CHAPTER 28: MAGNETIC FIELDS DUE TO CURRENTS

Dr Reem M. Altuwirqi

What we will learn

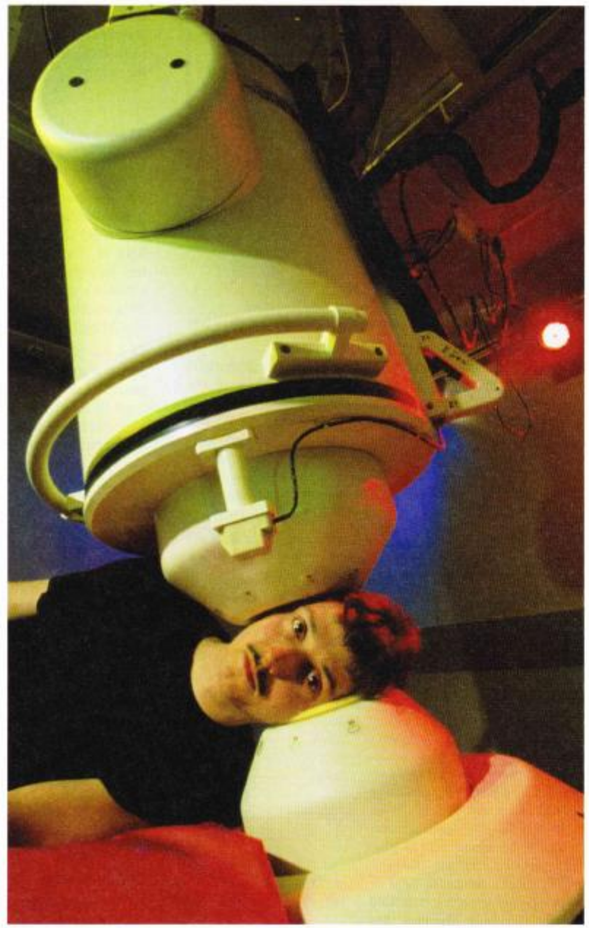
- Calculating the magnetic field due to a current
- Force between two parallel currents
- Ampere's law
- Magnetic fields of a solenoid and toroids

Magnetic Fields Due to Currents

When you read this sentence, a certain region of your brain is activated. When you smell a rose or feel a pencil in your grip, other regions are activated. One of the best ways to determine which regions are activated is to detect the magnetic field produced by the activation. The apparatus in the photograph can detect the magnetic field set up by a person's brain so that a map of brain activity can be correlated with what the person does. However, there are no magnetic materials in the brain.

So, how can brain activation produce a magnetic field?

The answer is in this chapter.



Jurgen Scriba/Photo Researchers

Calculating the magnetic field due to a current

The magnitude of the field ***dB*** produced at point *P* at distance *r* by a current length element *i* ***ds*** turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2},$$

where θ is the angle between the directions of and , a unit vector that points from *ds* toward *P*. Symbol μ_0 is a constant, called the permeability constant, whose value is

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}.$$

Therefore, in vector form

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s} \times \hat{\mathrm{r}}}{r^2} \qquad \text{(Biot–Savart law).}$$

This element of current creates a magnetic field at *P*, into the page.

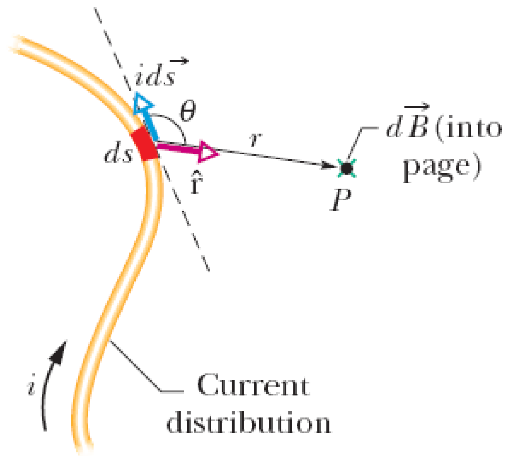
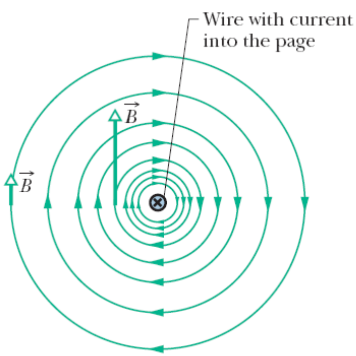


Fig. 29-1 A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green × (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

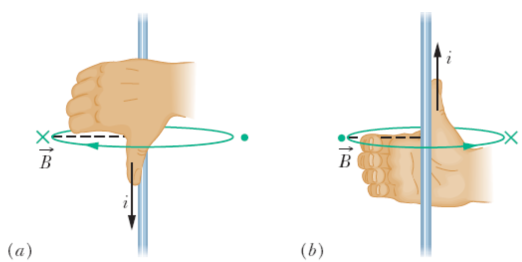
Calculating the magnetic field due to a current

In a straight wire

The magnitude of the magnetic field at a perpendicular distance *R* from a long (infinite) straight wire carrying a current *i* is given by



$$B = \frac{\mu_0 i}{2\pi R}$$



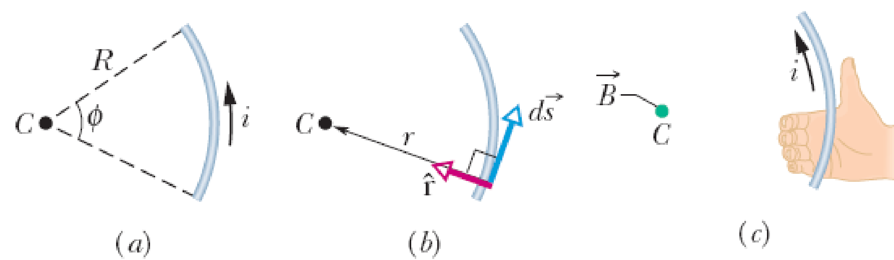
In an circular arc of a wire

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}.$$

$$B = \int dB = \int_0^{\phi}\frac{\mu_0}{4\pi}\frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi.$$

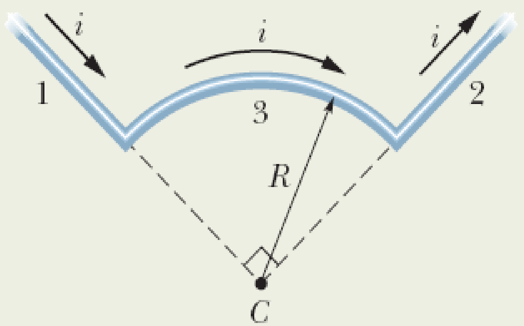
$$B = \frac{\mu_0 i\phi}{4\pi R} \qquad \text{(at center of circular arc).}$$

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R} \qquad \text{(at center of full circle).}$$



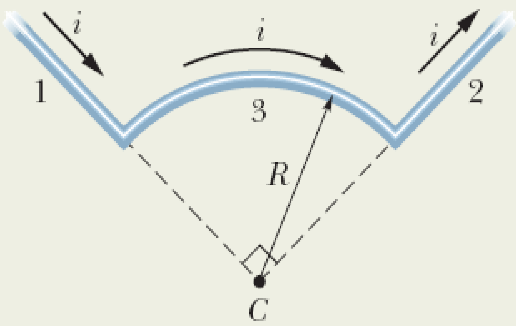
Calculating the magnetic field due to a current

The wire in Fig. 29-7*a* carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at C?



(*a*)

The wire in Fig. 29-7a carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at C?



(a)

Straight sections: For any current-length element in section 1, the angle θ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-7b); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C:

$$B_1 = 0.$$

The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle ϕ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center C is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

To find the direction of $\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-4. Mentally grasp the circular arc with your right hand as in Fig. 29-7c, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point *into the plane* of the page. Thus, $\vec{B}_3$ is directed into that plane.

Net field: Generally, when we must combine two or more magnetic fields to find the net magnetic field, we must combine the fields as vectors and not simply add their magnitudes. Here, however, only the circular arc produces a magnetic field at point C. Thus, we can write the magnitude of the net field $\vec{B}$ as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

Calculating the magnetic field due to a current

Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.

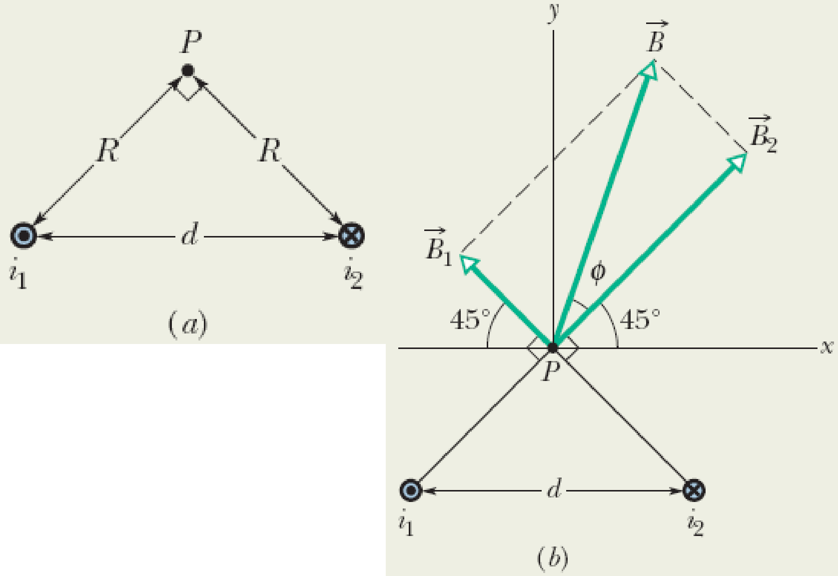
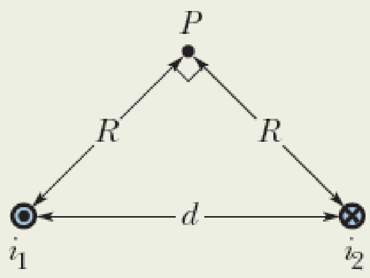
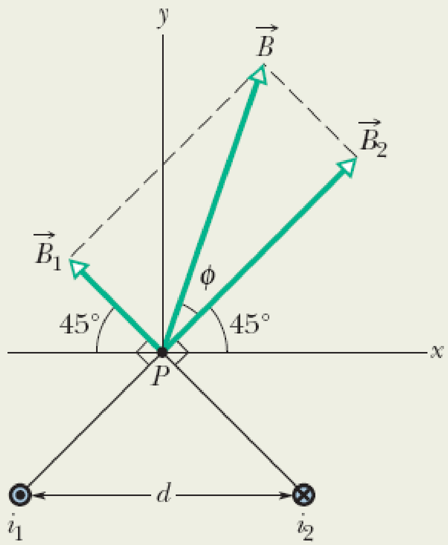


Figure 29-8a shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.



(a)



(b)

The two currents create magnetic fields that must be added as vectors to get the net field.

Finding the vectors: In Fig. 29-8a, point P is distance R from both currents i_1 and i_2. Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8a, note that the base angles (between sides R and d) are both 45°. This allows us to write $\cos 45° = R/d$ and replace R with $d \cos 45°$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point P, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-8b, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean theorem then gives us

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)}\sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})\sqrt{(15\ \mathrm{A})^2 + (32\ \mathrm{A})^2}}{(2\pi)(5.3 \times 10^{-2}\ \mathrm{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\ \mathrm{T} \approx 190\ \mu\mathrm{T}. \qquad \text{(Answer)}$$

The angle ϕ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-8b follows from

$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\ \mathrm{A}}{32\ \mathrm{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in Fig. 29-8b is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$

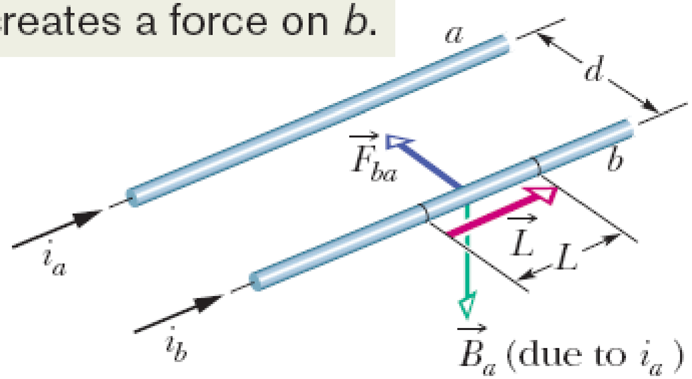
Force between two parallel currents

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

The field due to *a* at the position of *b* creates a force on *b*.



To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Parallel currents attract each other, and antiparallel currents repel each other.

Force between two parallel currents

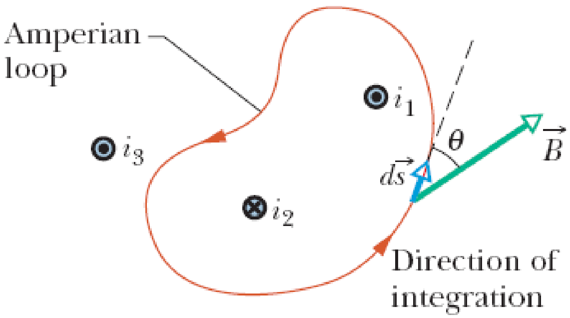
CHECKPOINT 1 The figure here shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. Rank the wires according to the magnitude of the force on each due to the currents in the other two wires, greatest first.

Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).}$$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_o}$$

Only the currents encircled by the loop are used in Ampere's law.



This is how to assign a sign to a current used in Ampere's law.

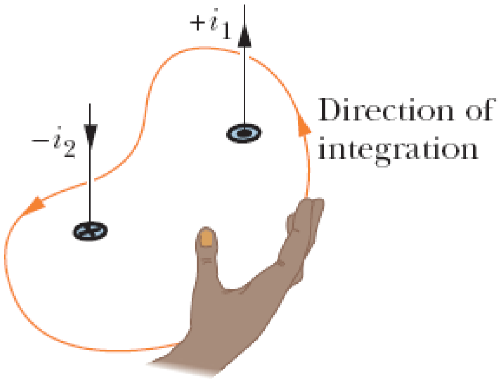


Fig. 29-12 A right-hand rule for Ampere's law, to determine the signs for currents encircled by an Amperian loop. The situation is that of Fig. 29-11.

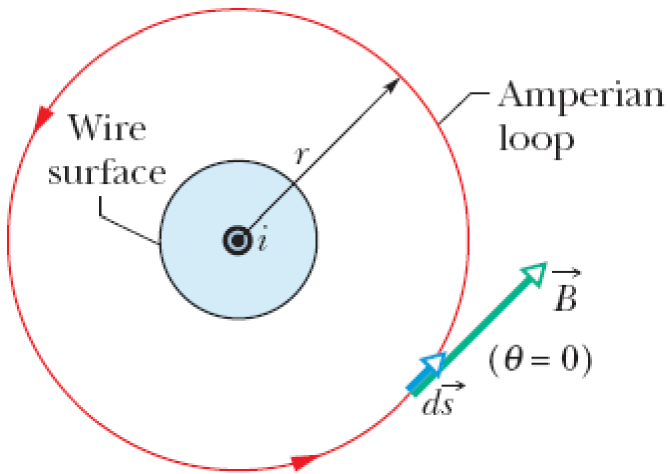
Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Ampere's Law

Ampere's Law, Magnetic Field Outside a Long Straight Wire Carrying Current:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).}$$

All of the current is encircled and thus all is used in Ampere's law.

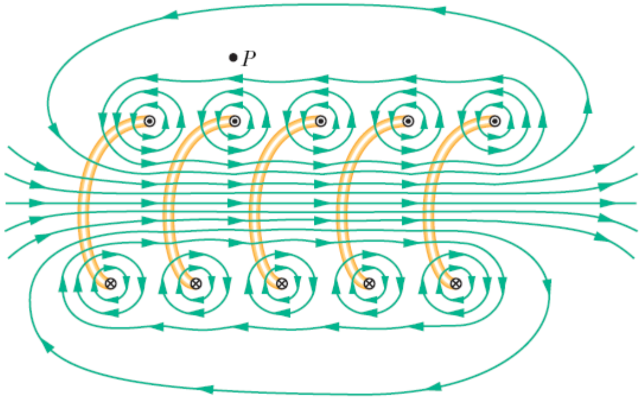


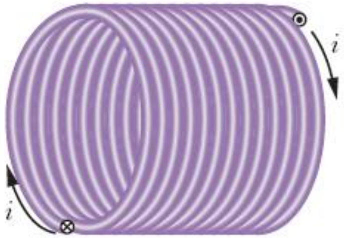
$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r).$$

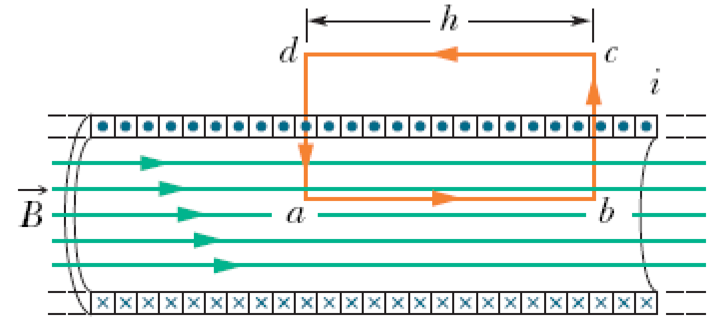
$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{(outside straight wire).}$$

Solenoid







Each turn produces circular magnetic field lines near itself. Near the solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc},$$

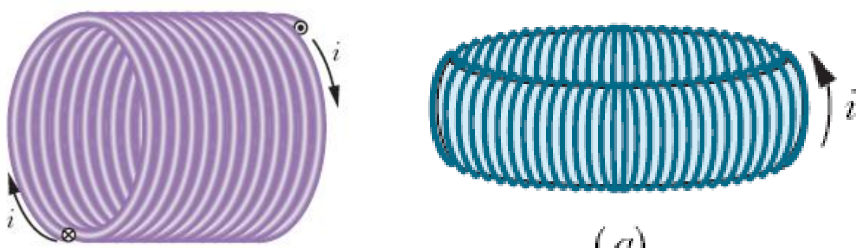
$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

$$i_{enc} = i(nh).$$

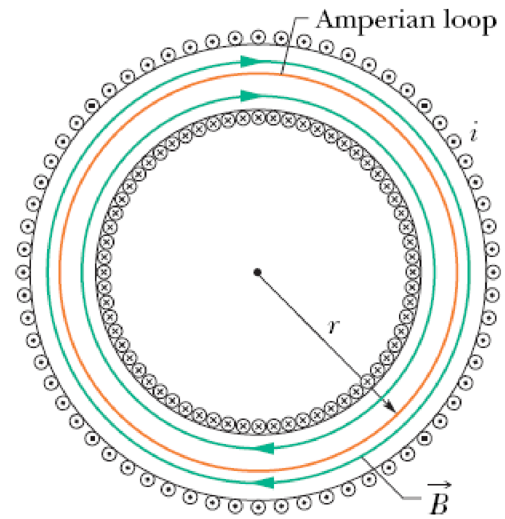
$$Bh = \mu_0 inh$$

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$

Toroids



(a)



(b)

$$(B)(2\pi r) = \mu_0 i N,$$

where *i* is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and *N* is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \quad \text{(toroid).}$$

Solenoid

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

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A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in = (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(5.57\ \mathrm{A})\frac{5 \times 850\ \text{turns}}{1.23\ \mathrm{m}}$$

$$= 2.42 \times 10^{-2}\ \mathrm{T} = 24.2\ \mathrm{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.

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