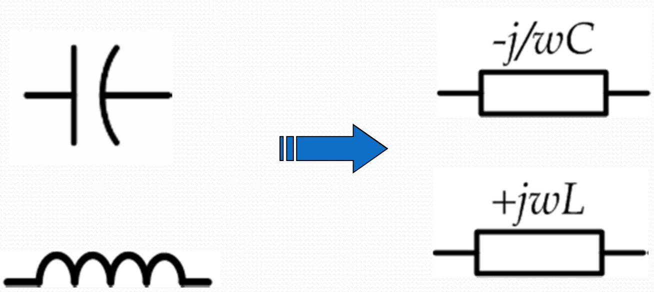
Complex Impedance

Section 05

Impedance

- treat all passive components as resistors
  - but with *complex* resistances



Impedance

- What is the impedance of a 10μF capacitor when operated at 60Hz?

$$Z_C = -\frac{j}{wC} = -\frac{j}{2\pi \times 60 \times 10 \times 10^{-6}} = -j265.25\Omega$$

- What is the impedance of a 2mH inductor when operated at 60Hz?

$$Z_L = jwL = j2\pi \times 60 \times 2 \times 10^{-3} = +j0.754\Omega$$

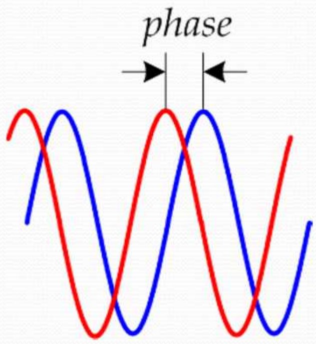
Laplace Domain

- When
  - Mixing AC and DC sources
  - Multiple different frequencies

- use Laplace instead of Fourier
  - *jw* → *s*
  - *Initial conditions*

Complex AC Source

- AC Volt or Current has:
  - Amplitude
  - Frequency
  - Phase



- Phase can be expressed in Complex Number

$$A\cos(2\pi f \cdot t + \varphi) \quad \rightarrow \frac{A}{\sqrt{2}}\angle\varphi$$

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Complex AC Source

$$A\cos(2\pi f \cdot t + \varphi) \quad \rightarrow$$

$$\underbrace{\frac{A}{\sqrt{2}}\angle\varphi}_{polar} \qquad \underbrace{\frac{A}{\sqrt{2}}\cdot(\cos\varphi + j\sin\varphi)}_{rectangular}$$

$$110\angle 30^\circ \quad \rightarrow 110\cdot(\cos 30^\circ + j\sin 30^\circ)$$

$$= 95.26 + j55$$

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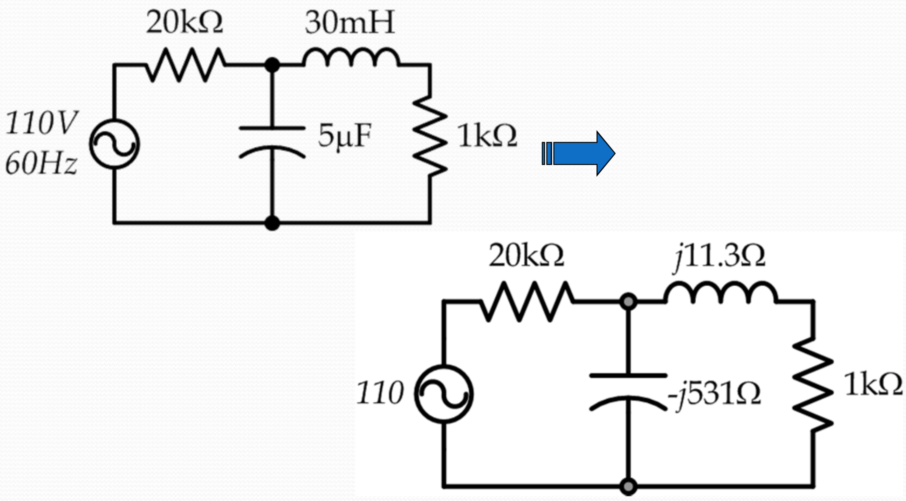
Example



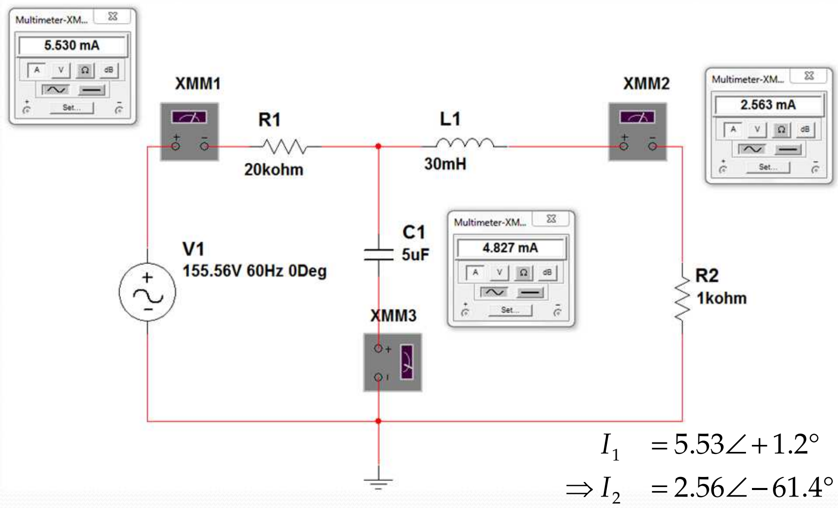
Solution

$$KCL: \quad I_1 = I_2 + I_3$$

$$KVL: \begin{aligned} -110 + 20{,}000 \times I_1 - j531 \times I_3 &= 0 \\ + j531 \times I_3 + j11.3 \times I_2 + 1000 \times I_2 &= 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} I_1 &= 5.44\angle +1.2^\circ \quad mA \\ I_2 &= 2.56\angle -61.4^\circ \quad mA \\ I_3 &= 4.83\angle +29.3^\circ \quad mA \end{aligned}$$

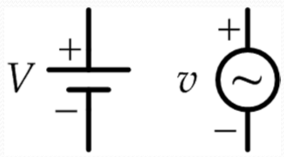
Simulation Solution



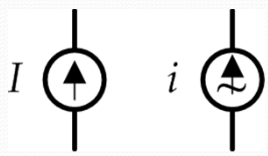
$$I_1 = 5.53\angle{+1.2^\circ} \quad mA$$
$$\Rightarrow I_2 = 2.56\angle{-61.4^\circ} \quad mA$$
$$I_3 = 4.83\angle{+29.3^\circ} \quad mA$$

Current Source

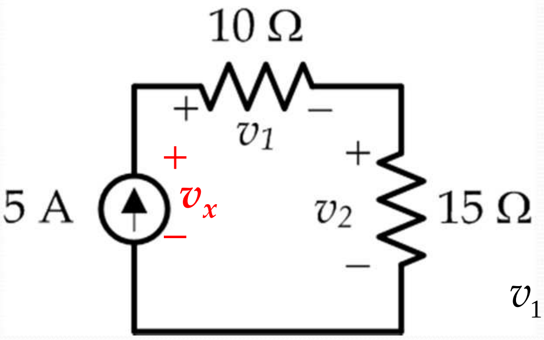
- Voltage Source
  - Generates constant volt regardless of the load



- Current Source
  - Generates constant current regardless of the load



KVL & KCL

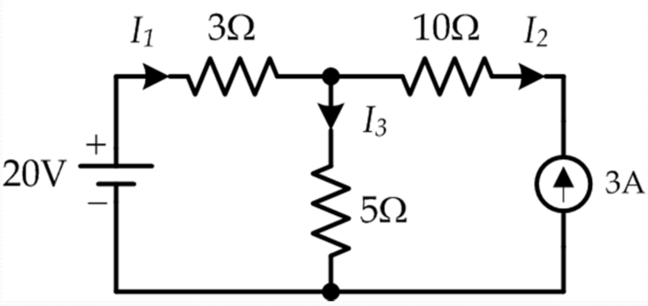


$$v_1 = 5 \times 10 = 50V$$

$$v_2 = 5 \times 15 = 75V$$

$$v_x = v_1 + v_2$$

Example



$$KCL: \quad I_1 = I_2 + I_3$$

$$KVL: \begin{array}{ll} -20 + 3I_1 + 5I_3 & = 0 \\ -5I_3 + 10I_2 + v_i & = 0 \end{array} \Rightarrow \begin{array}{lll} I_1 & = 0.625 & A \\ I_2 & = -3 & A \\ I_3 & = 3.625 & A \\ v_i & = 48.125 & V \end{array}$$

$$IS: \quad I_2 = -3A$$

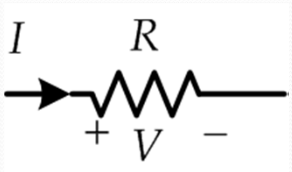
Power

- Power = rate of energy transfer
  - measured in Watts (W)

$$P = I \cdot V$$

$$P = I \cdot V = I^2 \cdot R$$

$$P = I \cdot V = \frac{V^2}{R}$$



Instantaneous Power

- instantaneous power, $p(t)$

$$p(t) = v(t) \times i(t)$$

- for general AC signals:

$$\begin{aligned} v(t) &= V_p \cos\left(2\pi f \cdot t + \theta_V\right) \\ i(t) &= I_p \cos\left(2\pi f \cdot t + \theta_I\right) \end{aligned}$$

Average Power

$$P = \frac{1}{T}\int_0^T v(t)\times i(t)\,dt$$
$$= \frac{1}{T}\int_0^T V_p \cdot I_p \times \left[\cos(2\pi f \cdot t + \theta_V)\cdot \cos(2\pi f \cdot t + \theta_I)\right]dt$$
$$= \frac{1}{T}\int_0^T \frac{V_p \cdot I_p}{2} \times \left[\cos(\theta_V - \theta_I) + \cos(4\pi f \cdot t + \theta_V + \theta_I)\right]dt$$
$$= \frac{V_p \cdot I_p}{2T}\cos(\theta_V - \theta_I)\times \int_0^T \left[1 + \cos(4\pi f \cdot t + \theta_V + \theta_I)\right]dt$$
$$= \frac{V_p \cdot I_p}{2T}\cos(\theta_V - \theta_I)\times T$$
$$= V_{RMS} \cdot I_{RMS} \times \cos(\theta_V - \theta_I)$$

Real Power

- thus, the real power is:

$$P = V_{rms} \cdot I_{rms} \times pf$$

- where, the power factor is:

$$pf = \cos(\theta_V - \theta_I)$$

Reactive Power

- Define '*S*' as the Complex Power:

$$S = I^{*}.V = P + jQ$$

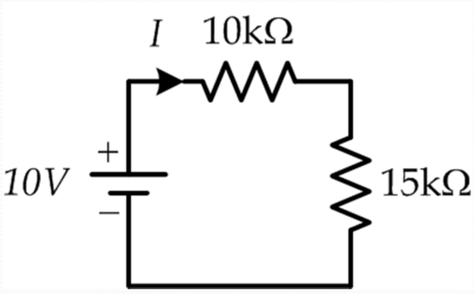
where (V) and (I) in complex form and RMS values

- Example:

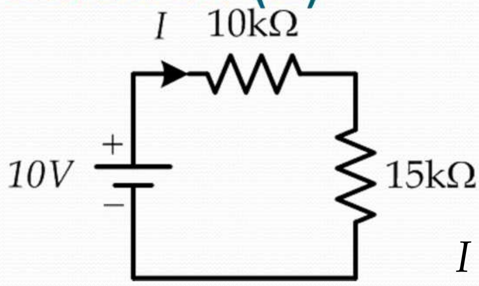
$$\begin{aligned} I &= 3 - j5 \\ V &= 9 - j7 \end{aligned} \quad \Rightarrow \quad \begin{aligned} S &= (3 + j5)\cdot(9 - j7) = 62 + j24 \\ P &= 62W \\ Q &= 24VAR \end{aligned}$$

Process Check

- Solve the circuit shown and find the power consumption of each component



Solution (1)



$$I = \frac{10}{10k + 15k} = 400\mu A$$

$$P_1 = I^2 \cdot R_1 = \left(400\times10^{-6}\right)^2 \cdot 10\times10^3 = 1.6mW$$

$$P_2 = I^2 \cdot R_2 = \left(400\times10^{-6}\right)^2 \cdot 15\times10^3 = 2.4mW$$

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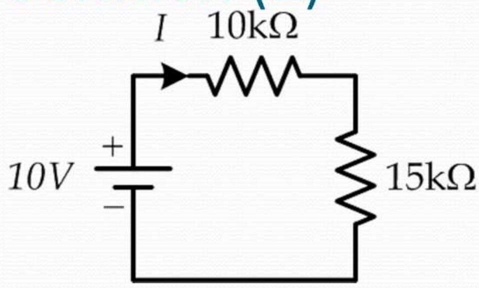
Solution (2)



$$V_1 = 10\frac{10k}{10k + 15k} = 4V$$

$$V_2 = 10\frac{15k}{10k + 15k} = 6V$$

$$P_1 = \frac{V_1^2}{R_1} = \frac{4^2}{10\times10^3} = 1.6mW$$

$$P_2 = \frac{V_2^2}{R_2} = \frac{6^2}{15\times10^3} = 2.4mW$$

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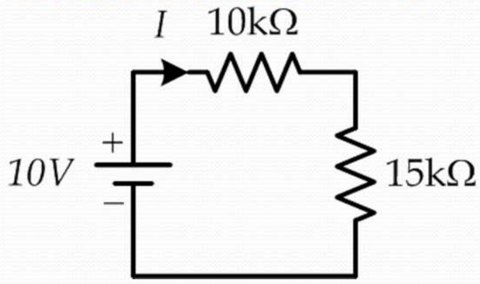
Total Power = 0

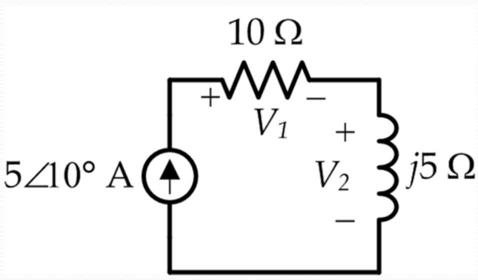


$$P_S = -I \cdot V = -400 \times 10^{-6} \cdot 10 = -4mW$$

$$P_1 + P_2 = 4mW$$

Total Power = Zero

Example

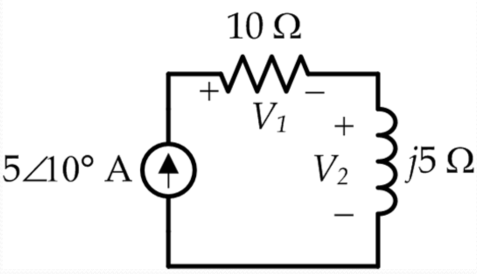


$$V_1 = 50\angle 10^\circ \quad V$$
$$V_2 = 25\angle 100^\circ \quad V$$

$$V_S = V_1 + V_2 = 56\angle 36.6^\circ \qquad V$$

$$S_S = -I_S^* \cdot V_S = -280\angle 26.6^\circ = -250 - j125 \qquad VA$$

Example



$$S_R = I_S^*.V_1 = (5\angle -10°)\times(50\angle 10°) = 250 + j0$$

$$S_L = I_S^*.V_2 = (5\angle -10°)\times(25\angle 100°) = 0 + j125$$

Total Power = Zero