Lec. 4 +
Lec. 5

CE 371 Surveying
Error analysis_3_4

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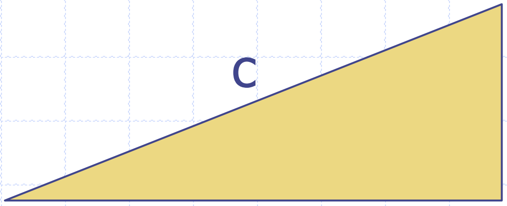
Overview

- Error Propagation
- Propagation of Standard Errors
- Propagation of Systematic Errors and Mistakes

Error Propagation

- In surveying we have quantities measured and quantities required. If there are errors in the measured quantities, then automatically there will be errors in the required quantities.
- Error propagation is the evaluation of the errors in the required quantities as functions of the errors in the measurements.
- Examples
- In a rectangle, length and width are measured, while the area is required.

- In the right angle triangle ABC, side c and angle A are measured, side b is required



Propagation of Standard Errors

- To compute the standard error of the required quantity, do the following:

1. Establish the function relating required quantity to measured ones.

   $$y = f(\,x_1, x_2, \ldots.\,., xn)$$

2. Find partial derivatives of the function with respect to all measurements.

   $$\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \;\ldots\ldots, \frac{\partial y}{\partial xn}$$

3. Apply the following formula to find the error σ_y of the required quantity:

$$\sigma_y = \sqrt{\left(\frac{\partial y}{\partial x_1}\sigma_{x1}\right)^2 + \left(\frac{\partial y}{\partial x_2}\sigma_{x2}\right)^2 + \cdots \ldots\ldots\ldots. + \left(\frac{\partial y}{\partial xn}\sigma_{xn}\right)^2}$$

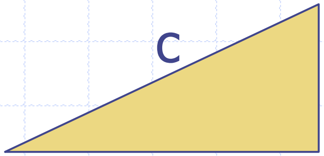
Example 1

- Line AC was measured in 2 parts, AB=100.40±0.03 m, BC=175.18±0.04 m. Compute the length of line AC and its standard error.
- Solution:
- 1)The function is: AC = AB + BC =100.40+175.18 = 275.58 m
- 2)Partial derivatives: $\frac{\partial AC}{\partial AB}, = 1 \quad \frac{\partial AC}{\partial BC} = 1$
- 3)Standard error:

$$\sigma_{AC} = \sqrt{(1x\ 0.03)^2 + \ (1x0.04)^2} = \pm 0.05 \text{ m}$$

Example 2

- In right angle triangle ABC, side c=200.85±0.07 m, and angle A=20.18±0.05 grads. Compute side b and its standard error.
- Solution:



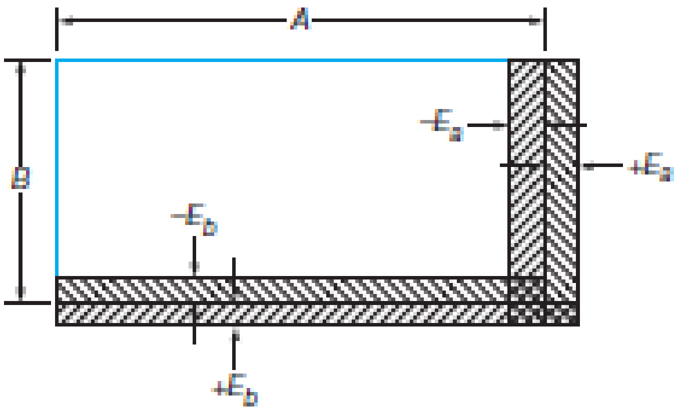
- Error in angle should be converted to radians
- 0.05x2$\pi/400 = 0.000785$
- 1)The function is: b = c cos A=200.85 cos (20.18) = 190.84 m
- 2)Partial derivatives: $\frac{\partial b}{\partial c} = \cos(A) = 0.95 \quad \frac{\partial b}{\partial A} = -c\sin(A) = -62.606$
- 3)Standard error:

$$\sigma_b = \sqrt{(0.95x\ 0.07)^2 + \ (-62.606x0.000785)^2} = \pm 0.08 \text{ m}$$

Example 3

- For the rectangular lot illustrated in Figure, observations of sides A and B with their 95% errors are (252.46 ±0.053) and (605.08 ±0.072) respectively. Calculate the parcel area and the expected 95% error in the area.
- Solution:
- Area = 252.46 * 605.08 = 152,760 ft^2
- $E_{95} = \sqrt{(252.46x\ 0.072)^2 + (605.08x0.053)^2}$ = ±36.9 ft^2

- What if the needed is the standard error of the area?



Error Propagation in a Sum or a Difference of Measurements

- When two or more quantities are added or subtracted, the error in result (E_s) is the square root of the sum of the square of the errors (e_1, e_2, ...) of the individual quantity i.e.,

- $E_s = \sqrt{(e_1)^2 + (e_2)^2 + \cdots \dots \dots \dots . + (e_n)^2}$

Example1:

- the total distance of AB=100.40±0.03 m, BC=175.18±0.04 m.
- AC = (100.40+175.18) $\pm\sqrt{0.03^2 + 0.04^2}$= 275.58 ±0.05 m

Example2:

- To compute BC if AC=275.58±0.05 m, AB=100.40±0.03 m
- BC = (275.58-100.40) $\pm\sqrt{0.05^2 + 0.03^2}$= 175.18 ±0.06 m

Errors Propagation in a Product of Measurements

- When two or more quantities are multiplied, the error in result ($E_{product}$) is the square root of the sum of the square of the fractional errors of the individual quantity. Thus,
- $E_{Product} = \pm A.B\sqrt{(\frac{e_A}{A})^2 + (\frac{e_B}{B})^2}$
- A rectangle is measured 160.881 ± 0.026 cm long and 75.007 ± 0.001 cm wide. The error in its area (12,067 cm2) is
- $E_b = \pm 160.881\ x\ 75.007\sqrt{(\frac{0.026}{160.881})^2 + (\frac{0.001}{75.007})^2} = \pm 1.957\ cm^2$

Errors Propagation in a Division

- When two or more quantities are divided, the error in result is the square root of the sum of the square of the fractional errors in the individual quantity.
- $E_{Product} = \pm \frac{A}{B}\sqrt{(\frac{e_A}{A})^2 + (\frac{e_B}{B})^2}$
- For example: If the area of a rectangular plot is somehow known to be 49,650 ± 10 m2 and the width dimension measured several times found to be 175.66 ± 0.46 m, the calculated length dimension is
- $= \frac{49650}{175.66} \pm \frac{49650}{175.66}\sqrt{(\frac{10}{49650})^2 + (\frac{0.46}{175.66})^2}$ = 282.72±0.74 m

Propagation of Systematic Errors & Mistakes

- To compute the propagation of systematic errors and mistakes, do the following:

1. Establish the function relating required quantity to measured ones.

   $$y = f(x_1, x_2, \ldots.., xn)$$

2. Find partial derivatives of the function with respect to all measurements.

   $$\frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \ \ldots\ldots, \frac{\partial y}{\partial xn}$$

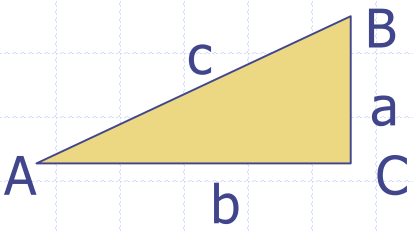
3. Apply the following formula to find the error σy of the required quantity:

   $$d_y = \left(\frac{\partial y}{\partial x_1} d_{x1}\right) + \left(\frac{\partial y}{\partial x_2} d_{x2}\right) + \cdots \ldots \ldots + \left(\frac{\partial y}{\partial xn} d_{xn}\right)$$

   $$y_c = y - dy$$

Example 4

- In right angle triangle ABC, side c=200.85 m, and angle A=20.18^g. Systematic error in c=0.07 m, and in A=0.05^g. Compute corrected side b and its error.



- Solution:
- Error in angle should be converted to radians
- 0.05x2$\pi/400 = 0.000785$

1)The function is: b = c cos A=200.85 cos (20.18) = 190.84 m

2)Partial derivatives: $\frac{\partial b}{\partial c} = \cos(A) = 0.95$ $\frac{\partial b}{\partial A} = -c\sin(A) = 62.606$

3)Standard error:

$d_b = (0.95x\ 0.07) + (62.606x0.000785)$ = 0.12 m

b_c= 190.84-0.12=190.72 m

Propagation of Systematic Errors and Mistakes

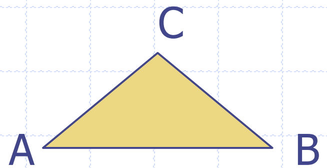
- Notice how the resulting errors are different from the previous section, that is because we were dealing before with standard errors which are related to random errors, while here we deal with systematic errors and mistakes.

Conditional Adjustment of Measurements

- In conditional adjustment, no quantities need to be computed.
- However, measurements, must satisfy a certain mathematical relation in order to be correct.

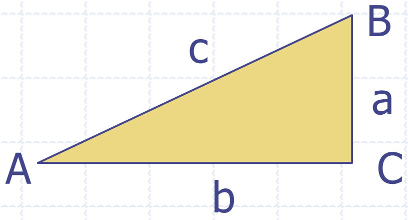
Examples of conditional adjustments

1. Measuring the three interior angles of a triangle where ($A+B+C-180^o=0$).

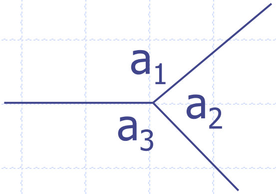


2. Measuring line AC, then measuring it again in parts as AB and BC, where($AC-AB-BC=0$).

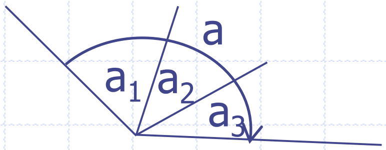
3. Measuring sides b and c of a right triangle and the included angle A, where ($b - c\cos(A)=0$).



4. Measuring the 3 sides (a,b,c) of a right triangle, where ($c-\sqrt{a^2+b^2}=0$).
5. Measuring all three angles around a point such that the horizon is closed where ($a_1+a_2+a_3-360^o=0$).



6. Measuring an angle (such as angle a case 5), then measuring it again in parts (as a_1, a_2, and a_3) where ($a_1+a_2+a_3-a=0$).



Conditional Adjustment of Measurements

- If there are errors in the measurements, the right hand side of the above equations will not be equal to zero, but rather equal to a "closure error "(δ). For correcting such measurements the steps are:

1. Establish the function relating measured quantities

   $$f(x_1, x_2, \ldots.., xn) = \delta$$

2. Compute the correction for each measurement

   $$\delta_{xi} = \frac{\delta}{n} \quad \text{(equal weight)}$$

   $$\delta_{xi} = \frac{\delta}{w_i . \sum \frac{1}{w}} \quad \text{(unequal weight)}$$

3. Compute adjusted measurements

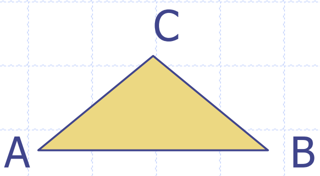
   $xi_{\ adjusted} = x_i \pm \delta_{xi}$ opposite to the sign of x_i in the equation

4. Use the adjusted measurements back as a check

   $$f(x_1, x_2, \ldots.., xn) = 0$$

Example 5

- The interior angles of triangle ABC were measured with equal precisions as $A=39.12^g$, $B=87.69^g$, and $C=73.25^g$. Adjust the three angles.



- Solution:

1. Closure error δ = A+B+C - 200^g =39.12+87.69+73.25-200 = $+0.06^g$
2. Error per angle δ_{xi} = $0.06^g/3$ = 0.02^g (angles are of equal precision).
3. Adjusted angles are:

   $A = 39.12^g - 0.02^g = 39.10^g$,

   $B = 87.69^g - 0.02^g = 87.67^g$

   $C = 73.25^g - 0.02^g = 73.23^g$
4. Check: $39.10^g + 87.67^g + 73.23^g - 200.00^g = 0.00^g$ OK.

Example 6

Line AC is measured as AC=8.23±0.04 m. Line AC is measured again in 2 parts as AB=6.45±0.03 m, BC=1.75±0.02 m. Adjust all 3 measurements.

Solution:

1. Closure error δ = AC- AB - BC = 0.03 m.
2. Measurements have unequal precisions. Weights must be calculated.

Assume σ_0= 0.04 m, therefore, the weights will be:

w_{AC} = $.04^2/.04^2$=1.000, w_{AB} =$.04^2/.03^2$=1.778, and w_{BC} =$.04^2/.02^2$= 4.000

$\Sigma(1/w_i)$= 1.812

closure error for AC = δ_{AC} = 0.03/(1.00 x1.812) = 0.017 m,

closure error for AB = δ_{AB} = 0.03/(1.78 x 1.812) = 0.009 m,

closure error for BC = δ_{BC} = 0.03/(4.00 x 1.812) = 0.004 m.

3. Adjusted AC=8.23-0.017=8.213 m,

   Adjusted AB=6.45+0.009=6.459 m.

   Adjusted BC = 1.75 + 0.004 = 1.754 m.

4. Check 8.213 - 6.459 - 1.754 = 0.000 OK.

Summary

- Error Propagation
- Propagation of Standard Errors
- Propagation of Systematic Errors and Mistakes