

Complex Numbers Review

1. ($j \triangleq \sqrt{-1}$) is an imaginary number; it is the base of complex numbers.
2. a complex number has a *real* part and an *imaginary* part as: $(x + j y)$

1. Addition

$$(a + j b) \pm (c + j d) = (a \pm c) + j (b \pm d)$$

Examples:

1. $(2 + j 3) + (4 + j 5) = (6 + j 8)$
2. $(2 + j 3) - (4 + j 5) = (-2 - j 2)$
3. $(2\pi + j \sqrt{3}) - (-4 + j 5 \sin(\frac{\pi}{4})) = (2\pi + 4) + j (\sqrt{3} - 5 \sin(\frac{\pi}{4})) \cong 10.28 - j 1.80$

2. Multiplication

$$(a + j b) \times (c + j d) = (ac - bd) + j (bc + ad)$$

note: $j^2 = -1$

Examples:

1. $(2 + j 3) \times (4 + j 5) = (8 - 15) + j(12 + 10) = -7 + j 22$
2. $(2\pi + j \sqrt{3}) \times (-4 + j 5 \sin(\frac{\pi}{4})) \cong (6.28 + j 1.73) \times (-4 + j 3.54) = -31.24 + j 15.31$

3. Polar Format

$$(x + j y) \rightarrow \begin{cases} \sqrt{x^2 + y^2} \angle \tan^{-1}(\frac{y}{x}) & x \geq 0 \\ \sqrt{x^2 + y^2} \angle 180 + \tan^{-1}(\frac{y}{x}) & x < 0 \end{cases}$$
$$a \angle \theta \rightarrow a \cos \theta + j a \sin \theta$$

Examples:

1. $(2 + j 3) \rightarrow \sqrt{2^2 + 3^2} \angle \tan^{-1} \frac{3}{2} = 3.61 \angle 56.31^\circ$
2. $5 \angle 30^\circ \rightarrow 5 \cos 30 + j 5 \sin 30 = 4.33 + j 2.5$

4. Division

$$\frac{(a + j b)}{(c + j d)} = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}} \angle \left(\tan^{-1} \left(\frac{b}{a} \right) - \tan^{-1} \left(\frac{d}{c} \right) \right)$$

5. Exercises

$$\begin{aligned}a &= 3 + j\,5 \\b &= 8 - j\,7 \\c &= -2 + j\,3 \\d &= -10 - j\,4 \\e &= 15\angle 30^\circ\end{aligned}$$

Calculate the followings:

1. $a + b$
2. $c \times d$
3. $(a + c) \div (b - d)$
4. polar forms of a , b , c , and d
5. Cartesian form of e
6. $(e - c) \times \frac{a}{d}$

Answers:

1. $11 - j\,2$
2. $32 - j\,22$
3. $-0.018 + j\,0.44$
4. $a = 5.83\angle 59.04^\circ$, $b = 10.63\angle -41.19^\circ$, $c = 3.61\angle 123.7^\circ$, $d = 10.77\angle -158.2^\circ$
5. $12.99 + j\,7.5$
6. $-4.99 - j\,6.85$