Optimizing inventory decisions in a multi-stage supply chain under stochastic demands

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1. Introduction

Supply chain management can be defined as a set of approaches utilized to efficiently integrate suppliers, manufactures, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide cost while satisfying service level requirements [20].

In recent years numerous articles in supply chain modeling have addressed the issue of inventory coordination. Banerjee [1] introduced the concept of joint economic lot sizing problem (JELS). He considered the case of a single vendor and a single purchaser under the assumption of deterministic demand and lot for lot policy. He analyzed the effects of each party's optimal lot size on case of independent optimization and developed a JELS model that focused on the joint total relevant cost (JTRC). Goyal and Szendrovits [9] presented a constant lot size model where the lot is produced through a fixed sequence of manufacturing stages, with a single setup and without interruption at each stage. Transportation of partial lots, called batches, is allowed between stages. This model mainly, relaxes the constraint that batches must be of equal size at any particular stage. Goyal [6] provided a more general model for the case of single vendor single buyer through relaxing the lot-for-lot policy. He assumed that the whole production lot should be produced before shipments take place. He showed that his model provides a lower or equal total joint relevant cost compared to [1].

Goyal and Gupta [8] extensively reviewed the literature which deals with the interaction between a buyer and vendor. They classified the literature dealing with the integrated models into four main classes. The first class represents models which deal with joint economic lot sizing policies. The second class characterizes models which deal with the coordination of inventory by simultaneously determining the order quantity for the buyer and the vendor. The third class is a group of models which deal with integrated problem but do not determine simultaneously the order quantity of the buyer and the vendor. The last class represents models which deal with buyer vendor coordination subject to marketing considerations. Lu [15] developed a one-vender multi-buyer integrated inventory model with the objective of minimizing the vendor's total cost.

This paper considers the case of a three-stage non-serial supply chain system. The supply chain system involves suppliers, manufactures, and retailers. Production and inventory decisions are made at the suppliers and manufactures levels. The production rates for the suppliers and manufactures are assumed finite. In addition the demand at each end retailer is assumed to be stochastic. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized. For this purpose, we develop a model to deal with different inventory coordination mechanisms between the chain members. We present a numerical example for illustrative purposes.

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annual cost subject to the maximum cost that the buyer may be prepared to incur. They have found the optimal solution for
the single vendor single buyer case under the stated assumptions and presented a heuristic approach for the one-vendor
multi-buyer case. Goyal [7] in his short paper revisited the single vendor single buyer where he relaxed the constraint of
equal sized shipments of Goyal [6] and suggested that the shipment size should grow geometrically. Lu [15] considered a
single manufacturer single supplier supply chain where the manufacturer orders its raw materials from its supplier, the con-
verts the raw materials into finished goods, and finally delivers the finished goods to its customers. He proposed an inte-
grated inventory control model that comprises of integrated vendor–buyer (IVB) and integrated procurement–production
(IPP) systems.
Hooke and Jeeves [12] extended the idea of producing a single product in a multi-stage serial production system with
equal and unequal sized batch shipments between stages. Moutaz Khouja [17] considered the case a three-stage non-serial
supply chain and developed the model to deal with three inventory coordination mechanisms between the chain members.
Bendaya and Al-Nassar [3] relaxed the assumption of Moutaz Khouja [17] regarding the completion of the whole production
lot before making shipments out of it and assumed that equal sized shipments take place as soon as they are produced and
there is no need to wait until a whole lot is produced.
Cárdenas-Barrón [4] formulated and solved an n-stage-multi-customer supply chain inventory model where there is a
company that can supply products to several customers. The production and demand rates were assumed constant and
known. This model was formulated for the simplest inventory coordination mechanism which is referred to as the same
cycle time for all companies in the supply chain. It was concluded that it is possible to use an algebraic approach to optimize
the supply chain model without the use of differential calculus.
Han [10] established a strategic resource allocation model to capture and encapsulate the complexity of the modern glo-
bal supply chain management problem. He constructed a mathematical model to describe the stochastic multiple-period
two-echelon inventory with the many-to-many demand-supplier network problem. He applied Genetic algorithm (GA) to
derive optimal solutions through a two-stage optimization process. His model simultaneously constitutes the inventory con-
trol and transportation parameters as well as price uncertainty factors.
Chung and Wee [5] considered an integrated three-stage inventory system with backorders. They formulated the prob-
lem to derive the replenishment policies with four-decision-variables algebraically. Long et al. [14] studied a supply chain
model in which a single supplier sells a single product to a single retailer who faces the newsvendor problem. The retailer
is loss averse. The results showed that the optimal production quantity with decentralized decision making with a whole-
sale price contract is less than that with centralized decision making. The supply chain can achieve channel coordination
with buy back and target rebate contracts. With buy back contracts, the supply chain system profits can be allocated arbi-
trarily between the supplier and retailer. A new kind of contract, the incremental buy back contract, gives similar results as
with the buy back contract. They analyzed advantages and drawbacks of these three types of contracts via numerical
examples.
Barnes-Schuster et al. [2] studied a system composed of a supplier and buyer(s). They assumed that the buyer faces ran-
dom demand with a known distribution function. The supplier faces a known production lead time. Their main objective was
to determine the optimal delivery lead time and the resulting location of the system inventory. For a system with a single
supplier and as single buyer, they showed that system inventory should not be split between a buyer and supplier. They also
derived the conditions indicating when the supplier or buyer(s) should keep the system inventory, based on systems param-
eters of shortage and holding costs, production lead times, and standard deviations of demand distributions. Man-Yi and
Xiao-Wo [16] studied how to evaluate the safety stock of node enterprise given desired product availability when market
demand of the node enterprise in supply chain is described by Gauss fuzzy variable. They discussed the impact of required
product availability and demand uncertainty on safety stock, compared the correlative issues with stochastic demand, and
got some useful results. Rau and OuYang [19] presented an integrated production–inventory policy under a finite planning
horizon and a linear trend in demand. They assumed that the vendor makes a single product and supplies it to a buyer with a
non-periodic and just-in-time (JIT) replenishment policy in a supply chain environment. They first, developed a mathema-
tical model and proved that it has the optimal solution. Then, they described an explicit solution procedure for obtaining the
optimal solution and they provided two numerical examples to illustrate both increasing and decreasing demands in the
proposed model. Nagarajan and Sošić [18] surveyed some applications of cooperative game theory to supply chain managen-
ment. They first, described the construction of the set of feasible outcomes in commonly seen supply chain models, and then
used cooperative bargaining models to find allocations of the profit pie between supply chain partners. They analyzed and
surveyed several models. Then they discussed the issue of coalition formation among supply chain partners. They presented
an exhaustive survey of commonly used stability concepts.
As mentioned earlier numerous articles in supply chain modeling have been written in response to the global
competition. However, most of the developed supply chain inventory models deal with two-stage supply chains. Even when
multi-stage supply chains are considered, most of the developed models are based on restrictive assumptions such as of the
deterministic demand. Therefore, there is a need to analyze models that relax the usual assumptions to allow for a more real-
istic analysis of the supply chain inventory coordination. In this paper, we extend [17] by relaxing the deterministic demand
and assume that the end retailers face stochastic demand.
The remainder of this paper is organized as follows. The next section presents problem Definition, Notations and assump-
tions. Section 3 describes the development of the model. A numerical example is presented in Section 4. Finally, Section 5
contains some concluding remarks.
2. Problem definition

Consider the case of a three-stage supply chain where a firm can supply many customers. This supply chain system involves suppliers, manufacturers and retailers. Production and inventory decisions are made at suppliers and manufacturers levels. The production rates for the suppliers and manufacturers are assumed finite. In addition the demand for each firm is assumed to be stochastic. The problem is to coordinate production and inventory decisions across the supply chain so that the total cost of the system is minimized.

The following notations are used in developing the models:

\[ T \] Basic cycle time, cycle time at the end retailer

\[ P_i \] Cycle time at the stage \( i \)

\[ A_i \] Setup cost at stage \( i \)

\[ K_i \] Integer multiplier at stage \( i \)

\[ h_i \] Inventory holding cost at stage \( i \)

\[ n_i \] Number of firms at stage \( i \)

\[ D_{ij} \] The mean demand rate of firm \( j \) at stage \( i \)

\[ P_{ij} \] Production rate of firm \( j \) at stage \( i \)

\[ x \] A random variable describing the demand at retailer \( j \)

\[ f_{ij}(x,T) \] The continuous probability density of the customer demand received at retailer \( j \) in stage \( i \) during the period \( T \)

\[ \pi \] The shortage penalty per unit short

3. Model development

In this work we deal with two coordination mechanisms. The first is the simple equal cycle time coordination mechanism where the same cycle time is adopted at all stages. The second coordination mechanism is the integers multipliers in which firms at the same stage of the supply chain use the same cycle time and the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage.

The formulation of the three multi-stage, multi customers, non-serial supply chain according to the two coordination mechanism is presented in the following two subsections.

3.1. Equal cycle time coordination

Let \( i = 1, 2, \) and 3 denote the stage index in the supply chain. And let \( f_1, f_2, \ldots, f_i \) be an index denoting firms within each stage. As we can see from Fig. 1 the expected total cost per unit time for a downstream retailer can be approximated as

\[ \text{TC}_3 = h_3 \int_0^{T_3} \left( TD_{3j} \right) f_{3j}(x,T)dx + h_3 \int_{TD_{3j}}^{\infty} \frac{(TD)^2_{3j}}{2x} f_{3j}(x,T)dx + \pi \int_{TD}^{\infty} \frac{1}{2x} (x - TD)^2 f_{3j}(x,T)dx + \frac{A_3}{T}, \tag{1} \]

where \( f_{3j}(x,T) \) is the continuous probability density of the demand at the \( j \)th retailer, during the period time of length \( T \). The first and second terms in Eq. (1) represents the average carrying cost at an end retailer, while the third term is the average shortage cost. The last term is the replenishment cost, which occurs in every period of length \( T \) when the product is ordered.

The annual total cost for a manufacturer at the second stage is made up of two parts: the first is cost of carrying raw material as they are transformed to final products; and the second is the cost of holding finished goods. This occurs only during the production portion of the cycle. During the non-production portion of the equal cycle the inventory drops to zero because the whole amount produced is immediately shipped to the retailer stage. During the production portion, the average raw material and finished products inventory is \( TD_{2j}/2 \). Since the production rate is \( P_{2j} \), the per unit time average raw material and finished goods holding costs are \( h_1(TD_{2j}^2)/2P_{2j} \) and \( h_2(TD_{2j}^2)/2P_{2j} \), see [17]. Hence the expected total cost for a firm at stage 2 (i.e. manufacturer) is

![Inventory level](image-url)

Fig. 1. Inventory level at a retailer.

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155\[ TC_{2j} = \frac{TD^2_{2j}}{2P_{2j}} (h_1 + h_2) + \frac{A_2}{T}. \]

156Similarly the expected total cost for a firm at stage 1 (i.e. supplier) is

158\[ TC_{1j} = \frac{TD^2_{1j}}{2P_{1j}} (h_0 + h_1) + \frac{A_1}{T}. \]

3.2. Integers multipliers coordination

159For the integers multipliers coordination mechanism, firms at the same stage of the supply chain use the same cycle time and the cycle time at each stage is an integer multiplier of the cycle time used at the adjacent downstream stage. In this case, the cycle time of a retailer is $T$ and that of a manufacturer is $K_2T$ where $K_2$ is a positive integer. The cycle time of a supplier is a multiple of that of the manufacturer and is equal to $K_1K_2T$, see [17].

160The total cost per unit time for retailer $j$ is given by the same expression (1) in the previous section. The total cost for a firm at stage 2 (i.e. manufacturer) is given by

162\[ TC_{2j} = \frac{K_2TD^2_{2j}}{2P_{2j}} h_1 + \frac{A_2}{K_2T} + \frac{TD_{2j}}{2} (K_2(1 + D_{2j}/P_{2j}) - 1)h_2. \]

164The cost of carrying raw material as it is being converted into finished goods is given by the first term on the right-hand side of Eq. (5). However, the second term is made up of two parts. The first part is $K_2TD_{2j} h_2/2P_{2j}$ which is the per unit time holding cost of the finished goods for the non-production portion of the cycle and similar to the cost in the equal cycle mechanism. The second part is $TD_{2j} (K_2 - 1)h_2/2$ which represents the per unit time holding cost for the non-production portion of the cycle, see [17]. Similarly the total cost for a firm at stage 1 (i.e. supplier) is

166\[ TC_{1j} = \frac{K_1K_2TD^2_{1j}}{2P_{1j}} h_0 + \frac{K_2TD^2_{1j}}{2} (K_1(1 + D_{1j}/P_{1j}) - 1)h_1 + \frac{A_1}{K_1K_2}. \]

4. Numerical analysis

177In this section, we consider an example of a three-stage supply chain having one supplier, three manufacturers, and seven retailers. The relevant data is shown in Table 1. This is similar to the example used in [17] except that the demand is stochastic in our case. The demand at the end retailers is assumed to follow normal distributions. In addition to the data in Table 1, it is assumed that the shortage cost per unit time per unit short is 0.08. A direct search program based on Hooke and Jeeves as described in [11] is developed to find the optimal solution. Under the equal time mechanism, the optimal cycle time is 0.0697 years and the expected total cost $TC = $61647.2365 per year. Under the integer multipliers mechanism, the basic cycle time at the retailers is 0.062 years. The integer multiplier for the manufactures is $K_2 = 1$, and hence the cycle time at this stage equals the basic cycle time. However the integer multiplier for the supplier is $K_1 = 2$ and consequently the cycle time at this stage is 0.124 years and the total cost drops by 7159.5726 to $TC = $54487.6639 per year.

Table 1

179| $j$ | Order cost | Holding cost | Demand |
<table>
<thead>
<tr>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td></td>
<td></td>
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</tbody>
</table>
Retailers |  |  |  |
| 1 | 50 | 5.0 | 10,000 | 500 |
| 2 | 50 | 5.0 | 20,000 | 500 |
| 3 | 50 | 5.0 | 40,000 | 500 |
| 4 | 50 | 5.0 | 12,000 | 500 |
| 5 | 50 | 5.0 | 24,000 | 500 |
| 6 | 50 | 5.0 | 9,000 | 500 |
| 7 | 50 | 5.0 | 18,000 | 500 |

181Manufacturers

182| $j$ | Set up cost | Holding cost | Production rate |
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>(0.8,2.0)</td>
<td>140,000</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>(0.8,2.0)</td>
<td>108,000</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>(0.8,2.0)</td>
<td>108,000</td>
</tr>
</tbody>
</table>

185Supplier

186| $j$ | Set up cost | Holding cost |
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<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>800</td>
<td>(0.08,0.8)</td>
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We perform some sensitivity analysis on saving from using integer multipliers mechanism over the equal time cycle mechanism. We first decrease the current values of the setup cost at the suppliers and manufacturers stages by 25%, 50% and finally 75%. The results are given in Table 2. We then increase the values of \((h_0, h_1, \text{and } h_2)\) by using multipliers of (1.25, 1.50 and 2.00) of the original values. The results are presented in Table 3.

5. Conclusion

In this paper we consider the case of a three-stage supply chain. This supply chain system involves suppliers, manufacturers, and retailers. Production and inventory decisions are made at the suppliers and manufacturers levels. The production rates for the suppliers and manufacturers are assumed finite. In addition the demand for each firm is assumed to be stochastic. We formulated a model to deal with two inventory coordination mechanisms between the chain members. The numerical results show that the integer multipliers coordination mechanism has lower cost than the equal cycle time coordination mechanism.

6. Uncited reference

[13].

References


Table 2

<table>
<thead>
<tr>
<th>((A_1,A_2)) decrease</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
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<tr>
<td>(T^<em>_\text{ETC}^</em>)</td>
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<td></td>
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<tr>
<td>Equal time cycle</td>
<td>0.0617</td>
<td>56317.16</td>
<td>0.0526</td>
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<tr>
<td>Integer multipliers</td>
<td>0.0554</td>
<td>50226.47</td>
<td>0.0479</td>
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<tr>
<td>Saving</td>
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Table 3

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<tr>
<th>((h_0, h_1, \text{and } h_2)) multipliers</th>
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<th>2.00</th>
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<td>(T^<em>_\text{ETC}^</em>)</td>
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<tr>
<td>Equal time cycle</td>
<td>0.0673</td>
<td>63269.21</td>
<td>0.0652</td>
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<tr>
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