

Chapter 3

VECTORS

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Physical Quantities

Vector Quantities

magnitude

+

direction



Follow certain rules
of addition and
multiplication

Scalar Quantities

+Ve Number

-Ve Number



Follow the rules of
ordinary algebra

Vectors Addition

Adding Vectors Geometrically

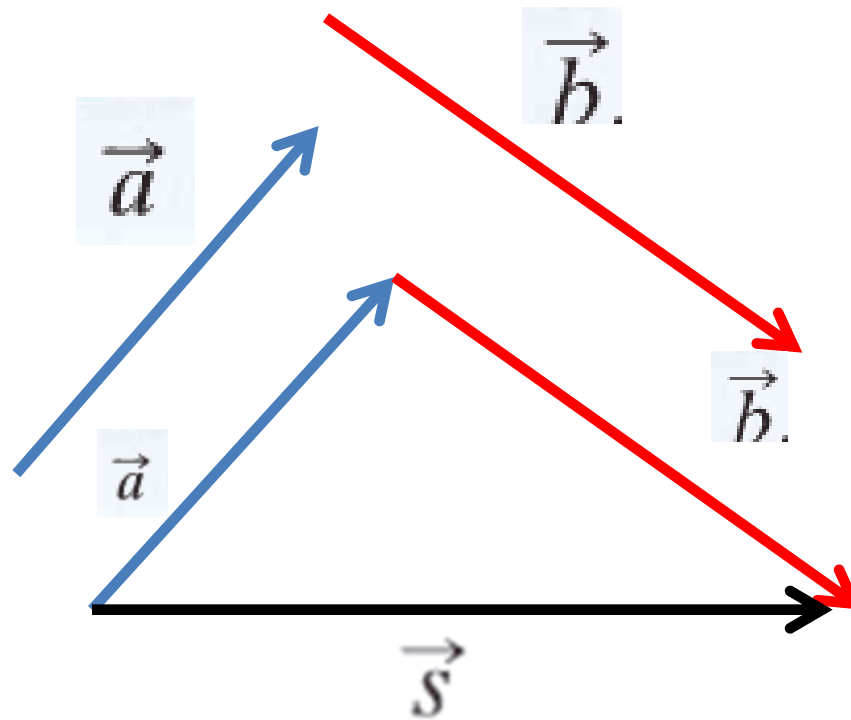
- Vector equation
- Commutative Law
- Associative Law
- Vector Subtraction

Adding Vectors by Components

- Components
- resolving the vector
- writing a vector in magnitude- angle notation

- Unit Vectors
- writing a vector in Unit vector notation

Adding Vectors Geometrically

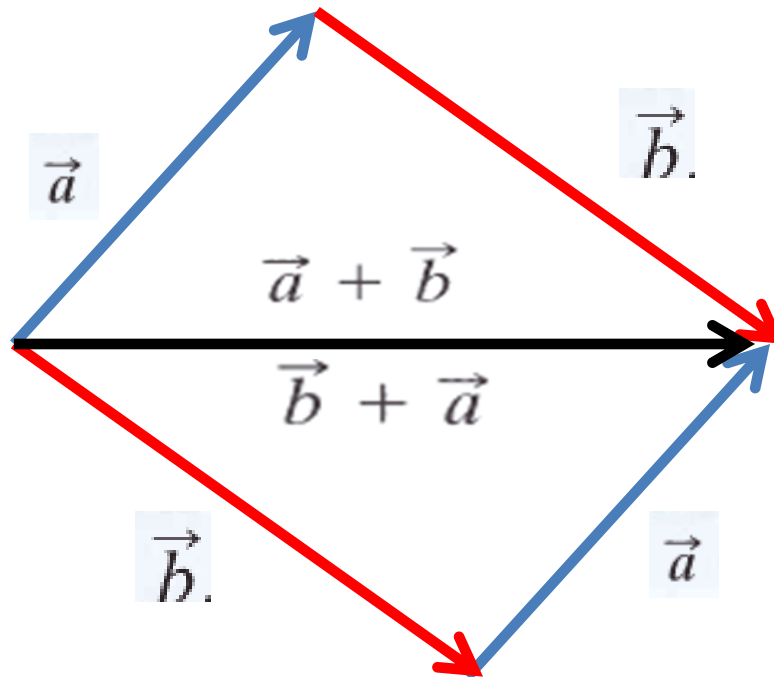


• Vector equation



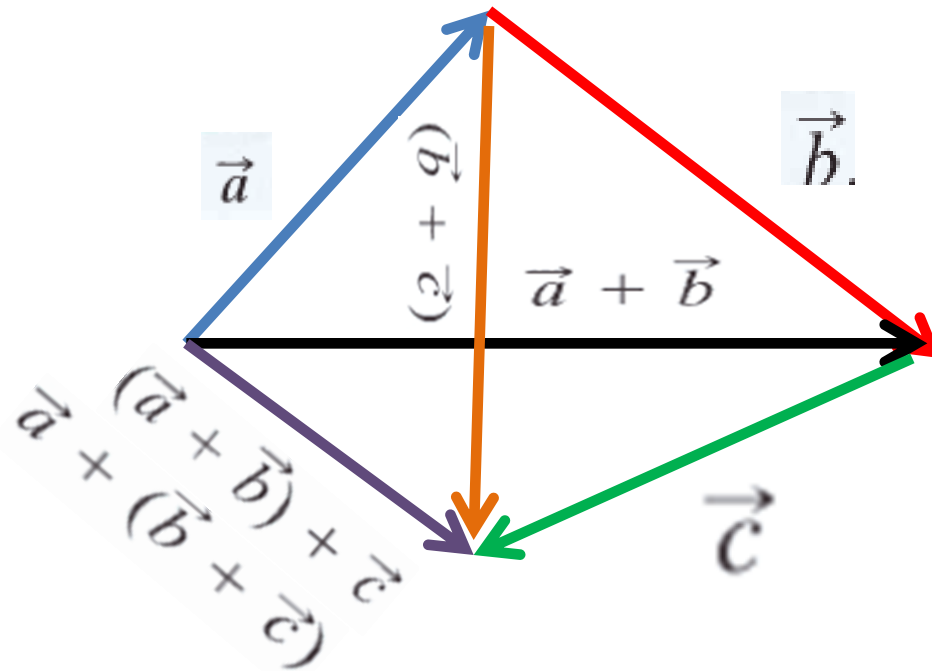
$$\vec{s} = \vec{a} + \vec{b},$$

- Commutative Law



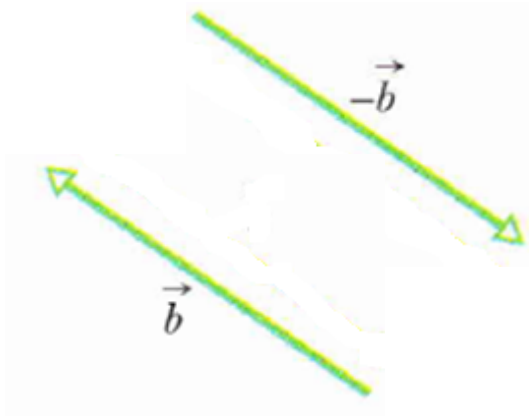
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

- Associative Law

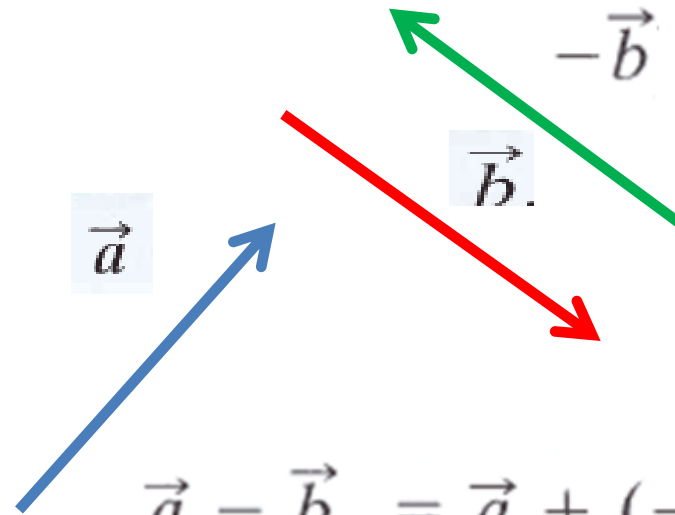


$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

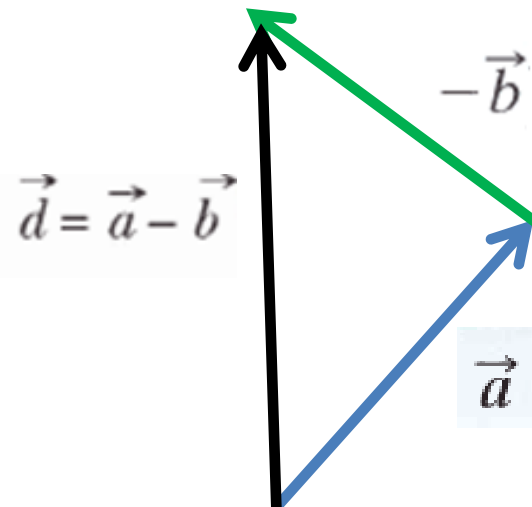
• Vector Subtraction



$$\vec{b} + (-\vec{b}) = \mathbf{0}$$



$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

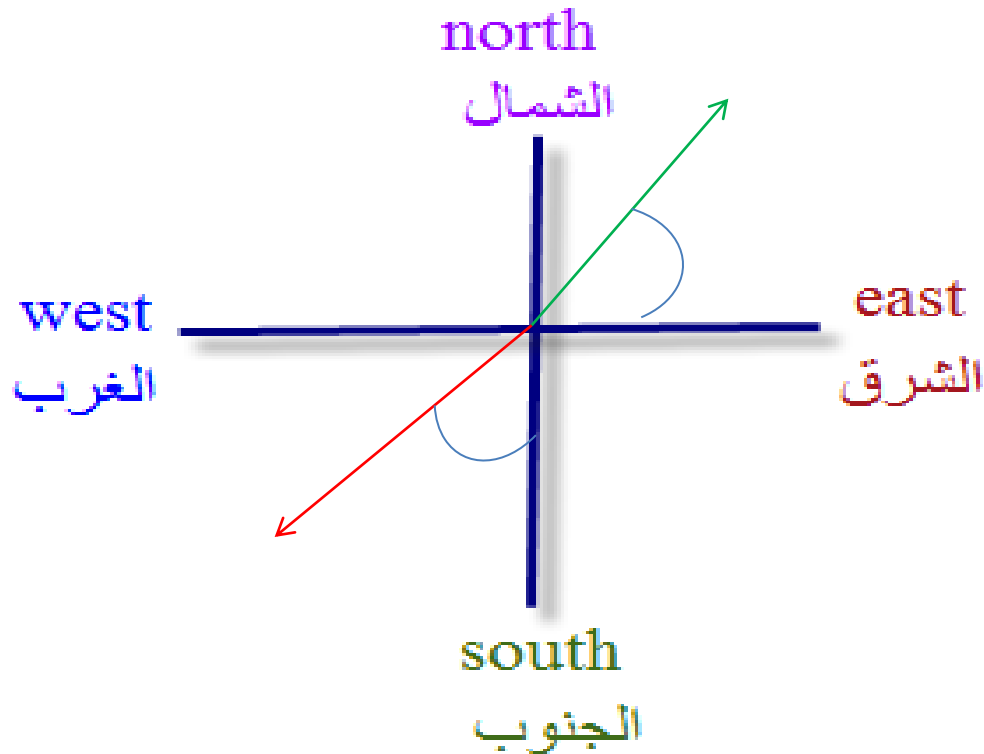


$$\vec{d} = \vec{a} - \vec{b}$$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order: (a) \vec{a} , 2.0 km due east (directly toward the east); (b) \vec{b} , 2.0 km 30° north of east (at an angle of 30° toward the north from due east); (c) \vec{c} , 1.0 km due west. Alternatively, you may substitute either $-\vec{b}$ for \vec{b} or $-\vec{c}$ for \vec{c} . What is the greatest distance you can be from base camp at the end of the third displacement?





North of **east** = toward the north from due **east**

West of **south** = toward the west from due **south**

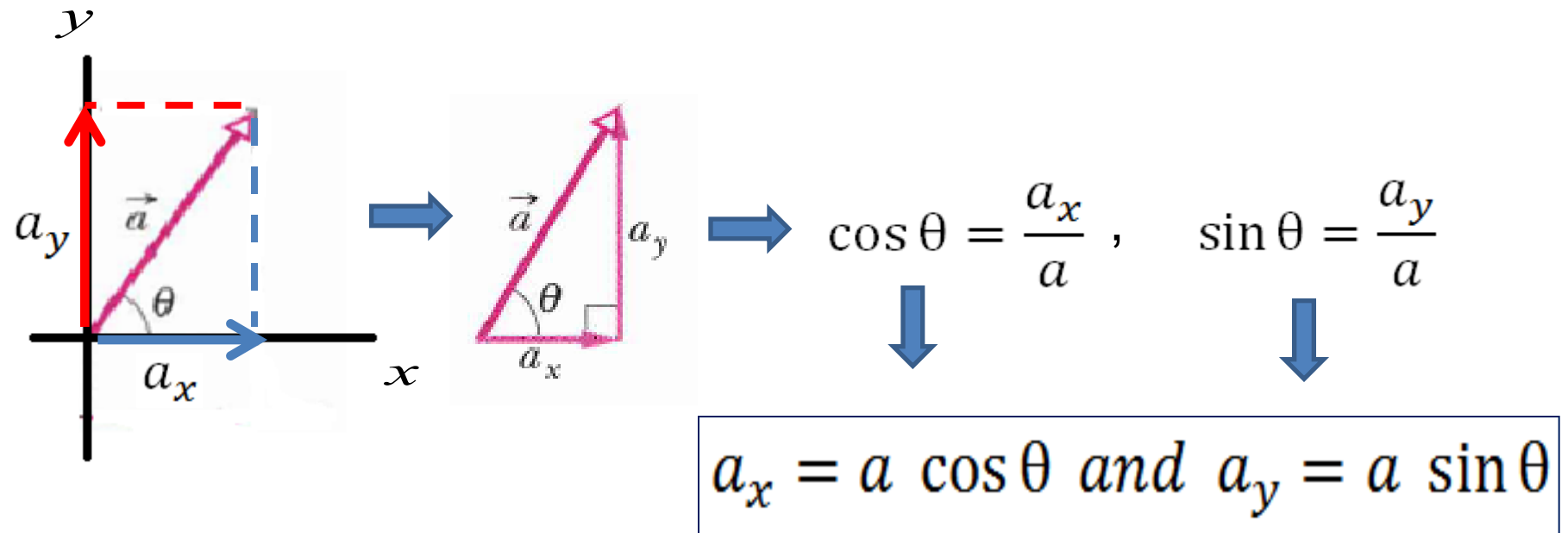


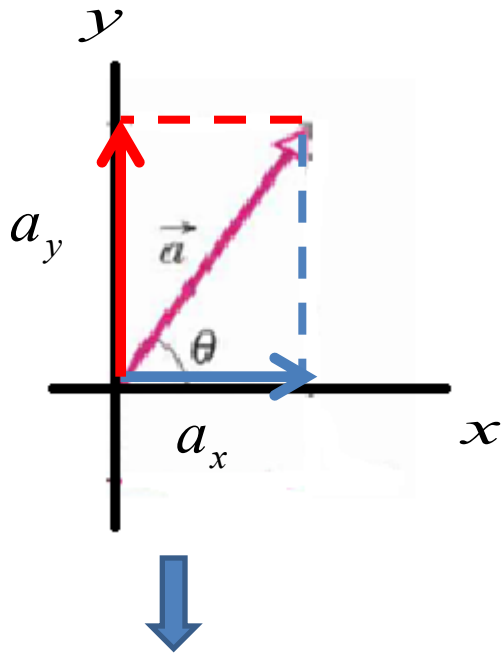
Components of Vectors

- Resolving the vector is the process of finding the components

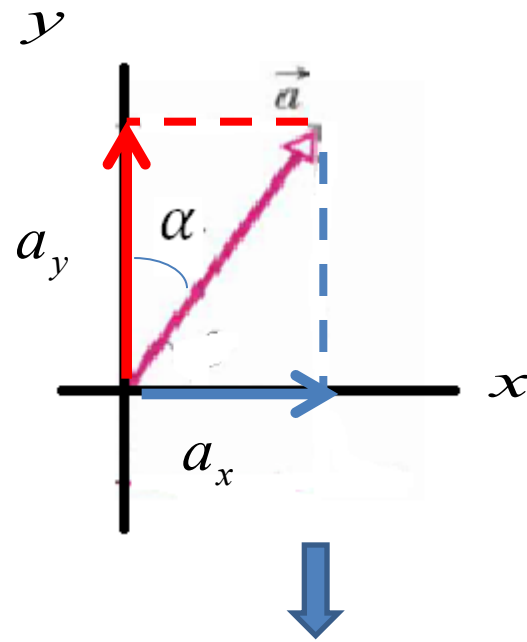


- Component is the projection of the vector on an axis





$$a_x = a \cos \theta \text{ and } a_y = a \sin \theta$$



$$a_x = a \sin \alpha \text{ and } a_y = a \cos \alpha$$

- Writing a vector in magnitude- angle notation

$\vec{a} : a_x \text{ and } a_y$

Magnitude

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

Angle (Direction)

$$\tan \theta = \frac{a_y}{a_x} \Rightarrow \theta = \tan^{-1} \frac{a_y}{a_x}$$

\vec{a}

a and θ

a_x and a_y

• Finding the components.

• Writing a vector in magnitude-angle notation

$$a_x = a \cos \theta$$

$$a_y = a \sin \theta$$

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$\tan \theta = \frac{a_y}{a_x}$$

Rem : When use these formulas to find the components, the angle must be measured from positive X-axis, if clockwise put θ -ve if counterclockwise put θ +ve.

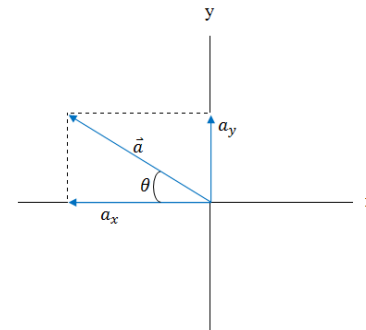
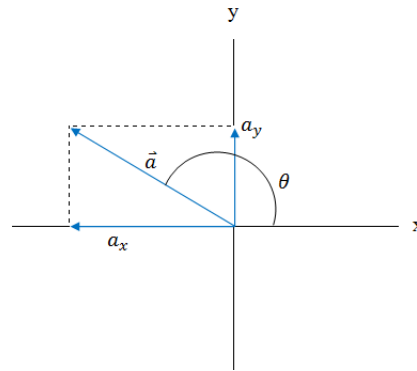
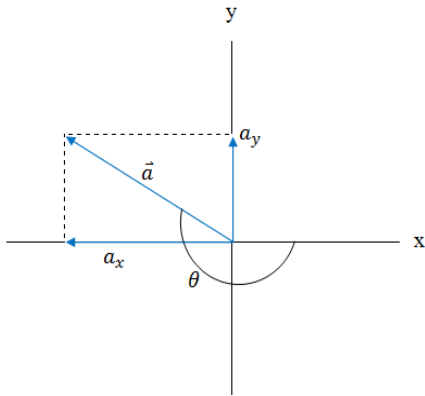
How to find the components of a vector in different positions?

When the angle is from the +ve x-axis

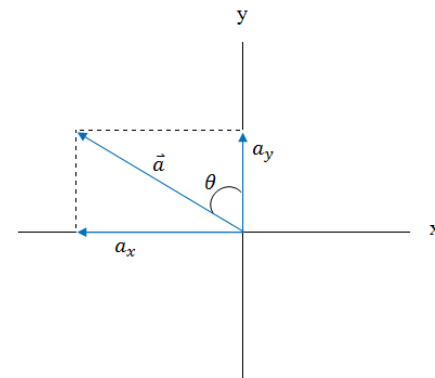
When the angle is from any different axis

Clockwise

Counter-clockwise



$$a_x = -a \cos \theta$$
$$a_y = +a \sin \theta$$



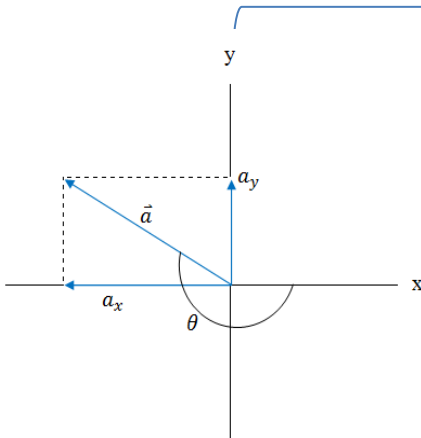
$$a_x = -a \sin \theta$$
$$a_y = +a \cos \theta$$

$$a_x = a \cos -\theta$$
$$a_y = a \sin -\theta$$

$$a_x = a \cos \theta$$
$$a_y = a \sin \theta$$

To find the components, when the angle is measured from the +ve x-axis use

$$a_x = a \cos \theta$$
$$a_y = a \sin \theta$$

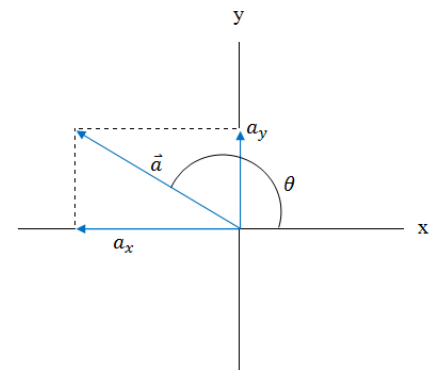


Put $\theta = -ve$, When the angle is measured clockwise from the +ve x-axis



$$a_x = a \cos -\theta$$

$$a_y = a \sin -\theta$$



Put $\theta = +ve$, When the angle is measured Counter-clockwise from the +ve x-axis

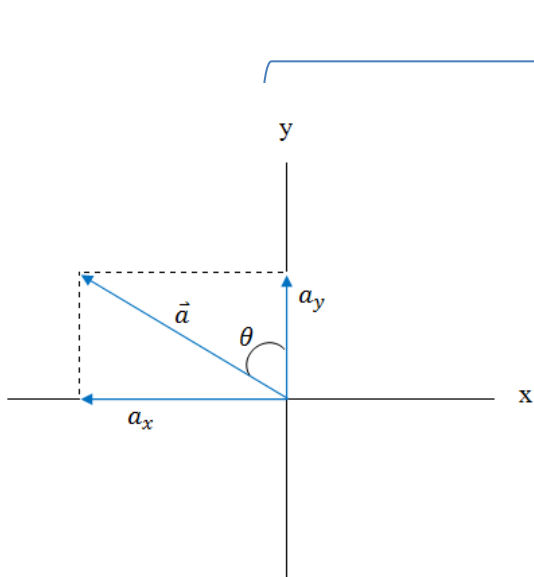


$$a_x = a \cos \theta$$

$$a_y = a \sin \theta$$

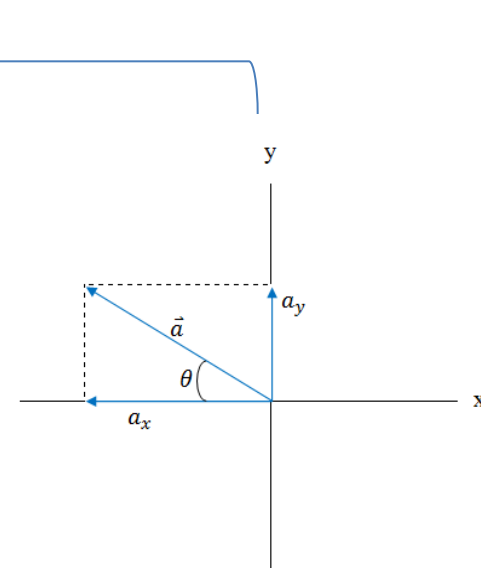
To find the components , when the angle is measured from any axis even the +ve x-axis

- 1- Take the given angle with the axis
- 2- put the signs of the components according to their positions on the axes
- 3- put sine or cosine according to the angle position.



$$a_x = - a \sin \theta$$

$$a_y = + a \cos \theta$$



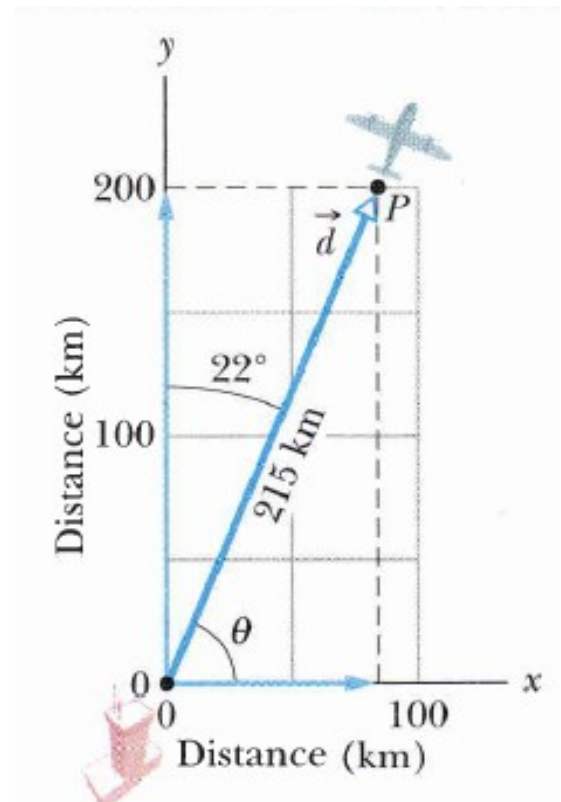
$$a_x = - a \cos \theta$$

$$a_y = + a \sin \theta$$

Sample Problem

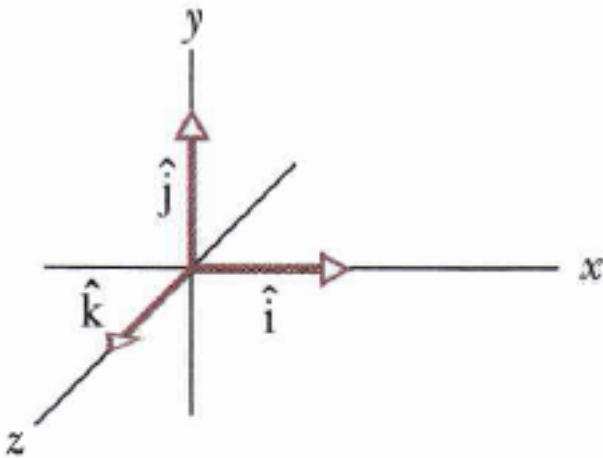
3-2

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



Unit Vectors

- Unit vector is a vector of magnitude 1 and points in a particular direction



- Writing a vector in Unit vector notation



$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Vector Components

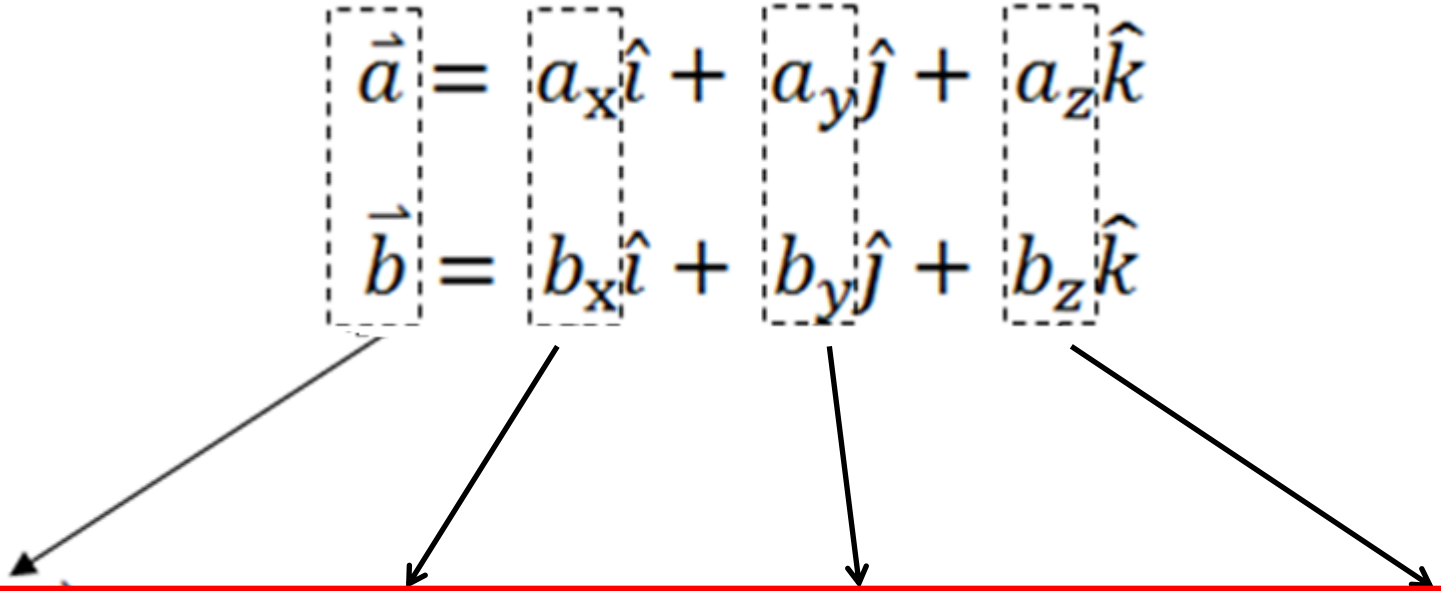
$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Scalar components

Adding vectors by Components

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\begin{array}{l} \vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \\ \vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k} \end{array}$$


$$\vec{a} + \vec{b} = (a_x + b_x) \hat{i} + (a_y + b_y) \hat{j} + (a_z + b_z) \hat{k}$$

Sample Problem

3-4

Figure 3-16a shows the following three vectors:

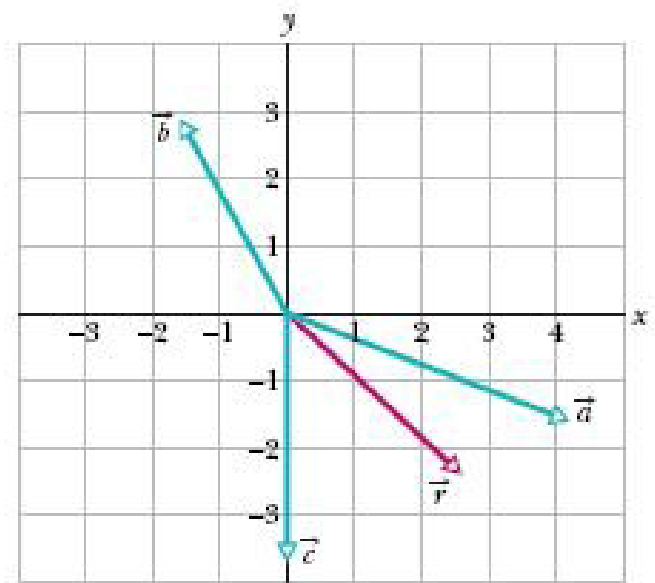
$$\vec{a} = (4.2 \text{ m})\hat{i} - (1.5 \text{ m})\hat{j},$$

$$\vec{b} = (-1.6 \text{ m})\hat{i} + (2.9 \text{ m})\hat{j},$$

and

$$\vec{c} = (-3.7 \text{ m})\hat{j}.$$

What is their vector sum \vec{r} which is also shown?



Vectors Multiplication

Multiplying a vector by a scalar

+ve scalar

will produce a new vector in the same direction as the started vector

$$\vec{a} = 2\hat{i} + 3\hat{j}$$
$$2\vec{a} = 4\hat{i} + 6\hat{j}$$

-ve scalar

will produce a new vector in the opposite direction of the started vector

$$\vec{a} = 2\hat{i} + 3\hat{j}$$
$$-2\vec{a} = -4\hat{i} - 6\hat{j}$$

Multiplying a vector by a vector

Scalar product
(or Dot product)

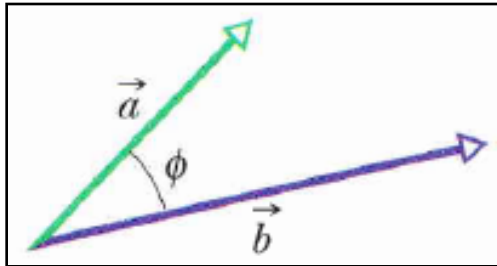
will produce a scalar

Vector product
(or cross product)

will produce a new vector

Scalar (or Dot product)

If the two vectors are given in magnitude and the angle between them



$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

If the two vectors are given in unit vector notation

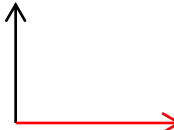
$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$
$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

1- The scalar product is commutative $\Rightarrow \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

2- If the two vectors are parallel $\Rightarrow \theta = 0 \Rightarrow \vec{a} \cdot \vec{b} = ab$ 

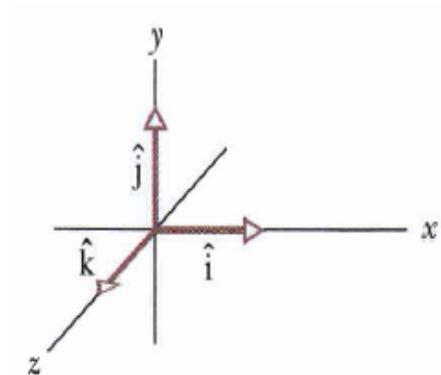
3- If the two vectors are perpendicular $\Rightarrow \theta = 90 \Rightarrow \vec{a} \cdot \vec{b} = 0$ 

4- If the two vectors are Antiparallel $\Rightarrow \theta = 180 \Rightarrow \vec{a} \cdot \vec{b} = -ab$ 

5- Multiplying Unit vectors

$$\hat{i} \cdot \hat{i} = (1)(1) \cos 0 = 1 \quad \Rightarrow \quad \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = (1)(1) \cos 90 = 0 \quad \Rightarrow \quad \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$



The scalar product is commutative

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

the angle between two vectors can be found

$$\vec{a} \cdot \vec{b} = ab \cos \theta$$

any two similar unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

any two different unit vectors

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

Properties Of the scalar product

If $\theta = 0 \Rightarrow \vec{a} \cdot \vec{b} = ab \Rightarrow$ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} \cdot \vec{b} = -ab \Rightarrow$ vectors are anti parallel

$\theta = 90 \Rightarrow \vec{a} \cdot \vec{b} = 0 \Rightarrow$ vectors are perpendicular

Sample Problem**3-7**

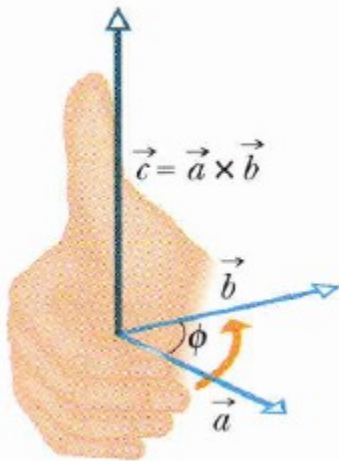
What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$?

Vector (or Cross product)

If the two vectors are given in magnitude and angle between them

$$|\vec{a} \times \vec{b}| = |c| = ab \sin \phi$$

The direction of the result vector



If the two vectors are given in unit vector notation

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \\ &= \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \\ &= (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} \\ &\quad + (a_x b_y - b_x a_y) \hat{k} \end{aligned}$$

$$|\vec{a} \times \vec{b}| = |c| = ab \sin \phi$$

1- The vector product is Anti-commutative $\Rightarrow \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

2- If the two vectors are parallel $\Rightarrow \theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$ 

3- If the two vectors are perpendicular $\Rightarrow \theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = ab$ 

4- If the two vectors are Anti-parallel $\Rightarrow \theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$ 

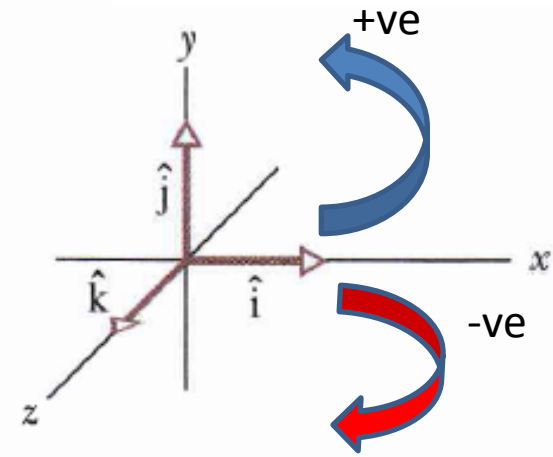
5- Multiplying Unit vectors

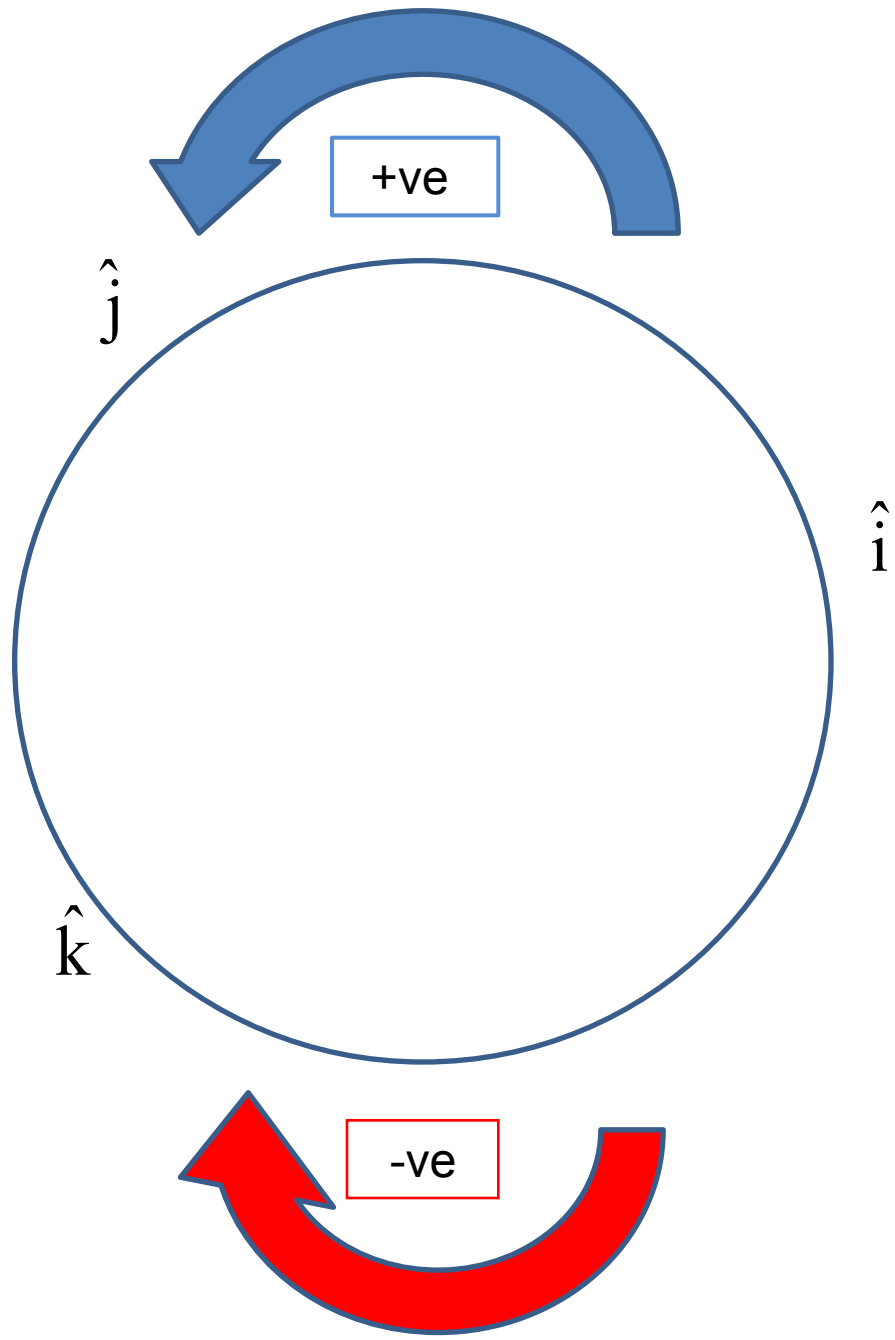
$$|\hat{i} \times \hat{i}| = (1)(1) \sin 0 = 0 \Rightarrow \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$|\hat{i} \times \hat{j}| = (1)(1) \sin 90 = 1 \Rightarrow \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \quad \hat{i} \times \hat{k} = -\hat{j}$$





Anti- commutative

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

any two different
unit vectors

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \\ \hat{k} \times \hat{i} = \hat{j}$$

any two similar unit
vectors

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

Properties of the Vector product

The small angle
between the two
vectors must be used
because the odd
property of the sin
function

$$|\vec{a} \times \vec{b}| = |c| = ab \sin \phi$$

If $\theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$ \Rightarrow vectors are parallel
 $\theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$ \Rightarrow vectors are anti parallel
 $\theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = ab$ \Rightarrow vectors are perpendicular

Sample Problem**3-9**

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

The End