King Abdulaziz University

Faculty of Science

Department of Mathematics

Postgraduate Studies in Mathematics

Master Degree (M.Sc.)

1420 – 1999

Postgraduate Studies in Mathematics

(M.Sc.)

M.Sc. studies were started in the mathematics department in 1399-1400 AH.

Aims:

1- To initiate and activate research.

2- To prepare qualified academic staff in the domain of mathematics.

3- To make available opportunity for acquiring sufficient knowledge of subject.

4- To enable the students of the universities of the Kingdom of Saudi Arabia to pursue advanced studies in the field of mathematics.

5- To cooperate and coordinate with other scientific institutions and manufacturing organizations in solving problems which require mathematical techniques.

6- To help universities and academic institutes by providing them with qualified teaching staff.

M.Sc. Admission Requirements:

To become eligible for admission to M.Sc. in Mathematics an applicant has to fulfill the following conditions:

1- He (or she) should hold B.Sc. in Mathematics from any recognized university of good academic repute.

2- He (or she) should have obtained overall Grade C (=good) and at least accumulative Grade B (=very good) in mathematics courses in his (or her) B.Sc.

3- He (or she) should qualify both the written admission test and the oral examination which are organized by the department of mathematics at the time of admission.

Scheme of Studies for M.Sc. Degree:

To earn M.Sc. in Mathematics, a student is required to pass advanced courses in mathematics with at least Grade B together with a dissertation. For completing the M.Sc.degree, the minimum

Numbers of credit hours is 30 while the maximum number of credit hours is 40 with the following breakup (subdivision):

1- Total numbers of credit hours for the advanced courses of study shall lie between 22 and 30, namely, the minimum credit hours 22 while the maximum credit hours 30.

2- The minimum number of credit hours for the M.Sc. dissertation shall be 8 while the maximum credit hours shall be 10.

Study plan for M.Sc. in Mathematics:

Every student registered for M.Sc. degree in mathematics shall have to follow the following plan of studies:

I Compulsory Courses

13 credit hours compulsory courses should be selected from the following:

MATH 601, MATH 611, MATH 615, MATH 622, MATH 641, MATH 661, MATH 692.

II Selected Courses

12 credit hours courses may be selected in consultation with the supervisor and with the approval of the department from the list of courses for M.Sc. in Mathematics.

III

Every student is required to earn at least 8 credit hours through a dissertation written on a topic approved by the department.

M.Sc. Courses

<u>Mathematics Department</u>

Course No.	Course Title	Credit hours	Pre-requisites
601	Theory of Ordinary Differential Equations	3	305, 311
602	Partial Differential Equations I	3	407
603	Stability Theory of Ordinary Differential Equations	3	305, 601
604	Nonlinear Differential Equations	3	601
605	Partial Differential Equations II	3	602
606	Generalized Solutions of Partial Differential Equations	3	602, 605
611	Functional Analysis I	3	417
612	Functional Analysis II	3	611
613	Spectral Theory	3	417
614	Generalized Functions	3	417
615	Real Analysis I	3	416
616	Real Analysis II	3	615
617	Summability Theory	3	417
621	Numerical Treatment of Simultaneous Linear Equations	3	321
622	Numerical Treatment of Ordinary Differential Equations	3	321, 423
623	Numerical Treatment of Partial Differential Equations	3	321, 423

Course No.	Course Title	Credit hours	Pre-requisites
624	Approximation Theory	3	321
625	Polynomial Approximations	3	624
631	Axiomatic Set Theory	3	Dept. Approval
632	Category Theory	3	641, 661
633	Mathematical Logic	3	Dept. Approval
634	Algebra of Proofs	3	641
635	Universal Algebra	3	641
641	Abstract Algebra	3	344
642	Group Theory	3	641
643	Ring Theory	3	641
644	Field Theory	3	641
645	Theory of Modules	3	641
646	Multilinear Algebra	3	641
647	Topological Groups	3	641, 661
648	Lie Groups and Lie Algebras	3	641, 651
651	Differentiable Manifolds	3	364
652	Riemannian Geometry	3	651
653	Fibre Bundles	3	651
654	Morse Theory	3	651
655	Geometry I	3	641, 661
656	Geometry II	3	655
657	Geometry III	3	656

Course No.	Course Title	Credit hours	Pre-requisites
658	Algebraic Geometry	3	641, 661
661	General Topology I	3	466
662	General Topology II	3	661
663	Homology Theory	3	661
664	Homotopy Theory	3	661
691	Selected Topics	3	Dept. Approval
692	Seminar	1	Dept. Approval
693	Thesis	8	Dept. Approval

Courses' Description

601 Theory of Ordinary Differential Equations

Systems, Existence proofs, Singularities, Asymptotic behavior of solutions. Boundaries of solutions. Eigen values and Eigen functions. Rayleigh- Rietz methods, Perturbation theory.

602 Partial Differential Equations I

Cauchy-Kowalevski theorem. Standard facts about harmonic functions. Proof of the existence of solutions for the Dirichlet problem by Perron's method. Poisson's equations. Estimates at the boundary . Parabolic equations, existence and uniqueness theorem. Hyperbolic equations, characteristics, the method of spherical means.

603 Stability Theory of Differential Equations

General theory of systems. Properties of linear systems. Stability and boundedness of systems. Perturbation techniques. Liapunov functions. Periodic and almost periodic solutions.

604 Nonlinear Differential Equations

Existence theorems. Theory of linear systems of differential equations. Stability and asymptotic behavior of nonlinear systems of differential equations. Poincare-Bendixson theory of two dimensional systems. Liapunov's method. Perturbation methods.

605 Partial Differential Equations II

Weak and strong maximum principles for elliptic equations. A priori bounds. Harnak inequality. Operators in divergence form. Shouder estimates, boundary and global estimates, the Dirichlet problem and proof of the existence of classical solutions. Interior and boundary regularity. The oblique derivative problem.

606 Generalized Solutions of Partial Differential Equations

Modifiers. Regularization and approximation by smooth functions in L^{p} spaces.$W^{\mathrm{m,p}}$ Spaces. Imbedding theorems, traces on the boundary. Existence of weak solutions for elliptic equations, weak and strong maximum principles. Differentiability. Local properties, Harnak inequality.

611 Functional Analysis I

Open mapping and closed graph Theorems for Banach spaces. Topological vector spaces- linear manifolds, hyperplanes, bounded sets. Metrizability. Completeness. Locally convex spaces, Hahn-Banach Theorem, Separation of convex sets, compact convex sets. Linear and bilinear mappings. Duality.

612 Functional Analysis II

Banach Algebra. Spectrum and spectral radius. Radical and semi-simplicity. B^* - algebra. The Gelfand Mapping. The Gelfand-Naiumark Theorem. Topological vector algebra. Locally m-convex algebras, spectrum.

613 Spectral Theory

Spectrum of self-adjoint operators, point spectrum and continuous spectrum, spectral decomposition of self-adjoint operators, operator functions, spectral. Measures, spectral integrals, spectral decomposition of normal operators, spectrum of regular and singular differential operators, qualitative study of spectrum of Schrodinger's operator, perturbation of the spectrum.

614 Generalized Functions

Problem of extending the collection of ordinary functions, test functions of one variable. Generalized functions of one variable. Examples of generalized functions. Ordinary differential equations. Partition of unity. Convergence in the space of generalized functions, the structure of generalized functions. Convolution of generalized functions. Fourier transforms of generalized functions.

615 Real Analysis I

Abstract measure theory using the Daniell approach. Fundamental system of functions and extensions, continuous and semicontinuous functions. Integrals on the fundamental systems and extension, upper and lower integrals and summable functions. Integrable functions. Measurable functions. Measures and the abstract Lebesgue integral. Measures in a topological space. Measures induced by integrals, integrals induced by measures in a topological space, integration over measureable sets. Abstracts Fubini Theorm, product measures in a topological space.

616 Real Analysis II

Complex measures. Decomposition theorems. Absolute continuity and singularity of complex measures. Radon-Nikodym Theorem. Regular measures. Measure in a locally compact group. Haar measure.

617 Summability Theory

Matrix transformations, regular and conservative transformations. Conditions for regularity. Sequence to function transformations, function to function transformations. Special methods of Summability-Narlund, Cesars, Holder, Abel, moment constant methods, Riesz typical means, method defined by integral functions. Cesaro methods-convexity and Taubenan theorem for power series methods. Hausdorff means. Euler and Berel methods of Summability.

621 Numerical Treatment of Simultaneous Linear Equations

Direct and indirect methods. Elimination methods, Iterative methods. The conjugate gradients methods III conditioned systems. Error Analysis. Bank systems. Eigenvalues and eigenvectors. Iterative methods. Transformation to tridiagonal and Hessenberg forms. Localization theorems for eigenvalues. Perturbation theory.

622 Numerical Treatment of Ordinary Differential Equations

One step methods. Runge-Kutta methods. Multistep and predictor-corrector methods. Numerical analysis including stability. Convergence and error analysis. Boundary-value problems. Integral equations.

623 Numerical Treatment of Partial Differential Equations

Finite difference techniques for elliptic equations. Treatment of boundary conditions. Iterative methods. Successive ever relaxation method. Explicit and implicit method for parabolic equations. Error analysis, stability analysis and convergence. The method of characteristics for quasilinear hyperbolic equations.

624 Approximation Theory

Least squares approximation. Orthogonal polynomials. Techebychev approximation, characterization and uniqueness. The de la Valee Poisson’s theorems. Approximation by rational functions using continued fractions and linear programming. Approximation in the norm. Spline functions and approximation of functionals. Multivariate approximation.

625 Polynomial Approximation

The Weierstrass Theorems. Fourier Series. Tchebychev polynomials. Bernstein polynomials. Preliminaries. Moduli of continuity. Zeros of trigonometric polynomials. Derivatives and limits of polynomials. Polynomials of best approximation. Existence, characterization, uniqueness. Best trigonometric approximation. Jackson's Theorems. Gaussian Quadrature. Jackson's- Favards theorems. Inverse theorems of periodic functions. Bernstein's Theorems. The Zygmund Class.

631 Axiomatic Set Theory

ZF-Theory, independence alternative theory, independence ordinals ' limit ordinals, Zorns-lemma, axiom of choice.

632 Category Theory

Functors. Monics. Epics and Zeros Hom-Sets. Large Categories. Duality. Functor categories, Graphs and free categories. Groups in categories. Yeneda Lemma. Beck's theorem.

633 Mathematical Logic

Formal languages. Deductive system. Semantics. Syntax. Monoidal categories.Labeled deductive systems.

634 Algebra of Proofs

Algebraic properties of the usual operations of logic. Algebraic model for proof.Completeness. Church-Rosser theorem.

635 Universal Algebra

Basic concepts, Set theoretic relations (without proof). Algebras and homomorphism. Polynomials, congruence relations, subalgebra, congrnence lattice of algebras, direct and inverse limit. Free algebras and independence algebraic construction. Determination of properties preserved under certain algebraic construction.

641 Abstract Algebra

Introduction (subgroups, homomorphisms, orders of elements). Direct product (internal and external). Fundamental theorem of finitely generated commutative groups. Sylow theorems (without proofs). Rings and Modules (ideals, factorization, Division, direct product, polynomials). Fields (separable extensions). Galois Theorems(without proof).

642 Group Theory

Sylow theorems. Composition series. Nilpotent groups and solvability. Galois Theorems.

642 Ring Theory

Ideal (localization, decomposition). Radical and representation. Near rings.

644 Field Theory

Extensions theorems. Galois theorems. Valuation theorems.

645 Theory of Modules

Representation theorems of algebra (matrix representation). Dimension Theorems. Noetherian and Artirian Modules. Projective and injective modules.

646 Multilinear Algebra

Multilinear algebra. Tensor algebra. Tensor products. Exterior algebra. Syummetric algebra. Graded algebra.

647 Topological Groups

The concept of topological groups. System of neighbourhoods of the identity. Subgroups. Homomorphisms. Direct product. Character groups, Locally compact groups (introduction). Discrete and compact groups (introduction).

648 Lie Groups and Lie Algebra

Lie groups, Lie subgroups. Left and right ranslations. Left and right invariant vector fields. Lie algebra, Lie subalgebra. Product of Lie subgroups.

651 Differentiable Manifolds

Topological Preliminaries. Topological manifolds. Differentiable manifolds. Submanifolds. Tangent and cotangent spaces. Vector fields and differential forms. Covariant differentiation or infinitesimal connections. Torison tensor and curvature tensor of connection.

652 Riemannian Geometry

Riemannian metric. Minkowski metric . Poincare metric. Riemann curvature tensor and its properties. Einstein spaces. Schur's Theorem. Parallelism and geodesics in Riemannian spaces.

Distributions. Integrable distributions. Involutive distributions.

653 Fibre Bundles

Concept of fibre bundle. Tangent and cotangent bundles. Grassmann Manifolds. Cross-section of bundles. Connection in bundles. Horizontal and Vertical lifts.

654 Morse Theory

Critical points. Hessian, Nullity. The Morse Lemma and its implications. Reeb's Theorem. Morse Inequalities. Focal points. Sard's Theorem. Relation between critical points and focal points.

655 Geometry I

Projective geometry, duality, subspaces, closure operator, homomorphism, transitivity, Desargues. Axiom, six-point theorem. Affine geometry, coordinaization in plane, Ternary-ring-Skalarsystem, Lenz-Barlotti classification. Polarities-Ovals.

656 Geometry II

Division rings and their plans. Non-desargian plane. Collineation groups. Configuration polarities. Ovals. Mobius geometry. Laguere-geometry.

657 Geometry III

Mobius plane, Miquel-theorem, Mobius plane of $\sum(K,Q)$ configuration. Papus-theorem. Principal theorem. Finite Mobius plane. Block-geometry.

658 Algebraic Geometry

Hibert nullstel lensalz, Krull-haupt ideal-theorem. Field-extension. Local ring, evaluated field and ring. Geometrical topics: affine, projective geometry over field, Mobius geometry connected with a field. Ovals. Topological topics: Koratowisk's problem, Seperation axioms, filter, density, topological rings and groups Cauchy-filter, Completation of groups. Algebraic curves, Prime curves. Topology of affine space, ideals connected to curves. Regular curve and function. Rational curves. Varieties.

661 General Topology I

Cardinality axioms. Connectedness (connected, pathwise, locally connected topological spaces). Compactness (compact, locally compact, countably compact and sequentially compact topological spaces).Compactification (one point and Stone-Cech compactification). Nets and Filters. Uniform spaces.

662 General Topology II

Paracompactness, metrization. Uryohn's lemma, Tietz's extension theorem. Completion of metric spaces. Topological n-manifolds. Direct and inverse systems of topological spaces.

663 Algebraic Topology I (Homology Theory)

Simplexes, Complexes, Chains and Cycles, Homology groups of simplicial complexs, Betti Numbers, Euler characteristic, Realative homology groups, Exact sequences, Singular homology theory introduction).

664 Algebraic Topology II (Homotopy Theory)

Retraction and deformation. Homotopy extension property. Fundamental groups of S^1 , T^2,p^2 ,homotopy type. Homotopy groups. Invariance of homotopy groups under continuous maps.

691 Selected Topics

692 Seminar

Group meetings for review and discussion of current literature in mathematics.

693 Thesis

Original investigation in selected problems in one of the branch of Mathematics approved by the department and faculty.