are caused due to variations in nature i.e., variations in wind, temperature, humidity, pressure, earth curvature, and atmospheric refraction.
2. **Instrumental**:
   - These result from imperfection in the construction or adjustment of surveying instruments, and movement of their individual parts.
   - e.g. incorrect length of tape, graduation error of the theodolite circles, and non-perpendicularity of axes.
3. **Human**:
   - These arise from limitations of the human senses of sight, touch and hearing. e.g. centering, leveling, and focusing of the theodolite, tape is not held horizontal, misreading values.

Mistakes

- Mistakes ≠ errors
- Mistakes or blunders are usually caused by
  - misunderstanding the problem,
  - carelessness,
  - fatigue,
  - missed communication,
  - poor judgment,
  - Faults in equipments,
  - Adoption of wrong technique.
- Mistakes could be large or small

Large mistakes

- Examples of large mistakes
  - transposition of numbers, such as recording 73.96 instead of the correct value of 79.36;
  - reading an angle counterclockwise, but indicating it as a clockwise angle in the field notes;
  - sighting the wrong target; or
  - recording a measured distance as 682.38 instead of 862.38.
- Large mistakes can easily be detected
- They must be detected by careful and systematic checking of all work, and eliminated by repeating some or all of the measurements.

Small mistakes

- Examples of small mistakes
  - recording a measured distance as 682.10 instead of 682.01
- It is very difficult to detect small mistakes because they merge with errors.
- When not exposed, these small mistakes will therefore be incorrectly treated as errors.

Types of Errors

1. Systematic errors
2. Random errors

Systematic Errors

- Systematic errors occur according to a system.
- These errors follow a definite pattern.
- Thus, if an experiment is repeated, under the same conditions, same pattern of systematic errors reoccur.
- These errors are dependent on the observer, the instrument used, and on the physical environment of the experiment.
- Any change in one or more of the elements of the system will cause a change in the character of the systematic error.
- Systematic errors are dealt with mathematically using functional relationships or models.
- Examples are:
  1. Tape expansion due to temperature.
  2. Tape sag due to gravity.

Random Errors

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- After mistakes are eliminated and systematic errors are corrected, a survey measurement is associated with random error only.
- This error is small and is equally liable to be plus or minus thus partly compensating in nature.
- Random errors are unpredictable and they cannot be evaluated or quantified exactly.

Elimination of Errors

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1. Mistakes: can be eliminated by
   1. common sense,
   2. repeating measurements,
   3. following proper measuring procedures
   4. Errors > $\pm$ 3.29 σ
2. Systematic: can be eliminated
   1. After calculating its value using certain formulas,
   2. following certain measuring procedures.
3. Random:
   1. can't be totally eliminated,
   2. can be minimized by using high precision instruments,
   3. by following careful measuring procedures.

Probability

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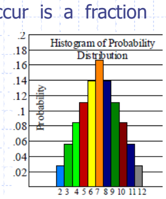
- Probability may be defined as the ratio of the number of times a result should occur to its total number of possibilities.
- In general, if a result may occur in m ways and fail to occur in n ways, then the probability of its occurrence is *m/(m+n)*
- The probability that any result will occur is a fraction between 0 and 1.
- Example of 2 dices

Sum	Frequency	Probability Distribution	Cumulative Probability
2	1	1/36	1/36
3	2	2/36	3/36
4	3	3/36	6/36
5	4	4/36	10/36
6	5	5/36	15/36
7	6	6/36	21/36
8	5	5/36	26/36
9	4	4/36	30/36
10	3	3/36	33/36
11	2	2/36	35/36
12	1	1/36	36/36=1.0
	Σ = 36	Σ = 1.0	



Surveying Measurements and Probability

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1. True value of a measurement is never known due to random errors.
2. What about
   - error = measured value – true value.
3. No measurement is exact, every measurement contains random errors.
4. The exact random error present is always unknown.
5. The best value for a set of measurements is its average (Mean value).

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Example

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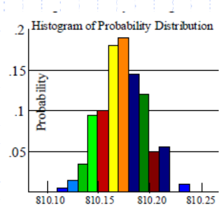
- A distance of about 810 m is measured 200 times. The measurements and their frequencies are in the table below. Draw the probability histogram.

Value of measurement	Frequency	Probability Distribution
810.11	1	0
810.12	3	0.01
810.13	7	0.03
810.14	19	0.09
810.15	20	0.1
810.16	36	0.18
810.17	38	0.19
810.18	29	0.14
810.19	24	0.12
810.2	10	0.05
810.21	11	0.05
810.22	0	0
810.23	2	0.01
	Σ = 200	Σ = 1.0



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Normal Distribution Function

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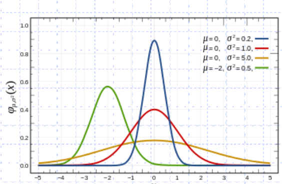
- Instead of representing probability by a histogram it is more convenient to use a mathematical function in which probability is a function of the measurement.
- In Surveying, the Normal Distribution Function is the most common one. It has a bell-shaped curve as shown in the figure. The mathematical form is:
- $F(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(M-M')^2}{2\sigma^2}}$



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Normal Distribution Function

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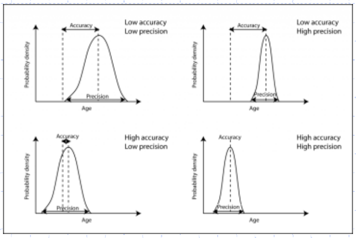
- A low value for σ indicates a high precision curve where most measurements are close to the mean value.
- High precision curve also indicates low random error values.



Example

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- The mean and standard error of a set of measurements are M′=38.45 m, $\sigma = \pm 0.08$ m. What is the probability that a measurement is

a) Below 38.35 m,

b) Between 38.37 and 38.53 m?

- Solution
- To find the probability associated with a normal random variable, use a graphing calculator, an online normal distribution calculator, or a normal distribution table.

http://onlinestatbook.com/2/calculators/normal_dist.html

- a) P(M<38.35) = 10.56 %
- b) P(38.37<M<38.53) = 68.27 %

Summary

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