Behavior of Traveling Waves at Junctions

Note Title 2/19/2014

* Traveling waves (V_1, I_1) at junctions (discontinuity) are divided into two waves:
    1. Reflected Waves (Backward) (V_2, I_2)
    2. Refracted Transmitted waves (Forward) (V_3, I_3)

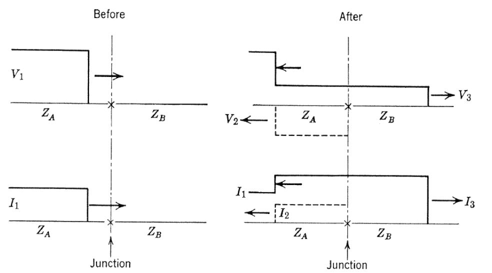
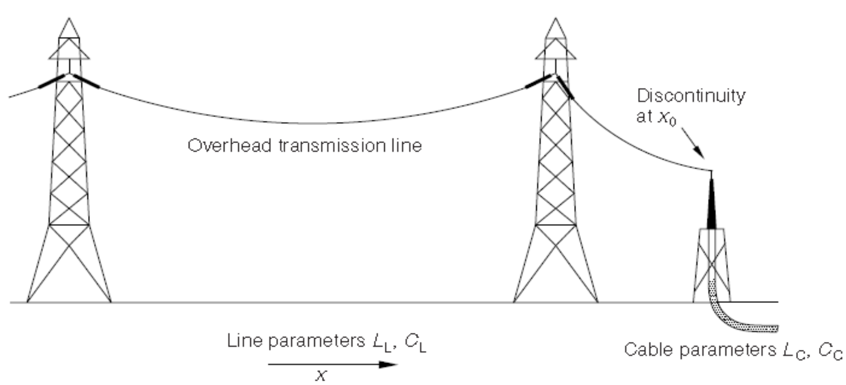


Fig. 9.7. Voltage and current waves being reflected and refracted at a junction between two lines.

* Junction (discontinuity): is a point where the characteristic impedance (Z) is unequal on the juction sides (e.g. $Z_A \neq Z_B$)



$$I_1 = \frac{V_1}{Z_A} \quad , \quad I_2 = -\frac{V_2}{Z_A} \quad , \quad I_3 = \frac{V_3}{Z_B}$$

* If the current and the voltage are contineous at the junction:

$$V_3 = V_1 + V_2 \quad \& \quad I_3 = I_1 + I_2$$

$$\frac{V_3}{Z_B} = \frac{V_1}{Z_A} - \frac{V_2}{Z_A}$$

$$V_2 \left(\frac{Z_B + Z_A}{Z_B Z_A}\right) = V_1 \left(\frac{Z_B - Z_A}{Z_B Z_A}\right) \Longrightarrow V_2 = V_1 \left(\frac{Z_B - Z_A}{Z_B + Z_A}\right) = a V_1$$

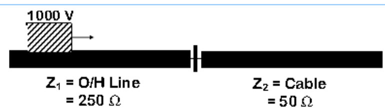
where a is the reflection coefficient , $-1 \leqslant a \leqslant 1$

$$V_3 \left(\frac{Z_B + Z_A}{Z_B Z_A}\right) = \frac{2V_1}{Z_A}$$

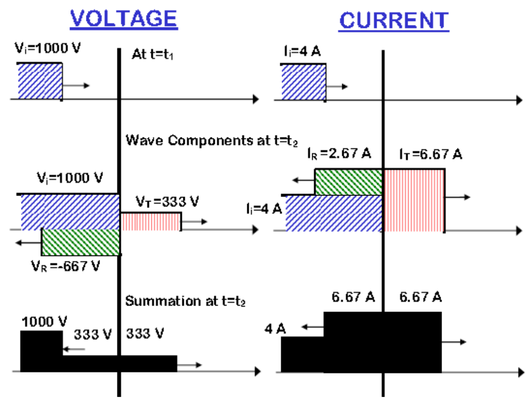
$$\Rightarrow V_3 = \left(\frac{2 Z_B}{Z_B + Z_A}\right) V_1 = b V_1$$

where b is the refraction coefficient , $a = 1 + b$

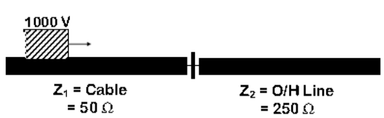
* Termination $(Z_1 > Z_2)$



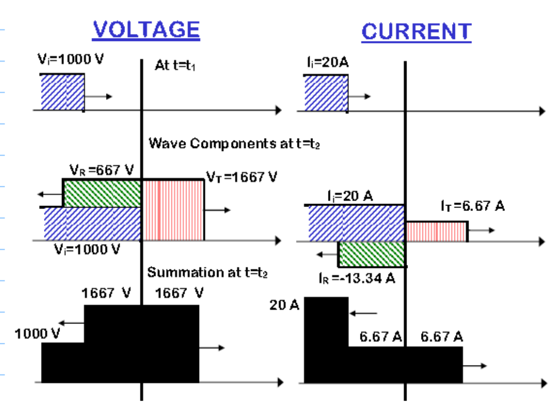
$$\alpha = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{50 - 250}{250 + 50} = -0.667 \qquad 1 + \alpha = \frac{2Z_2}{Z_1 + Z_2} = \frac{2x50}{250 + 50} = 0.333$$



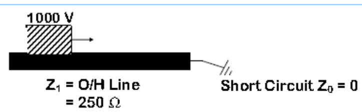
* Termination $(Z_1 < Z_2)$:



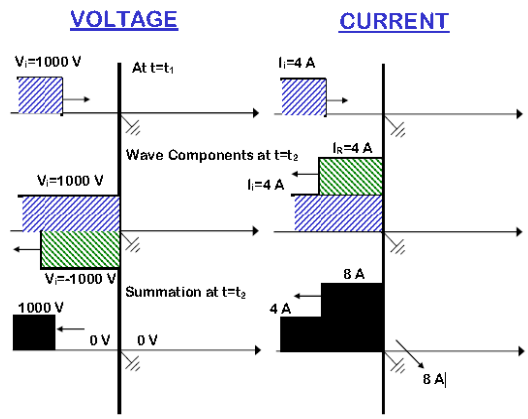
$$Q = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{250 - 50}{50 + 250} = 0.667 \qquad 1 + \alpha = \frac{2Z_2}{Z_1 + Z_2} = \frac{2x250}{50 + 250} = 1.667$$



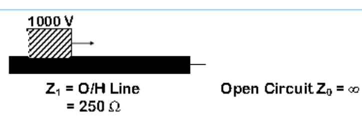
* Termination into a short circuit ($Z_0 = 0$)



$a = -1 \qquad 1 + a = 0$



* Termination into open circuit ($Z_o = \infty$)



$a = 1 \quad , \quad b = 2$