King Abdulaziz University
Faculty of Science
Department of Mathematics

Ph. D. Program
(1434-35 / 2013-14)

The academic requirements for the degree of Doctor of Philosophy in Mathematics:

1- Total Number of Hours

Minimum number of credit hours for the degree of Doctor of Philosophy in Mathematics is 43.These are divided as under:

A.	Number of Credit Hours for Compulsory Courses	**15**
B.	Number of Credit Hours for Optional Courses	**18**
C.	Number of Credit Hours for the Dissertation	**10**
D.	Total Number of Credit Hours	**43**

Compulsory Courses are following:
1. Math 701: Differential Equations
2. Math 711: Functional Analysis-I
3. Math 713: Real Analysis-I
4. Math 741: Abstract Algebra
5. Math 771: General Topology-I

The student will take the remaining courses with the advice of the academic supervisor. The passing grade for each of these courses is 70%.

2- Comprehensive Exams

The department offers comprehensive exams in six sequences; a student must pass comprehensive exams in three sequences, one of the exams from his/her own field of research. These sequences are in the areas of :
1. Differential Equations,
2. Functional Analysis,
3. Real Analysis,
4. Abstract Algebra,
5. General Topology, and
6. Numerical Analysis.

Each exam depends on at least two courses in an area offered in two consecutive semesters. The passing grade of a comprehensive exam is 70%. If a student fails to obtain the passing grade in a comprehensive exam, then he/she will be given one more chance to pass the comprehensive exam in the same subject. If a student does not pass the exam in the second attempt, he/she must be permanently dropped out from the program. The comprehensive exams will be held in the summer term of each year. The re-sit exam will be offered in the beginning of the second semester of the following year.

3- Dissertation:

After completing the requirement of the compulsory courses and the comprehensive exams in three areas, the student with the consultation of the Committee of Higher Studies will select a research supervisor. The research supervisor will advise the student for taking further optional courses. When the student will complete 33 credit hours of the course work, he/she will immediately be assigned a research topic by his/her research supervisor. The student will conduct original research related to that topic and at the end of the research he/she will write a research dissertation. From this dissertation he/she must produce at least one research paper that may appear in a well reputed research journal of Mathematics.

Course Code	Course Title	Course Type	Credits			Prerequisite
			Th.	Pr.	Total	
Math 701	Differential Equations	Compulsory	**3**	**-**	**3**	-
Math 702	Ordinary Differential Equations	Optional	**3**	**-**	**3**	Math 701
Math 703	Partial Differential Equations I	Optional	**3**	**-**	**3**	Math 701
Math 704	Partial Differential Equations II	Optional	**3**	-	**3**	Math 703
Math 711	Functional Analysis I	Compulsory	**3**	**-**	**3**	-
Math 712	Functional Analysis II	Optional	**3**	**-**	**3**	Math 711
Math 713	Real Analysis I	Compulsory	**3**	**-**	**3**	-
Math 714	Real Analysis II	Optional	**3**	**-**	**3**	Math 713
Math 715	Complex Analysis	Optional	**3**	-	**3**	-
Math 721	Numerical Analysis I	Optional	**2**	**2**	**3**	-
Math 722	Numerical Analysis II	Optional	**2**	**2**	**3**	Math 721
Math 741	Abstract Algebra	Compulsory	**3**	**-**	**3**	-
Math 742	Group Theory	Optional	**3**	**-**	**3**	Math 741
Math 743	Ring Theory	Optional	**3**	**-**	**3**	Math 742
Math 744	Field Theory	Optional	**3**	-	**3**	Math 743
Math 745	Module Theory	Optional	**3**	**-**	**3**	Math 743
Math 746	Multilinear Algebra	Optional	**3**	**-**	**3**	-
Math 751	Geometry I	Optional	**3**	**-**	**3**	-
Math 752	Geometry II	Optional	**3**	-	**3**	Math 751
Math 753	Algebraic Geometry	Optional	**3**	-	**3**	Math 743
Math 771	General Topology I	Compulsory	**3**	**-**	**3**	-
Math 772	General Topology II	Optional	**3**	**-**	**3**	Math 771
Math 773	Algebraic Topology I	Optional	**3**	**-**	**3**	Math 743
Math 774	Algebraic Topology II	Optional	**3**	-	**3**	Math 773
Math 795	Seminar	Optional	**1**	**-**	**1**	
Math 796	Selected topics in Mathematics-I	Optional	**3**	**-**	**3**	
Math 797	Selected topics in Mathematics-II	Optional	**3**	-	**3**	Math 796
Math 799	Thesis	Compulsory	**10**	-	**10**	

Course Code	Course Title	Credits	Prerequisite
Math 701	Differential Equations	3	

Course Description

Dynamic models, stable and unstable motion, stability of linear and nonlinear systems, Liapunov functions, feedback, growth and decay, the logistic model, population models, cycles, bifurcation, catastrophe, biological and biomedical models, chaos, strange attractors, deterministic and random behavior. Asymptotic solutions of ordinary differential equations, matched asymptotic expansions, multiple scales, WKB and related methods, asymptotic approximations of integrals, stationary phase/steepest descent methods.

Course Code	Course Title	Credits	Prerequisite
Math 702	Ordinary Differential Equations	3	Math 701

Course Description

Linear autonomous and periodic systems, reducible, almost reducible and regular systems, stability and small perturbations of the coefficients of linear systems, basic theory of integro-differential equations. Nonlinear differential equations in abstract spaces, existence theory with continuous and discontinuous right hand sides, global existence of solutions of differential equations in Banach spaces, fundamental properties, stability and asymptotic behavior, perturbing Lyapunov functions, existence theory and the topological methods, abstract nonlinear boundary value problems, monotone iterative method, semigroups and resolvent operators.

Course Code	Course Title	Credits	Prerequisite
Math 703	Partial Differential Equations I	3	

Course Description

Classical weak and strong maximum principles for second order elliptic and parabolic equations, Hopf boundary point lemma, Sobolev spaces, weak derivatives, approximation, density theorem, Sobolev inequalities, L_2 -theory for second order elliptic, parabolic and hyperbolic equations, existence via Lax-Milgram Theorem, Fredholm alternative, a brief introduction to L_2 -estimates, Spectral theory and applications to partial differential equations, semigroup theory applied to second order parabolic and hyperbolic equations. A brief introduction to elliptic and parabolic regularity theory, variational methods, method of upper and lower solutions, fixed point method, bifurcation method.

Course Code	Course Title	Credits	Prerequisite
Math 704	Partial Differential Equations II	3	Math 703

Course Description

The nonlinear Schrodinger (NLS) equation and solitary waves. Structural properties: Lagrangian and Hamiltonian structure, Noether theorem, invariances and conservation laws. The nonlinear Klein-Gordon and averaging techniques, The Sine-Gordon equation and solitary, and anti-solitary solutions. The optimal transportation problem of Monge and Kantorovich. Nonlinear diffusion-reaction phenomena: Burgers' and Fisher's equations. Analysis of the blow-up: self-similarity, modulation analysis, rate of blow-up. Basic theory of scheme consistency, convergence and stability; various methods including finite difference methods (explicit and implicit), finite volume methods and finite element methods.

Course Code	Course Title	Credits	Prerequisite
Math 711	Functional Analysis I	3	
Course Description	Review: Banach space theory, topological vector spaces (TVSs), locally convex spaces. Metrizable and locally bounded TVSs, Barrelled and Bornological locally convex spaces, spaces of continuous and bounded linear mappings, Hahn-Banach Theorem for locally convex spaces, duality theory in TVS(weak topologies), hyperplanes in TVSs, open mapping and closed graph theorems in TVSs, principle of uniform boundedness. nuclear spaces and nuclear operators.		

Course Code	Course Title	Credits	Prerequisite
Math 712	Functional Analysis II	3	Math 711
Course Description	Differential and integral calculus in Banach Spaces. Banach fixed point theorems and its applications to differential and integral equations. Brouwer and Schauder fixed point theorems. Compact, monotone, accretive and nonexpansive operators. Metric projections and Chebychev approximation. Optimization. Variational Inequalities. Degree theory. Semigroup of linear operators.		

Course Code	Course Title	Credits	Prerequisite
Math 713	Real Analysis I	3	
Course Description	Topological Spaces, Metric Spaces, Completeness and Compactness in Metric Spaces, Baire's Category Theorem, Baire's Category Theorem, Banach Contraction Principle, Arzela-Ascoli Theorem. Abstract Measure Theory, Positive Measures-Outer Measures, Extension Theorem of Positive Measures (without proof) , Regular Measures on Topological Spaces , The Lebesgue Measure on $\mathbb{R}$. Integration, Measurable functions-Approximation by simple functions, The integration process for positive functions, The Banach Space L_1 of integrable functions, limit Theorems. L_p-Spaces, Convexity inequalities, The Banach L_p -Spaces, Signed measures-Total variation, The Radon-Nikodym-Lebesgue Theorem , Duality in L_p -Spaces. Multiple Integration, Product of σ –finite Measures, Multiple integrals –Fubini's Theorem , Lebesgue measure on $\mathbb{R}^n$, Convolution.		

Course Code	Course Title	Credits	Prerequisite
Math 714	Real Analysis II	3	Math 713
Course Description	Elements from Banach Spaces, Duality in Banach spaces , Weak and Strong convergence, Hahn Banach Theorem , Uniform Boundedness Theorem , Hilbert Spaces-Orthonormal Bases. The Bochner Integral, Measurability of Banach valued functions: Petti's Theorem, Bochner Integral : Construction, Integrability criterion – The space $L_1 (\mu , X)$,The case of $\mathbb{R}^n$ -valued functions. The Daniell Integral, Positive linear functionals on Function Spaces, Extension of linear functionals, Measurability and set functions defined by functionals , Measures defined by positive linear functionals, The Daniell Representation Theorem. The Riesze Integral Representation Theorem, The space $Cc(X)$ of continuous functions with compact support. The space $C_0(X)$ of continuous functions vanishing at infinity , Positive Linear Functionals on $Cc(X)$ and Positive Regular Borel Measures on X, Bounded Linear Functional on $C_0(X)$ and Complex Regular Borel Measures on X. Integral Representation of Bounded Operators, Vector Measures-Weak and Strong σ –additivity, Weakly Compact Operators on Banach Spaces, The integral Representation Theorem of Bartle-Dunford Schwartz.		

Course Code	Course Title	Credits	Prerequisite
Math 715	Complex Analysis	3	

Course Description

Complex numbers. Analytic functions. Power series. Integration along paths. Mobius transformations. Conformal mappings. Cauchy's theorem and Cauchy integral formula. Maximum modulus principle and the Schwarz lemma. Runge's theorem. Rouche's theorem. The logarithm. Singularities. Meromorphic functions. The residue theorem. Entire functions and the Weierstrass factorization theorem.

Course Code	Course Title	Credits	Prerequisite
Math 721	Numerical Analysis I	3	

Course Description

Gauss Elimination method, Factorization methods (Crouts , Doolittle,Cholesky), Numerical solution of block systems of band structure. Iterative methods (Jacobi, Gauss Seidel, and SOR), Numerical solution of nonlinear systems: Fixed points for function for several variables, Newton’s method and Quasi Newton’s method, Steepest decent techniques, Homotopy and continuation method. Eigenvalue Problems (Power method, Householder’s method, QR method).

Course Code	Course Title	Credits	Prerequisite
Math 722	Numerical Analysis II	3	Math 721

Course Description

Numerical solution of initial value problem (single step method and linear multistep method, Differential system and stiff problems, Numerical solution of boundary value problems: Finite difference method, Shooting method. Numerical solution of partial differential equations using finite difference method, Parabolic equations, Hyperbolic equations, Elliptic equations, Coupled systems of time dependent problems, Introduction to the finite element method.

Course Code	Course Title	Credits	Prerequisite
Math 741	Abstract Algebra	3	

Course Description

Group Representations: Representations, Equivalent representations, Kernels of representations and faithful representations. FG-Modules, FG-Submodules and Reducibility: FG-modules, Permutation modules, FG-modules and equivalent representations, FG-submodules, Irreducible FG-modules. Group Algebra and FG-Homomorphism: The Group algebra of a finite group, The regular FG-module, FG acts on an FG-module, FG-homomorphism, Isomorphic FG-modules, Direct sums. Maschke’s Theorem and Schur’s Lemma: Maschke’s theorem, Consequences of Maschke’s Theorem, Schur’s Lemma, Representation theory of finite abelian groups, Some further applications of Schur’s Lemma. Irreducible Modules and the Group Algebra: Irreducible submodules of a group algebra, The Space of a group algebra homomorphisms.

Course Code	Course Title	Credits	Prerequisite
Math 742	Group Theory	3	Math 741

Course Description

Introduction (Solvable and Nilpotent Groups): Abelian and Central Series, Characteristic Subgroups and Commutators, Solvable Groups, Nilpotent Groups, Complement of a Subgroup and the Normal p-complement. Supersolvable Groups: M-Groups, Supersolvable Groups, Frattini Subgroup, Fitting Subgroup, Generalized Fitting Subgroup and Quasisimple Groups. Extensions, Semidirect and Central Products, Hall Subgroups, Sylow Basis. Formations, Definitions and Examples, Saturated Formations, Proving Gaschutz-Lubeseder-Schmid Theorem which states that: "A formation of finite groups is saturated iff it is local", Examples about Saturated and Nonsaturated Formations.

Course Code	Course Title	Credits	Prerequisite
Math 743	Ring Theory	3	Math 742

Course Description

Basic terminology, notation, and examples of rings: Rings with generators and relations, semigroup & group rings, skew polynomial and Laurent series rings, differential polynomial rings, triangular rings, Weyl algebras and enveloping algebras. Basic Ring Theory: Rings with chain conditions, prime and semiprime rings and ideals, primitive and semi primitive rings, prime, nil, and Jacobson radicals, density theorems structure theorem for semi simple artinian ring.Local and semi local rings, the theory of idempotent, perfect and semiperfact rings. Noncommutative localization: Rings of quotients, Goldie's rings, Goldie's theorem, maximal rings of quotients. Frobenius and quasi-Frobenius rings.

Course Code	Course Title	Credits	Prerequisite
Math 744	Field Theory	3	Math 743

Course Description

Algebraic Preliminaries: Localization, integral extensions, polynomial rings, unique factorization domains, Dedekind domains, Euclidean domains, tensor products. Embedding and separability: Separable extensions, pure inseparability, perfect fields, simple transcendental extensions.Galois Theory: Normal subgroups and normal extensions, abelian and cyclic extensions, the Galois group of polynomial, the Fundamental Theorem of algebra, the Galois of a finite field. Wedderburn's Theorem, realizing groups as Galois groups, cyclic extensions, solvable extensions, radical extensions, solvability of Galois groups of radical extensions, a Galois correspondence for radical extensions, Duality of lattices for Gal(E/F) and cog(E/F), the lattice of intermediate fields of radical extensions.The Theory of Binomials: The Galois group of binomial, dual groups of Pairings, Kummer Theory.

Course Code	Course Title	Credits	Prerequisite
Math 745	Module Theory	3	Math 743

Course Description

Categories and Functors: Definitions, examples, natural transformations, natural equivalence. Generators and free modules, products and coproducts (with universal properties). Properties of hom and tensor functors in Mod-R and R-Mod.Projective and injective modules, projective covers and injective hulls, projective generators and injective cogenerators. Flat modules: Flatness, torsion freeness, Von Neumann regularity, faithfully flat modules, coherent modules, pure exact sequences. Homological dimensions: Projective and injective dimensions, weak dimensions and global dimensions of rings.

Course Code	Course Title	Credits	Prerequisite
Math 746	Multilinear Algebra	3	Math 743
Course Description	Multilinear mappings, Some properties of tensor products, Associative algebras, The tensor algebra of a module, The exterior algebra of a module, The symmetric algebra of a module, Coalgebras and Hopf algebras.		

Course Code	Course Title	Credits	Prerequisite
Math 751	Geometry I	3	
Course Description	Exterior Forms, Exterior Derivative, Lie Derivative and Integration, Anti-exact Differential Forms and Homotopy Operators, Isovector Methods for Second Order Partial Differential Equations, Calculus of Variations, Modern Thermodynamics, Electrodynamics with Electric and Magnetic Charge, Gauge Theori.		

Course Code	Course Title	Credits	Prerequisite
Math 752	Geometry II	3	Math 751
Course Description	Differential Manifolds, Riemannian Metrics, Affine Connections, Riemannian Connections, Geodesics, Convex Neighborhood, Curvature, Jacobi Fields.		

Course Code	Course Title	Credits	Prerequisite
Math 753	Algebraic Geometry	3	Math 743
Course Description	Preliminaries: Algebras over rings, ideals, prime ideals, ring of polynomials, chain conditions, rings of fractions, algebraic sets, algorithms for polynomials, Grobner bases, Hilbert basis theorem, Zariski topology, Hilbert nullstellensatz.Algebraic varieties: Ringed spaces, morphisms of ringed spaces, affine algebraic varieties, projective varieties, morphisms, rational maps, nonsingular varieties, products of varieties, fibred product, and dimension theory of affine and projective varieties. Local studies: Tangent spaces to plane curves, tangent cones to plane curves, tangent spaces of subvarieties. Schemes: Sheaves, schemes, separated and proper morphisms, sheaves of modules, projective morphisms, differentials. Introduction to descent theory.		

Course Code	Course Title	Credits	Prerequisite
Math 771	General Topology I	3	
Course Description	Cardinal numbers, order relations, ordinal numbers, the axiom of choice. Topology and topological spaces, open and closed sets. Closure and interior. Bases and subbases, first and second countability. Locally finite and locally discrete families. Weight and character of spaces. Methods of generating topologies. Derived set, dense and nowhere dense set, Borel sets. Continuous function, closed functions and open functions, homeomorphisms and homeomorphic spaces. Axioms of separation, Urysohn's Lemma and Vedenissoff theorem. Convergence in topological spaces, Nets and filters, sequential and Fréchet spaces. Operation on topological spaces: subspaces, sums, Cartesian product (Tychonoff topology and box topology), the diagonal theorem, quotient spaces and quotient mappings.		

Course Code	Course Title	Credits	Prerequisite
Math 772	General Topology II	3	Math 771
Course Description	Compact spaces, operations on compact spaces, the Tychonoff theorem, Wallace theorem, Alexandroff theorem. Locally compact spaces and k-spaces. Compactifications, Stone-Čech compactification. Wallman extension. Perfect mappings. Lindelöff spaces. Čech-complete spaces. Countably compact spaces, pseudo compact spaces, sequentially compact spaces. Realcompact spaces. Metric and metrizable spaces. Operation on metrizable spaces. Totally bounded and complete metric spaces. Compactness in metric spaces. Metrization theorems. Paracompact spaces, countably paracompact spaces. Weakly and strongly paracompact spaces. More Metrization theorems. Connected spaces. Various kinds of disconnectedness.		

Course Code	Course Title	Credits	Prerequisite
Math 773	Algebraic Topology I	3	743
Course Description	The Fundamental group, Covering spaces, Homotopy type and homotopy equivalence of spaces. The van Kampen Theorem. Fibrations and Cofibrations, Fibrations, Replacing a map by a fibration, Fiber homotopy equivalence, Cofibrations, Replacing a map by a cofibration, Fiber and cofiber sequences. Higher homotopy groups, Homotopy groups. Properties of the homotopy groups, Relative homotopy groups, A few calculations, n-Equivalences, weak equivalences, Fiber, Bundles, Fiber bundles, Principal bundles and associated bundles, Maps of bundles and pullbacks, Cohomology of Fiber Bundles, Vector Bundles.		

Course Code	Course Title	Credits	Prerequisite
Math 774	Algebraic Topology II	3	Math 773
Course Description	Homology and Cohomology, Homology groups, The Hurewicz Theorem, The Universal Coefficient Theorem for homology and Cohomology, Postnikov Towers, Spectra, Category of Spectra, Homotopy groups of a spectrum, Representation Theorems, K- Theory, The Bott periodicity theorem, The splitting principle and the Thom isomorphism, The Adams operations, The Hopf invariant one problem and its applications, Steenrod Algebra, Steenrod Algebra and its Dual.		

Course Code	Course Title	Credits	Prerequisite
Math 795	Seminar	1	
Course Description			

Course Code	Course Title	Credits	Prerequisite
Math 796	Selected topics in Mathematics-I	3	
Course Description			

Course Code	Course Title	Credits	Prerequisite
Math 797	Selected topics in Mathematics-II	10	Math 796
Course Description			

Course Code	Course Title	Credits	Prerequisite
Math 799	Dissertation	10	
Course Description	The dissertation must contain results of extensive research and make an original contribution to the field.		