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Dimensional Analysis via an Electromagnetically-Oriented Set of Fundamental Dimensions

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Abstract. This paper describes the electromagnetically-oriented $LTI\emptyset$ dimensional basis that is based on the reference dimensions of Length (L), Time (T), Electric Current (I), and Electric Potential ($\emptyset$). We utilize this basis in the matrix solution of dimensional-analysis (DA) problems involving mainly electromagnetic quantities. Representations of electromagnetic quantities in the $LTI\emptyset$ basis (compared with the standard $MLTI$ basis that uses Mass (M) instead of Potential ($\emptyset$)) are more informative, much simpler, and have salient duality features. Moreover, DA computations of electromagnetic problems via the Gauss-Jordan algorithm in the $LTI\emptyset$ basis are more efficient, much less error prone, and quicker to detect linear dependencies in the dimensional equations. Both details and advantages of the proposed method are explored via demonstrative examples, which are of obvious significance in the learning and teaching of electromagnetism.

Key words. Dimensional analysis**,** dimensionless products, basis and regime variables, Gauss-Jordan elimination, electromagnetically-oriented dimensional basis, learning and teaching electromagnetism.

1. Introduction

Dimensional analysis (DA), fundamentally related to the principle of similitude, is an effective way to analyze a physical phenomenon without explicit knowledge of its governing physical laws, provided we are confident that such laws exist. In fact, DA can be used along with experimental data to develop an empirical mathematical model of the physical phenomenon concerned. The use of DA is justified by the single premise that the phenomenon can be described by a dimensionally correct equation among the pertinent variables. We do not need or assume pre-knowledge of this equation or model, for, otherwise, DA application would not be warranted. The prominent advantage of DA is that it dispenses with any knowledge of the inner mechanisms of the physical phenomenon. Dimensional analysis reduces the number of variables used, thereby facilitating fitting of equations to data. Another major advantage of DA is that it can partition the original variables in a general way into appropriate sets of basis (independent) variables and regime (dependent) ones, with each of the regime variables appearing (possibly with basis variables) in a dimensionless grouping that enjoys a concrete physical interpretation.

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Successful DA application necessitates that we understand the underlying phenomenon well enough just to identify all the pertinent variables, which must be all quantifiable. While DA does not need to know the inner mechanisms of the underlying phenomenon, it does not offer to explain these mechanisms, either. A complete model of the phenomenon is not supplied by DA alone, since DA must be supplemented with further information, usually in the form of experimental data. Direct utility of DA is limited to product formulas, in which variables are raised to certain exponents.

Dimensional analysis has a long history extending for several centuries, but it was almost one century ago that Buckingham laid the mathematical foundation for classical DA [1, 2]. During those centuries electricity and magnetism were developing as independent phenomena, and it was only in the nineteenth century that they were realized to be different manifestations of a single underlying theory, collectively called Maxwell's equations of electromagnetism (EM). One might observe the existence of an intimate relation between dimensional analysis and electromagnetism, even from the times of their infancies. In fact, dimensional analysis aided Maxwell, the undisputed father of modern electromagnetism, in the formulation of his celebrated equations [3]. The utility of dimensional analysis in solving problems of electromagnetics is exemplified by an example in the seminal paper of Buckingham [1]. Casting Maxwell's equations in a dimensionless form facilitates utilizing them in mathematical proofs [4], and allows their solution without dealing with very large numbers [5]. A modern prominent technique in electromagnetics (and other branches of physics), viz., that of differential forms, relies heavily on DA concepts to classify various quantities as volume, area, line, or none densities or quotients [6-9]. There are many publications linking DA and EM units and dimensions [10-29], and many others applying the most fundamental DA theorem (Buckingham Pi Theorem) to EM problems [30-49].

This paper offers a detailed exposition on using the electromagnetically-oriented $LTI\emptyset$ dimensional basis (instead of the mechanically-oriented $MLTI$ dimensional basis associated with the SI system of units) for solving DA problems mainly involving electromagnetic quantities. We describe transformations between the two bases, and report novel observations on characteristic features of EM and non-EM quantities, which are revealed by their dimensional exponents in the $LTI\emptyset$ basis. We supplement the use of this basis by the utilization of a modern matrix approach for Dimensional Analysis [49-59] in the derivation of dimensionless products via the celebrated Gauss-Jordan algorithm [60], which possesses many obvious advantages for handling linear dependencies, and for partitioning variables into basis and regime ones. The task of this algorithm is to reduce the augmented dimensional matrix into a reduced row echelon form (RREF). This task is considerably facilitated when the submatrix associated with the basis variables is shaped as closely as possible to a unit matrix, a goal that can be achieved with the use of the $LTI\emptyset$ basis, provided electromagnetic quantities dominate the basis variables of the DA problem, and, not necessarily its regime variables.

Rushdi and Rushdi [54] list many salient features of the Gauss-Jordan algorithm that makes it our unrivaled choice for handling DA problems (whether in a manual or an automated fashion). This algorithm acts as a mechanism of switching from an initial basic set of agreed-upon fundamental dimensions to a final set of desirable base dimensions. That role resembles this algorithm's role as a part of the Simplex Method of linear programming. The Gauss-Jordan algorithm does not make any presupposition of the rank r of the dimensional matrix. Neither does it need any preparatory work to determine the matrix rank (for example, by evaluating determinants via Cramer's rule as suggested by Middendorf [33]). This algorithm integrates the step of rank determination with its own work. If it encounters a row whose entries are all 0 (an all-0 row) it avoids this row by interchanging it with a latter row that is not all-0. The procedure terminates if there is no remaining row that is not all-0. The matrix rank is simply the number of rows successfully processed by the procedure (for which, there exist non-zero pivoting elements). For our manual solutions here, if the algorithm creates an all-0 row, then we will simply omit it in the next stage to avoid notorious swapping operations.

The remainder of this paper is structured as follows. Section 2 discusses the issue of selecting fundamental dimensions for electromagnetics. The problem of transformations between the $LTI\emptyset$ basis and the $MLTI$ basis is subsequently explored, first by scalar techniques (Section 3), and later by a novel application of the Gauss-Jordan algorithm (Section 4). Section 5 lists the dimensional exponents for EM and non-EM quantities in both the $LTI\emptyset$ and $MLTI$ bases, and points out the superior features possessed by the $LTI\emptyset$ basis for handling electromagnetic quantities. Several illustrative examples are then presented in Section 6 to demonstrate the effectiveness of the proposed approach which makes the most of the Gauss-Jordan algorithm through the use of the electromagnetically-oriented basis for the dimensional analysis of EM problems. Section 7 concludes the paper.

2. Selection of Fundamental Dimensions for Electromagnetics

The fundamental dimensions *may* be chosen rather *arbitrarily*, but, for practical reasons, *should* be chosen *appropriately*, and should be scientifically justifiable. The first step in the selection of a dimensional system is to choose the number N of fundamental dimensions. There are two extreme values for N as it can be as small as *one* in a mono-dimensional system, and it might be large enough to allow *all* dimensions to be fundamental ones in an omni-dimensional system. Szirtes [43] demonstrates the *possibility* and (at the same time) the *impracticality* of these two extreme systems:

- In a mono-dimensional system, all dimensions, except a single fundamental one, are derived and expressed as positive, zero, or negative powers of that single fundamental dimension. Such a system is totally impractical, suffers from excessive ambiguities, and it forces its users to use dimensions which are terribly inappropriate. It is particularly inferior to a (moderately) multidimensional one, since it drastically undermines the utilization of the requirement of dimensional homogeneity in deriving and verifying formulas and in constructing dimensionless products.
- In an omni-dimensional system, no ambiguity is encountered, since every dimension is fundamental and none is derived. Therefore, variables of different dimensions must be measured entirely independently, and every physical relation would be a separate scientific discovery requiring at least one mandatory dimensional 'constant of nature,' that would have to be determined separately so as to make the formula dimensionally homogeneous.

Since many drawbacks result from the selection of too few or too many fundamental or reference dimensions, the best choice seems to be a traditional (moderately) multidimensional system [43]. A considerable convenience can stem from using three to five reference dimensions as rendered feasible by the problem at hand. Bridgman [61] emphasizes the fact that there is nothing sacrosanct about the number of reference dimensions and that dimensional analysis is merely a man-made tool that may be manipulated at will. This principle of free choice of the reference dimensions has been widely accepted, preached and practiced [62]. In fact, the International System of Units (SI System) uses seven fundamental dimensions ($N = 7$), namely mass, length, time, electric current, temperature, amount of substance, and luminous intensity. It also employs two supplementary fundamental units for two particular dimensionless quantities, namely: the plane angle (length/length) and the solid angle (area/length squared). Treatment of these dimensionless quantities as fundamental dimensions leads to a convenient and systematic way of conversion between different systems [11, 51]. The SI system suffers from few ambiguities such as its assignment of non-distinct designations for torque and energy or for pressure, normal stress and shear stress [43].

The SI system is an outgrowth of the MLT system covering the *kinetic* quantities of Mass (M), Length (L), and Time (T). This mass-based system was competitive with (and more popular, albeit less efficient than) another three-dimensional system, *viz.*, the FLT system, in which force (F) replaces Mass as a fundamental dimension. The MLT system can express all mechanical quantities in a unique way, but it experiences ambiguities with many electromagnetic quantities [29]. To extend the MLT system to cover *electromagnetic* quantities appropriately, it suffices to add only one additional quantity [11], which originally was the electric charge (Q) [63], but was later superseded with the electric current (I). Dimensional analysis involving electromagnetic/electromechanical quantities, therefore, is typically based on the use of the $MLTI$ multidimensional system. An alternative system using the same number of fundamental dimensions is the $LTI\emptyset$ system [43, 57, 64, 65], where $\emptyset$ stands for electric potential or voltage. Though this system has been known for more than a century, it has never been fully developed or adequately utilized. It starts as a system covering the *kinematic* quantities of Length (L) and Time (T), and augments it with the *two* electric (or electromagnetic) quantities of current and potential. Likewise, in the $LTQ\Phi$ system that was proposed by Kalantaroff in 1929 [12, 27, 29], electric charge (Q) and magnetic flux (Φ) are taken as fundamental dimensions, again in addition to Length and Time. All these modern multidimensional systems use four fundamental dimensions, but the split of these four dimensions to purely mechanical and purely electromagnetic ones is $3+1$ for mechanically-oriented systems ($MLTQ, MLTI, FLTQ$, and $FLTI$) and $2+2$ for the electromagnetically-oriented ones ($LTI\emptyset$ and $LTQ\Phi$). The two electromagnetic quantities employed in these two latter systems are *dual* quantities, where dual electromagnetic quantities are obtained by interchanging the dimensions of current and voltage (or, equivalently, by interchanging the dimensions of electric charge and magnetic flux) [64]. A more elaborate understanding of the concept of 'duality' might be secured by referring to any standard text on electromagnetics [66-68].

Thomas [64] offers a lucid justification for the introduction of the $LTI\emptyset$ system that essentially goes as follows. In a mechanical system, dimensional simplification is usually achieved by employing 'force' as a fundamental or basis quantity. By direct electromechanical analogy, the electrical analogous quantity for 'force', namely 'voltage', is suggested to be a fundamental quantity. Since 'voltage' acts as the '*forcing function'* in a series electric circuit, the '*response*' in such a circuit, namely 'current' is proposed as a second fundamental quantity. Alternatively, one might view a parallel electric circuit, in which 'current' is the 'forcing function' and 'voltage' is the 'response', thereby coming to the same conclusion. 'Time' is an indispensable choice for a third fundamental quantity, since many electric phenomena are *dynamic*, in which many prominent quantities stand for the time rate of change of other quantities. It is remarkable to note that the product $TI\emptyset$ of the three fundamental quantities chosen so far stands for 'energy', and that many other important physical quantities of electric circuits rely solely on the three-element basis of $TI\emptyset$. However, to extend our coverage from that of lumped electric-circuit phenomena to that of *distributed* electromagnetic phenomena, we need to add 'length' as a fourth fundamental quantity.

3. Transformation between Two Fundamental Dimensional Systems for Electromagnetics

The basic liaison between mechanical and electromagnetic quantities arises from the fact that mechanical energy and electromagnetic energy share the same nature and dimension. Now, the dimension of mechanical energy is given by

$$[Mechanical\ energy] = [Mechanical\ work] = [Force][displacement] = [Mass][Acceleration]L = M\,L^2\mathrm{T}^{-2}, \quad (1)$$

while the dimension of electromagnetic energy is given by

$$[Electromagnetic\ energy] = [Poynting\ vector][Area][Time] = [Electric\ field\ intensity][Magnetic\ field\ intensity]\ L^2T = \emptyset L^{-1} I L^{-1} L^2 T = \emptyset IT\ , \quad (2)$$

The result in (2) could be obtained by employing a lumped electric circuit rather than a distributed electromagnetic phenomenon, for then (2) could be replaced by the dimension of electric energy, which is [Electric voltage] [Electric current] [Time], which is again $\emptyset IT$. Equating (1) to (2) results in

$$M\,L^2\mathrm{T}^{-2} = \emptyset IT. \quad (3)$$

Hence, we express the dimension of Mass (M) in the $LTI\emptyset$ basis, and the dimension of electric potential ($\emptyset$) in the $MLTI$ basis as

$$M\ = L^{-2}\,T^3\,I\,\emptyset. \quad (4)$$

$$\emptyset = M\,L^2\,T^{-3}\,I^{-1}. \quad (5)$$

Now, we consider an arbitrary physical quantity Q expressed in the $MLTI$ and $LTI\emptyset$ bases by the vectors of exponents $\boldsymbol{r} = [r_1\ \ r_2\ \ r_3\ \ r_4]^T$ and $\boldsymbol{R} = [R_1\ \ R_2\ \ R_3\ \ R_4]^T$. Our aim is to find the transformation matrix $\boldsymbol{T}$ that transforms the vector of exponents $\boldsymbol{R}$ in the $LTI\emptyset$ basis to the vector of exponents $\boldsymbol{r}$ in the $MLTI$ basis. The dimension $[Q]$ of Q is given by

$$[Q] = M^{r_1}\,L^{r_2}\,T^{r_3}\,I^{r_4} = L^{R_1}\,T^{R_2}\,I^{R_3}\,\emptyset^{R_4} = (L^{-2}\,T^3\,I\,\emptyset)^{r_1}\,L^{r_2}\,T^{r_3}\,I^{r_4} = L^{R_1}\,T^{R_2}\,I^{R_3}\,(M\,L^2\,T^{-3}\,I^{-1})^{R_4},$$

and hence, the various exponents are related by

$$R_1 = -2r_1 + r_2,\ R_2 = 3r_1 + r_3,\ R_3 = r_1 + r_4,\ R_4 = r_1. \quad (6)$$

$$r_1 = R_4,\ r_2 = R_1 + 2\,R_4,\ r_3 = R_2 - 3\,R_4,\ r_4 = R_3 - R_4\,. \quad (7)$$

These scalar relations can be written as a pair of matrix equations

$$\boldsymbol{r} = \boldsymbol{TR},\ \boldsymbol{R} = \boldsymbol{T^{-1}\,r}. \quad (8)$$

The two matrix equations (8) are conveniently displayed in scalar form in Fig. 1, in which the column vector at the right of the square transformation matrix $\boldsymbol{T}$ or its inverse $\boldsymbol{T^{-1}}$ is written as a row vector on top of the matrix [49, 53-59, 69], and the equality sign is omitted and implicitly understood. The four vectors comprising the transformation matrix $\boldsymbol{T}$ are the vectors of exponents for the variables $L, T, I,$ and $\emptyset$ in the $MLTI$ basis, while the four vectors comprising the inverse transformation matrix $\boldsymbol{T^{-1}}$ are the vectors of exponents for the variables $M, L, T,$ and I in the $LTI\emptyset$ basis. In Fig. 1, $\boldsymbol{T\,T^{-1}} = \boldsymbol{T^{-1}\,T} = \boldsymbol{I}$, as required.

	R_1	R_2	R_3	R_4
r_1	**0**	**0**	**0**	**1**
r_2	**1**	**0**	**0**	**2**
r_3	**0**	**1**	**0**	**−3**
r_4	**0**	**0**	**1**	**−1**

	r_1	r_2	r_3	r_4
R_1	**−2**	**1**	**0**	**0**
R_2	**3**	**0**	**1**	**0**
R_3	**1**	**0**	**0**	**1**
R_4	**1**	**0**	**0**	**0**

Figure 1. Convenient display for the transformations $\boldsymbol{r} = \boldsymbol{TR}$ and $\boldsymbol{R} = \boldsymbol{T^{-1}\,r}$ from the $LTI\emptyset$ dimensional basis to the $MLTI$ dimensional basis and back.

The two matrix equations (8) can be used to construct a table of dimensional exponents for all electromechanical quantities of interest in both the mechanically-oriented $MLTI$ dimensional basis and the electromagnetically-oriented $LTI\emptyset$ dimensional basis. Such a table has appeared earlier in Thomas [64], and is split here into two tables (Table 2 and Table 3) in order to give further explanations (in forthcoming sections) of the concept of duality in electromagnetism.

4. Transformation Derivation via the Gauss-Jordan Algorithm

Throughout this paper, we consider that a sought product π_j of a set of physical variables is dimensionless if, and only if, the exponents of these variables are a solution of the set of p homogeneous linear equations (not necessarily linearly independent) in n unknowns, expressed in matrix form as [33, 43, 49, 53-59, 62]:

$$\boldsymbol{Dz} = \boldsymbol{0}, \tag{9}$$

where $\boldsymbol{D}$ is the $p \times n$ dimensional matrix. This matrix has p rows ($p \leq N$), which represent the adopted fundamental reference dimensions or elements of the dimensional basis, and n columns, which denote the variable exponents in the sought dimensionless product, or, with a gross (albeit common and appealing) abuse of notation, designate the variables themselves. We will designate a column twice: (a) by the correct exponent notation, and (b) by the common variable notation. A typical entry of this matrix is the exponent to which a reference dimension (row) is raised in the dimensional product formula representing the particular variable (column).

The vector $\boldsymbol{z}$ comprises the n variable exponents in the sought dimensionless product, which are unknown constants, yet to be interrelated (partially determined). The Gauss-Jordan algorithm [60] achieves this purpose of inter-relating the exponents by applying elementary row operations that transform the matrix $\boldsymbol{D}$ into a reduced row echelon form (RREF). The vector of exponents $\boldsymbol{z}$ is not written as a column vector to the right of the dimensional matrix as suggested by Eq. (9), but is written (in a non-conventional way) as a row vector on top of it [53-59, 69]. In addition, the equality sign in Eq. (9) is omitted and implicitly understood, while the zero vector in the R.H.S. of Eq. (9) is added as an extra vector for $\boldsymbol{D}$ resulting in an *augmented matrix*, to whose entire rows we apply the same elementary row operations in the tableaus of the Gauss-Jordan (GJ) algorithm. Such operations are explained by assignment operations written in the leftmost column of the algorithm tableaus, wherein $E_i^{(k)}$ denotes the equation of augmented row i at stage k of the algorithm. The structure so obtained constitute standard Gauss-Jordan tableaus and is exemplified by the tableaus in Tables 1 and 4-11.

As a prelude to employing the Gauss-Jordan procedure in solving DA problems, we report herein an unusual application for it as an alternative instructive means for deriving the transformations in Eqs. (6-8). Table 1 uses the Gauss-Jordan procedure for moving from the mechanically-oriented $MLTI$ dimensional basis to the electromagnetically-oriented $LTI\emptyset$ dimensional basis and back. The table involves six exponents (m, l, t, i, f and q), which correspond to the variables M, L, T, I, $\emptyset$ and Q, and hence it covers all variables of the $MLTI$ and $LTI\emptyset$ bases as well as the quantity of interest Q (not to be confused with the electric charge). The table consists of five stages, of which each of stages 0, 2, and 4 has a specified dimensional basis (while each of the remaining stages lacks such a basis). Stage 0 (and also its identical stage 4) is characterized by the $MLTI$ dimensional basis, since the submatrix of $\boldsymbol{D}$ under the M, L, T, and I variables is a unit matrix, while stage 2 is characterized by the $LTI\emptyset$ dimensional basis, since the submatrix of $\boldsymbol{D}$ under the L, T, I and $\emptyset$ variables is a unit matrix. Therefore, the column under the variable of interest Q is its vector of dimensional exponents, which is $\boldsymbol{r}$ in stages 0 and 4 (of the $MLTI$ basis) and $\boldsymbol{R}$ in stage 2 (of the $LTI\emptyset$ basis).

Table 1 elegantly recovers the transformations in Eqs. (6-8) that are displayed in Fig. 1. In fact, it does so *twice*, consistently giving the same result. On one hand, the scalar values expressed in the Q column recover Eq. (6) at stage 2 and recover Eq. (7) at stage 4. On the other hand, going in column Q from $\boldsymbol{r}$ at stage 0 to $\boldsymbol{R}$ at stage 2 amounts to a left multiplication of the entire matrix by the inverse transformation matrix $\boldsymbol{T^{-1}}$, and going in the same column from $\boldsymbol{R}$ at stage 2 to $\boldsymbol{r}$ at stage 4 amounts to a left multiplication, again of the entire matrix, by the transformation matrix $\boldsymbol{T}$ itself. Therefore, the submatrix of $\boldsymbol{D}$ under the M, L, T, and I variables (which is a unit matrix in stage 2) is the transformation matrix $\boldsymbol{T}$ in each of stages 0 and 4 (highlighted in pale bluish-green). Likewise, the submatrix of $\boldsymbol{D}$ under the L, T, I and $\emptyset$ variables (which is a unit matrix in stages 0 and 4) is the inverse matrix $\boldsymbol{T^{-1}}$ in stage 2 (highlighted in pale orange).

This present situation is reminiscent of the initial and final tableaus in *the Simplex Method* used for solving linear-programming problems [54]. In the Simplex Method, the $MLTI$ variables depart their list of basic variables in stage 0, so that the $LTI\emptyset$ variables can enter this list (replacing them one by one) in stage 2. The only difference between the two situations is that: in the present case these entering and departing variables are known *a priori*, while in the linear-programming case, each entering or departing variable is selected at a specific step, according to rules dictated by some objective function [54].

We stress that though Table 1 outlines a specific procedure that is apparently of a limited value (one for moving from the $MLTI$ dimensional basis to the $LTI\emptyset$ one, and *vice versa*), that table can be modified to handle the matrix transformation and inverse transformation (if any) between any two dimensional bases.

5. Dimensional Exponents for EM and non-EM Quantities

The electromagnetically-oriented $LTI\emptyset$ dimensional basis is a four-dimensional basis that devotes two of its four reference dimensions to two electromagnetic (EM) quantities, electric current and electric potential (or voltage), which happen to be dual. As a result, this basis relates the $LTI\emptyset$ vectors of indices

$\boldsymbol{R}^a = [{R_1}^a \;\; {R_2}^a \;\; {R_3}^a \;\; {R_4}^a\,]^T$ and $\boldsymbol{R}^b = [{R_1}^b \;\; {R_2}^b \;\; {R_3}^b \;\; {R_4}^b\,]^T$ of two dual EM quantities a and b as follows

$$ {R_1}^a = {R_1}^b \,, \qquad (10a) $$

$$ {R_2}^a = {R_2}^b \,, \qquad (10b) $$

$$ {R_3}^a = {R_4}^b \,, \qquad (10c) $$

$$ {R_4}^a = {R_3}^b . \qquad (10d) $$

Equations (10) indicate that any two dual EM quantities have identical L and T exponents, and swapped I and $\emptyset$ exponents in the $LTI\emptyset$ basis. Table 2 displays dimensional exponents of pairs of dual electromagnetic quantities in the mechanically-oriented $MLTI$ dimensional basis and the electromagnetically-oriented $LTI\emptyset$ dimensional basis.

A physical quantity such that

$r_4 = 0$, or equivalently,

$$ R_3 = R_4 \,, \qquad (11) $$

is a self-dual quantity (in the electromagnetic (EM) sense), i.e., a quantity that lacks an EM dimension genuinely or through cancellation. A self-dual quantity might be

a) A genuine non-EM quantity, such as any of the six fundamental quantities of mass, length, time, temperature, amount of substance, luminous intensity, as well as many quantities composed solely of these six quantities. Note that the aforementioned six fundamental quantities are precisely the seven fundamental SI dimensions with the electromagnetic dimension excluded.
b) A product of two dual EM quantities, such as $\epsilon\mu, LC, VI$, $\boldsymbol{E} \times \boldsymbol{H}$ and $Q\Phi$ (see Table 2), where each of the original EM quantities violates each of the two equivalent conditions (11). Note that two dual quantities of exponents $\boldsymbol{R}^a$ and $\boldsymbol{R}^b$ satisfying (10) have a product of indices $2{R_1}^a, 2{R_2}^a,\; {R_3}^a + {R_4}^a$ and ${R_4}^a + {R_3}^a$, and hence it satisfies each of the two equivalent conditions of self-duality (11).

In Table 2, the exponent R_1 for the length dimension in the $LTI\emptyset$ dimensional basis is such that:

- $R_1 = 0$ for the lumped quantities I, V, L, C, Z, R, Y and G, in addition to the quantities Q and Φ which are the time integrals of the fundamental quantities I and $\emptyset$.

Table 1. The Gauss-Jordan procedure for moving from the mechanically-oriented $MLTI$ dimensional basis to the electromagnetically-oriented $LTI\emptyset$ dimensional basis and back.

	Stage Number	M / m	L / l	T / t	I / i	$\emptyset$ / f	Q / q	
${E_1}^{(0)}$	0	1	0	0	0	1	r_1	0
${E_2}^{(0)}$		0	1	0	0	2	r_2	0
${E_3}^{(0)}$		0	0	1	0	−3	r_3	0
${E_4}^{(0)}$		0	0	0	1	−1	r_4	0
${E_1}^{(1)} \leftarrow {E_2}^{(0)}$	1	0	1	0	0	2	r_2	0
${E_2}^{(1)} \leftarrow {E_3}^{(0)}$		0	0	1	0	−3	r_3	0
${E_3}^{(1)} \leftarrow {E_4}^{(0)}$		0	0	0	1	−1	r_4	0
${E_4}^{(1)} \leftarrow {E_1}^{(0)}$		1	0	0	0	1	r_1	0
${E_1}^{(2)} \leftarrow {E_1}^{(1)} - 2{E_4}^{(1)}$	2	−2	1	0	0	0	$R_1 = -2r_1 + r_2$	0
${E_2}^{(2)} \leftarrow {E_2}^{(1)} + 3{E_4}^{(1)}$		3	0	1	0	0	$R_2 = 3r_1 + r_3$	0
${E_3}^{(2)} \leftarrow {E_3}^{(1)} + {E_4}^{(1)}$		1	0	0	1	0	$R_3 = r_1 + r_4$	0
${E_4}^{(2)} \leftarrow {E_4}^{(1)}$		1	0	0	0	1	$R_4 = r_1$	0
${E_1}^{(3)} \leftarrow {E_4}^{(2)}$	3	1	0	0	0	1	R_4	0
${E_2}^{(3)} \leftarrow {E_1}^{(2)}$		−2	1	0	0	0	R_1	0
${E_3}^{(3)} \leftarrow {E_2}^{(2)}$		3	0	1	0	0	R_2	0
${E_4}^{(3)} \leftarrow {E_3}^{(2)}$		1	0	0	1	0	R_3	0
${E_1}^{(4)} \leftarrow {E_1}^{(3)}$	4	1	0	0	0	1	$r_1 = R_4$	0
${E_2}^{(4)} \leftarrow {E_2}^{(3)} + 2\,{E_1}^{(3)}$		0	1	0	0	2	$r_2 = R_1 + 2\,R_4$	0
${E_3}^{(4)} \leftarrow {E_3}^{(3)} - 3\,{E_1}^{(3)}$		0	0	1	0	−3	$r_3 = R_2 - 3\,R_4$	0
${E_4}^{(4)} \leftarrow {E_4}^{(3)} - {E_1}^{(3)}$		0	0	0	1	−1	$r_4 = R_3 - R_4$	0

- $R_1 = -1$ for the per-line quantities H, E, μ and $\in$. Among these, the vector quantities $\boldsymbol{H}$ and $\boldsymbol{E}$ frequently appear in the form $\boldsymbol{H.dl}$

and $\boldsymbol{E}.\boldsymbol{dl}$ (with $\boldsymbol{dl}$ depicting infinitesimal length), which are conveniently represented as 1-(differential) forms Hdl and Edl, or as integrands over curves (quantities that can be integrated along a one-dimensional curve) [6-9].

- $R_1 = -2$ for the per-surface quantities $\boldsymbol{B}$ and $\boldsymbol{D}$. These frequently appear in the form $\boldsymbol{B}.\boldsymbol{ds}$ and $\boldsymbol{D}.\boldsymbol{ds}$ (with $\boldsymbol{ds}$ depicting infinitesimal area), which are conveniently represented as 2-(differential) forms Bds and Dds, or as integrands over surfaces (quantities that can be integrated over a two-dimensional surface) [6-9].
- $R_1 = -3$ for the per-volume quantities $\boldsymbol{B}.\boldsymbol{H}$ and $\boldsymbol{D}.\boldsymbol{E}$. These frequently appear in the form $\boldsymbol{B}.\boldsymbol{H}\, dV$ and $\boldsymbol{D}.\boldsymbol{E}\, dV$ (with dV depicting infinitesimal volume), which are conveniently represented as 3-(differential) forms $BHdl$ and $DEdl$, or integrands over volumes (quantities that can be integrated over a three-dimensional space) [6-9].

Table 2. Dimensions of pairs of dual EM quantities in the mechanically-oriented $MLTI$ dimensional basis and the electromagnetically-oriented $LTI\emptyset$ dimensional basis.

Physical Quantity	Symbol	The $MLTI$ Dimensional Basis				The $LTI\emptyset$ Dimensional Basis			
		r_1	r_2	r_3	r_4	R_1	R_2	R_3	R_4
Electric current	I	0	0	0	1	0	0	1	0
Electric voltage	V	1	2	−3	−1	0	0	0	1
Magnetic field intensity	H	0	−1	0	1	−1	0	1	0
Electric field intensity	E	1	1	−3	−1	−1	0	0	1
Permeability	μ	1	1	−2	−2	−1	1	−1	1
Permittivity	$\in$	−1	−3	4	2	−1	1	1	−1
Magnetic flux density	B	1	0	−2	−1	−2	1	0	1
Electric flux density	D	0	−2	1	1	−2	1	1	0
Magnetic flux	Φ	1	2	−2	−1	0	1	0	1
Electric Charge	Q	0	0	1	1	0	1	1	0
Inductance	L	1	2	−2	−2	0	1	−1	1
Capacitance	C	−1	−2	4	2	0	1	1	−1
Impedance, resistance or	Z, R or	1	2	−3	−2	0	0	−1	1
Admittance, conductance	Y, G or	−1	−2	3	2	0	0	1	−1
Magnetic dot product	B.H	1	−1	−2	0	−3	1	1	1
Electric dot product	D.E	1	−1	−2	0	−3	1	1	1

In line with the observations above, Thomas [64] point out that the $LTI\emptyset$ dimensional basis justifies the unit names of farad/m and henry/m assigned to permittivity $\in$ and permeability μ when compared with the unit names of farad and henry given to capacitance C and inductance L. Likewise, this basis justifies the unit names of field intensities: H (ampere/m) and E (volt/m), as well as those of flux densities: B (weber/m^2) and D (coulomb/m^2).

Table 3 displays dimensional exponents of self-dual physical quantities in the $MLTI$ dimensional basis and the $LTI\emptyset$ dimensional basis. The quantities in this table are partitioned as mass-independent quantities with rather simple $LTI\emptyset$ exponents (highlighted in pale blue) and mass-dependent ones with rather non-simple $LTI\emptyset$ exponents (highlighted in pale orange). According to Eq. (11), these quantities do not need an electromagnetic dimension ($r_4 = 0$), and can be described in the MLT basis (which was sufficient before the era of electricity and magnetism). For all quantities in Tables 2 and 3, the absolute value of each of the two electromagnetic exponents is bounded and does not exceed one ($R_3, R_4 \epsilon \{-1,0,1\}$).

Tables 2 and 3 clearly indicate that representations of electromagnetic quantities in the $LTI\emptyset$ basis (compared with the standard $MLTI$ basis) are more informative, much simpler, and partially self-checking (thanks to the boundedness of the two electromagnetic exponents and the inter-relations (10) among exponents of dual quantities). Our thesis herein is that the $LTI\emptyset$ basis should be the one of choice in the matrix solution of dimensional-analysis problems involving predominantly electromagnetic quantities.

The following section demonstrates, by way of examples, that DA computations of electromagnetic problems via the Gauss-Jordan algorithm in the $LTI\emptyset$ basis are more efficient, much less error prone, and quicker to detect linear dependencies (if any) in the dimensional equations.

6. Various Examples Comparing the $LTI\emptyset$ and $MLTI$ Dimensional Bases

6.1. Transient analysis of an RL parallel circuit

This subsection deals with a problem of electric-circuit theory, which models the lumped special case of the (generally distributed) electromagnetic phenomena.

Let us consider the situation in which a DC current source of constant value I is imposed for time $t \geq 0$ on a parallel combination of a resistance R and an inductance L (Figure 2).

The transient voltage $v(t)$ on this parallel combination is required for time $t \geq 0$, and hence the variable v must be a regime variable [53, 54, 70], and it is placed last in a proposed dimensionless product

$$\pi = k\ L^l\ R^r\ I^i\ t^\tau\ v^V, \qquad (12)$$

where k is a dimensionless constant. The most important variable among the remaining variables is t, and it is placed immediately before v. Table 4 demonstrates the Gauss-Jordan procedure for solving this problem in the $LTI\emptyset$ dimensional basis, while Table 5 demonstrates the same procedure for solving this problem in the $MLTI$ dimensional basis. Here, $p = 4$, $n = 5$, and $r = 3$. The same final solution is obtained in both tables. However, the $LTI\emptyset$-based solution is obviously more efficient, entails simpler numbers (all of which are integers belonging to $\{-1, 0,1\}$), and hence it is less error prone, and it also detects linear dependency (manifested by an all-0 row) from the outset. The operations involved in the two tables include the following operations (arranged in decreasing complexity): floating-point row computation (f), row summation or differencing (s), row negation (n), and row assignment (a). The $LTI\emptyset$-based solution requires 2 stages beyond the initial stage involving $(3s + 3a)$ operations. The $MLTI$-based solution requires also 2 stages beyond the initial stage involving $(3f + s + n + 2a)$ operations.

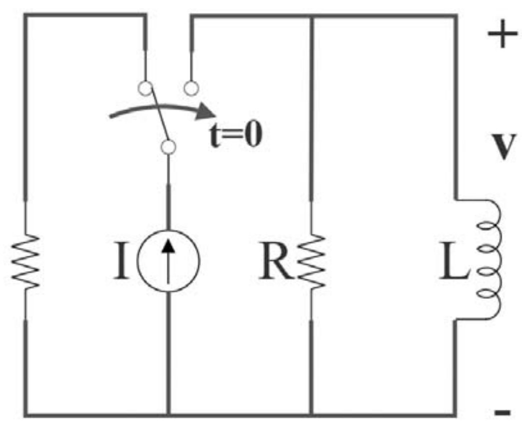

Figure 2. An electric circuit with a constant current source imposed at time 0 on a parallel combination of a resistor and inductor.

At the last stage of each solution, the $p \times n$ dimensional matrix $\boldsymbol{D}$ would be changed to an $r \times n$ matrix that is partitioned into two parts. The left part is an $r \times r$ unit matrix (shaded in light blue) and the right part is an $r \times (n - r)$ matrix $\boldsymbol{C}$ (shaded in dark blue). We now construct a full-rank $(n - r) \times n$ matrix $\boldsymbol{K}$ of exponents depicting the dimensionless products as shown at the bottom of Table 4 and Table 5. This product matrix is comprised of two juxtapositioned matrices: (a) the negative transpose $-\boldsymbol{C^T}$, an $(n - r) \times r$ matrix (shaded in dark green), which is obtained by negating and transposing $\boldsymbol{C}$ (the right part of the last stage of the dimensional matrix after implementing the

Gauss-Jordan algorithm, of course, after removing the all-zero row), and (b) an $(n-r) \times (n-r)$ unit matrix (shaded in light green) [54]. The matrix $\boldsymbol{K}$ is called the nullspace or kernel of $\boldsymbol{D}$, and is such that $rank(\boldsymbol{K}) = (n-r)$ is the nullity or defect of $\boldsymbol{D}$, the $(n-r)$ by p matrix $\boldsymbol{K}\boldsymbol{D}^{\mathrm{T}}$ is a zero matrix, and the $(n-r)$ rows of $\boldsymbol{K}$ form a basis for the nullspace of $\boldsymbol{D}$ [54, 62]

Table 3. Dimensions of individual self-dual ($r_4 = 0$ or $R_3 = R_4 \epsilon \{-1, 0, 1\}$) physical quantities in the $MLTI$ mechanically-oriented dimensional basis and the electromagnetically-oriented $LTI\emptyset$ dimensional basis.

Physical Quantity	Symbol	The $MLTI$ Dimensional Basis				The $LTI\emptyset$ Dimensional Basis			
		r_1	r_2	r_3	r_4	R_1	R_2	R_3	R_4
Mass	M	1	0	0	0	−2	3	1	1
Length	L	0	1	0	0	1	0	0	0
Time	t	0	0	1	0	0	1	0	0
Wavenumber	k	0	−1	0	0	−1	0	0	0
Frequency	ν	0	0	−1	0	0	1	0	0
Area	A	0	2	0	0	2	0	0	0
Volume	V	0	3	0	0	3	0	0	0
Moment of inertia	I	1	2	0	0	0	3	1	1
(Volumetric) density (of	ρ	1	−3	0	0	−5	3	1	1
Area density of mass	ρ_s	1	−2	0	0	−4	3	1	1
Linear density of mass	ρ_l	1	−1	0	0	−3	3	1	1
Specific volume	v	−1	3	0	0	5	−3	−1	−1
Humidity	η	1	−3	0	0	−5	3	1	1
Mass flow rate	q_m	1	0	−1	0	−2	2	1	1
Volumetric flow rate	q	0	3	−1	0	3	−1	0	0
Velocity	$\boldsymbol{v}$	0	1	−1	0	1	−1	0	0
Angular velocity	$\boldsymbol{\omega}$	0	0	−1	0	0	−1	0	0
Acceleration	$\boldsymbol{a}$	0	1	−2	0	1	−2	0	0
Angular acceleration	ω_a	0	0	−2	0	0	−2	0	0
Momentum	$\boldsymbol{p}$	1	1	−1	0	−1	2	1	1
Angular momentum	$\boldsymbol{L}$	1	2	−1	0	0	2	1	1
Force	$\boldsymbol{F}$	1	1	−2	0	−1	1	1	1
Pressure or stress	$\boldsymbol{P}$ or $\boldsymbol{\sigma}$	1	−1	−2	0	−3	1	1	1
Dynamic viscosity	μ	1	−1	−1	0	−3	2	1	1
Kinematic viscosity	μ	0	2	−1	0	2	−1	0	0
Work or energy	E	1	2	−2	0	0	1	1	1
Torque	T	1	2	−2	0	0	1	1	1
Power	P	1	2	−3	0	0	0	1	1
Volumetric density of en-	U	1	−1	−2	0	−3	1	1	1
Planck's constant	h	1	2	−1	0	0	2	1	1
Gravitational constant	h	−1	3	−2	0	5	−5	−1	−1
Hubble's constant	H	0	0	−1	0	0	−1	0	0

Each of the two identical versions of the matrix $\boldsymbol{K}$ at the bottom of Tables 4 and 5 indicates that there are two products:

$\pi_1 = tR/L = t/(L/R)$ and $\pi_2 = v/RI$, which constitute a (non-unique) complete set of dimensionless products. These two products are called regimes for the regime variables v and t

[53, 70]. According to Buckingham Pi Theorem, these two dimensionless products are related by an arbitrary function Φ equated to zero, namely:

$$\Phi\ (\pi_1\ , \pi_2) = 0. \quad (13)$$

Finally the mathematical model of the transient voltage v can be stated by expressing its regime π_2 as an arbitrary function Ψ (to be determined experimentally) of the other regime, namely

$$\pi_2 = \Psi\ (\pi_1). \quad (14)$$

It is known (outside the scope of dimensional analysis, through the theory of first-order linear ordinary differential equations) that the function Ψ is a decaying exponential, namely.

$$v/RI = exp\ (-t/(L/R)\ ). \quad (14a)$$

This result is usually referred to as the exponential relaxation of a first-order (single-time-constant) linear circuit or a first-order linear ordinary differential equation. The transients in the circuit in Fig. 1 are described by a single time constant $(\tau = L/R)$, where $[\tau] = [L]/[R] = (T\emptyset/I)/(\emptyset/I) = T$, i.e., by a parameter τ, which has the dimensions of time, indeed.

In retrospect, we might have not insisted on taking voltage and time as regime variables. Table 6 shows an alternative ordering of variables for the $LTI\emptyset$-based solution in Table 4, in which the regime variables are taken (arbitrarily, and ignoring the problem requirements) as the inductance and resistance. Here, the Gauss-Jordan algorithm does absolutely nothing beyond constructing its initial tableau. Now, we obtain two products: $\pi_3 = IL/vt$ and $\pi_4 = RI/v$ which constitute another complete set of dimensionless products. This new complete set is related to the old one via

$$\pi_3 = 1/(\pi_1\,\pi_2),\ \ \pi_4 = 1/\pi_2, \quad (15a)$$

$$\pi_1 = \pi_4/\pi_3, \quad \pi_2 = 1/\pi_4. \quad (15b)$$

Table 7 shows yet another ordering of variables for the $LTI\emptyset$-based solution in Table 4. Since the rank of the dimensional matrix is now known to be 3, this ordering suggests that the variables are partitioned into a set $\{t, L, R\}$ of basis variables and a set $\{v, I\}$ of regime variables. An advantage of the Gauss-Jordan algorithm is that it detects the impossibility of this partitioning and corrects it *en route*. Contrary to widespread belief, the Gauss-Jordan algorithm does not necessarily partition $\boldsymbol{D}$ into two matrices such that the first of which is a unit matrix. Generally, the Gauss-Jordan algorithm replaces $\boldsymbol{D}$ by its reduced row echelon form (RREF) [54], an example of which is shown in the second stage of Table 7. In this more general (albeit less appealing situation), the algorithm employs two correct sets of basis and regime variables as $\{t, L, v\}$ and $\{R, I\}$ by swapping the roles of the variables R and v as basis or regime variables.

Table 4. The Gauss-Jordan procedure for solving the circuit problem of Sec. 6.1 in the $LTI\emptyset$ dimensional basis. The final stage of matrix $\boldsymbol{D}$ is shaded in blue (partitioned into a unit matrix in light blue, followed by $\boldsymbol{C}$ in dark blue), while the matrix $\boldsymbol{K}$ is shaded in green (partitioned into the negative transpose of matrix $\boldsymbol{C}$ in dark green, followed by a unit matrix in light green).

	l	i	r	τ	V	
	L	I	R	t	v	
$E_1^{(0)}$	0	0	0	0	0	0
$E_2^{(0)}$	1	0	0	1	0	0
$E_3^{(0)}$	−1	1	−1	0	0	0
$E_4^{(0)}$	1	0	1	0	1	0
$E_2^{(1)} \leftarrow E_2^{(0)}$	1	0	0	1	0	0
$E_3^{(1)} \leftarrow E_3^{(0)} + E_2^{(0)}$	0	1	−1	1	0	0
$E_4^{(1)} \leftarrow E_4^{(0)} - E_2^{(0)}$	0	0	1	−1	1	0
$E_2^{(2)} \leftarrow E_2^{(1)}$	1	0	0	1	0	0

$E_3^{(2)} \leftarrow E_3^{(1)} + E_4^{(1)}$	0	1	0	0	1	0
$E_4^{(2)} \leftarrow E_4^{(1)}$	0	0	1	−1	1	0
π_1	−1	0	1	1	0	
π_2	0	−1	−1	0	1	

Table 5. The Gauss-Jordan procedure for solving the circuit problem of Sec. 6.1 in the $MLTI$ dimensional basis.

	l	r	i	τ	V	
	L	R	I	t	v	
$E_1^{(0)}$	1	1	0	0	1	0
$E_2^{(0)}$	2	2	0	0	2	0
$E_3^{(0)}$	−2	−3	0	1	−3	0
$E_4^{(0)}$	−2	−2	1	0	−1	0
$E_1^{(1)} \leftarrow E_1^{(0)}$	1	1	0	0	1	0
$E_2^{(1)} \leftarrow E_2^{(0)} - 2\,E_1^{(0)}$	0	0	0	0	0	0
$E_3^{(1)} \leftarrow E_3^{(0)} + 2\,E_1^{(0)}$	0	−1	0	1	−1	0
$E_4^{(1)} \leftarrow E_4^{(0)} + 2\,E_1^{(0)}$	0	0	1	0	1	0
$E_1^{(2)} \leftarrow E_1^{(1)} + E_3^{(1)}$	1	0	0	1	0	0
$E_3^{(2)} \leftarrow -E_3^{(1)}$	0	1	0	−1	1	0
$E_4^{(2)} \leftarrow E_4^{(1)}$	0	0	1	0	1	0
π_1	−1	1	0	1	0	0
π_2	0	−1	−1	0	1	0

Table 6. The Gauss-Jordan procedure for solving the circuit problem of Sec. 6.1 in the $LTI\emptyset$ dimensional basis with an alternative ordering of variables.

	τ	i	V	l	r	
	t	I	v	L	R	
$E_1^{(0)}$	0	0	0	0	0	0
$E_2^{(0)}$	1	0	0	1	0	0
$E_3^{(0)}$	0	1	0	−1	−1	0
$E_4^{(0)}$	0	0	1	1	1	0
π_3	−1	1	−1	1	0	
π_4	0	1	−1	0	1	

Now, the three basis variables $t, L,$ and v are not assigned to consecutive columns, and though the matrix under them is, in fact, a unit matrix, it might not readily appear as such (due to lack of visual adjacency). In the lower part of Table 7, we interchange the columns for R and v so as to place all columns with pivots consecutively at the left to form an identity matrix. Both parts of Table 7 yield the two products: $\pi_5 = tR/L$ and $\pi_6 = IL/vt$, which constitute yet another complete set of dimensionless products, again related to the earlier sets, since π_5 and π_6 are equal to π_1 and π_3, respectively. The total number of complete sets of dimensionless products is at most (here strictly less than) the number of choosing two regime variables out of five variables (without order or repetition), which is ten [53].

The non-uniqueness of the complete set of dimensionless products is occasionally cited as a limitation of dimensional analysis [54]. However, we note that Eq. (14) is the desirable solution of the problem, and it can be reached in a variety of ways, such as directly from Table 4, or via Table 6 together with Eqs. (15b).

Results similar to those of this subsection are obtained by Middendorf [33] and Rushdi & Rushdi [57] for the dual problem in which a DC *voltage* source of value V is imposed for time $t \geq 0$ on a *series* combination of a *resistance* R and a *capacitance* C, and wherein the transient *current* $i(t)$ is required. For both problems, the dimension of length L is dispensable with, in the sense that (a) the dimensional basis $LTI\emptyset$ can be replaced by its $TI\emptyset$ subset, since each of the pertinent variables has a zero exponent for the dimension of length L, and (b) an all-0 row appears right from the outset in the initial stage of the Gauss-Jordan algorithm. Generally, for circuit problems, the $LTI\emptyset$ dimensional basis (or its $TI\emptyset$ subset) has a definite advantage.

A very famous problem in electromagnetics is the problem of Coulomb, in which one seeks the inverse square law for the dependence of the far-field intensity of the electric field E on the distance from the origin r, at which a point source is located. To demonstrate the elegance and power of DA solutions, we solve a generalization of this problem. Specifically, we give a DA solution of a problem that comprises six primitive problems, one of which is the aforementioned problem of Coulomb. We will consider a general electric source S, located at (or in the vicinity of) the origin (practically located at the origin under the far-field assumption). This source has a dimension of $[S] = I\ T\ L^j$, and it might be

6.2. Far-Field Observations due to an Electric Source at the Origin

1. A point electric charge (monopole) $S = Q$ ($j = 0,\ [S] = I\ T$), located at the origin (0,0,0).
2. An electric dipole of moment $S = Qa$ ($j = 1,\ [S] = I\ T\ L$), comprising two charges of equal magnitudes and opposite signs: a positive charge $(+Q)$ located at $(a/2,0,0)$, and a negative one $(-Q)$ located at $(-a/2,0,0)$, where $a \ll r$.
3. An electric quadrupole of moment $S = Qa^2$ ($j = 2,\ [S] = I\ T\ L^2$), comprising four charges of equal magnitudes and alternating signs: a positive charge $(+Q)$ located at $(a/2, a/2,0)$, a negative one $(-Q)$ at $(a/2, -a/2,0)$, a second positive charge $(+Q)$ situated at $(-a/2, -a/2,0)$, and finally another negative charge $(-Q)$ located at $(-a/2, a/2,0)$, where $a \ll r$.
4. We also consider an observed quantity O of dimension $[O] = \emptyset\, L^{-i} = M\, L^{2-i}\, T^{-3}\, I^{-1}$, where this quantity can be the electric potential $\emptyset$ ($i = 0$), or the electric field intensity E ($i = 1$). The variable O must be a regime variable, and it is placed last in a proposed dimensionless product
5. $\pi = k\ r^R\ \epsilon^p\ S^s\ O^o$, (16)
6. where k is a dimensionless constant. Table 8 demonstrates the Gauss-Jordan procedure for solving this problem in the $LTI\emptyset$ dimensional basis, while Table 9 demonstrates the same procedure for solving this problem in the $MLTI$ dimensional basis. The same final solution is obtained in both tables. However, the $LTI\emptyset$-based solutions is obviously more efficient, and hence less error prone, and it is, once more, quicker to detect an all-0 row in the dimensional matrix. The $LTI\emptyset$-based solution requires 2 stages beyond the initial stage involving $(f + 4s + 2a)$ operations. The $MLTI$-based solution requires also 2 stages beyond the initial stage involving $(4f + s + n + 2a)$ operations.
7. Each of the two versions of the matrix $\boldsymbol{K}$ at the bottom of Tables 8 and 9 indicates that there is a single dimensionless product $\pi_1 = k\ r^{1+i+j}\ \epsilon^1\ S^{-1}\ O^1$. According to Buckingham Pi Theorem, this product must be a constant, and hence the observed quantity is
8. $O = k_{ij}\, S\, /(\epsilon\ r^{1+i+j})$. (17)
9. The far field potentials $\emptyset$ due to a charge ($S = Q$), a dipole ($S = Qa$), and a quadrupole ($S = Qa^2$) are $k_{00}\, Q\, /\, \epsilon\, r$, $k_{01}\, Qa\, /\, \epsilon\, r^2$, and $k_{02}\, Qa^2 / \epsilon\, r^3$, respectively, while the corresponding far electric field intensities E are $k_{10}\, Q\, /\, \epsilon\, r^2$, $k_{11}\, Qa\, /\, \epsilon\, r^3$, and $k_{12}\, Qa^2 / \epsilon\, r^4$, respectively. These results are in agreement with those derived by analytic techniques of electromagnetics [71]. In particular, we recover the celebrated inverse square law ($E = k_{10}\, Q\, /\, \epsilon\, r^2$) of Coulomb.

6.3. The leakage current through the electrolyte on a wet contaminated insulator

Piah and Darus [41] employed Dimensional Analysis to model the leakage current (I) due to the electrolyte formed on a wet contaminated insulator. The other variables included in the analysis were: the electrolyte conductivity (σ), the electrolyte volumetric flow rate (Q), environmental pressure (P), humidity (H), and the applied electric field (E). The variable I must be a regime variable [53, 54, 70], and it is placed last in a proposed dimensionless product

$$\pi = k\ \sigma^s\ Q^q\ \ P^p\ H^h\ E^e\ I^i, \qquad (18)$$

where k is a dimensionless constant. The most important variable among the remaining variables is E, and it is placed immediately before I. Table 10 demonstrates the Gauss-Jordan procedure for solving this problem in the $LTI\emptyset$ dimensional basis, while Table 11 demonstrates the same procedure for solving this problem in the $MLTI$ dimensional basis. The same final solution is obtained in both tables, and it is in agreement with the one obtained earlier in [41]. However, the $LTI\emptyset$-based solution is only slightly more efficient, and hence somewhat less error prone. The $LTI\emptyset$-based solution requires 4 stages beyond the initial stage involving $(7f + 4s + 2n + 3a)$ operations. The $MLTI$-based solution requires also 4 stages beyond the initial stage involving $(10f + 3s + 2n + 2a)$ operations. The computations for this problem are dramatically more complex than those in the earlier subsections, and in this case the dimensional matrix is of full rank. The superiority of the $LTI\emptyset$-based solution is less pronounced in the present case (compared with the cases in the earlier subsections), since the present problem is not dominantly an electromagnetic one. Anyhow, the superiority of the $LTI\emptyset$-based solutions for dominantly electromagnetic DA problems was extensively verified by considering such problems in many and diverse recent publications [72-80].

In passing, we observe that the dimensional bases associated with the international system of units (SI system) have been claimed (criticized!) to suffer from inherent redundancy [22, 81].

Table 7. The Gauss-Jordan procedure for solving the circuit problem of Sec. 6.1 in the $LTI\emptyset$ dimensional basis, repeated twice for two different orderings of variables.

	τ	l	r	V	i	
	t	L	R	v	I	
${E_1}^{(0)}$	0	0	0	0	0	0
${E_2}^{(0)}$	1	1	0	0	0	0
${E_3}^{(0)}$	0	−1	−1	0	1	0
${E_4}^{(0)}$	0	1	1	1	0	0
${E_2}^{(1)} \leftarrow {E_2}^{(0)} + {E_3}^{(0)}$	1	0	**−1**	0	**1**	0
${E_3}^{(1)} \leftarrow -{E_3}^{(0)}$	0	1	**1**	0	**−1**	0
${E_4}^{(1)} \leftarrow {E_4}^{(0)} + {E_3}^{(0)}$	0	0	**0**	1	**1**	0
π_5	**1**	**−1**	1	**0**	0	
π_6	**−1**	**1**	0	**−1**	1	

	τ	l	V	r	i	
	t	L	v	R	I	
	1	0	0	**−1**	**1**	0
	0	1	0	**1**	**−1**	0
	0	0	1	**0**	**1**	0
π_5	**1**	**−1**	**0**	1	0	
π_6	**−1**	**1**	**−1**	0	1	

Table 8. The Gauss-Jordan procedure for solving the far-field problem of Sec. 6.2 in the $LTI\emptyset$ dimensional basis.

	R	p	s	o	
	r	ϵ	S	O	
${E_1}^{(0)}$	1	-1	j	$-i$	0
${E_2}^{(0)}$	0	1	1	0	0
${E_3}^{(0)}$	0	1	1	0	0
${E_4}^{(0)}$	0	-1	0	1	0
${E_1}^{(1)} \leftarrow {E_1}^{(0)} + {E_2}^{(0)}$	1	0	$1+j$	$-i$	0
${E_2}^{(1)} \leftarrow {E_2}^{(0)}$	0	1	1	0	0
${E_3}^{(1)} \leftarrow {E_3}^{(0)} - {E_2}^{(0)}$	0	0	0	0	0
${E_4}^{(1)} \leftarrow {E_4}^{(0)} + {E_2}^{(0)}$	0	0	1	1	0
${E_1}^{(2)} \leftarrow {E_1}^{(1)} - (j+1){E_4}^{(1)}$	1	0	0	$-i-1-j$	0
${E_2}^{(2)} \leftarrow {E_2}^{(1)} - {E_4}^{(1)}$	0	1	0	-1	0
${E_4}^{(2)} \leftarrow {E_4}^{(1)}$	0	0	1	1	0
π_1	$1+i+j$	1	-1	1	

Table 9. The Gauss-Jordan procedure for solving the far-field problem of Sec. 6.2 in the $MLTI$ dimensional basis.

	p	R	s	o	
	ϵ	r	S	O	
${E_1}^{(0)}$	-1	0	0	1	0
${E_2}^{(0)}$	-3	1	j	$2-i$	0
${E_3}^{(0)}$	4	0	1	-3	0
${E_4}^{(0)}$	2	0	1	-1	0
${E_1}^{(1)} \leftarrow -{E_1}^{(0)}$	1	0	0	-1	0
${E_2}^{(1)} \leftarrow {E_2}^{(0)} - 3\,{E_1}^{(0)}$	0	1	j	$-1-i$	0
${E_3}^{(1)} \leftarrow {E_3}^{(0)} + 4\,{E_1}^{(0)}$	0	0	1	1	0
${E_4}^{(1)} \leftarrow {E_4}^{(0)} + 2\,{E_1}^{(0)}$	0	0	1	1	0
${E_1}^{(2)} \leftarrow {E_1}^{(1)}$	1	0	0	-1	0
${E_2}^{(2)} \leftarrow {E_2}^{(1)} - j\,{E_3}^{(1)}$	0	1	0	$-1-i-j$	0
${E_3}^{(2)} \leftarrow {E_3}^{(1)}$	0	0	1	1	0
${E_4}^{(2)} \leftarrow {E_4}^{(1)} - {E_3}^{(1)}$	0	0	0	0	0
π_1	1	$1+i+j$	-1	1	

Such redundancy is manifested in many problems with electromechanical/ electromagnetic problems, in which the dimensional matrix in the $MLTI$ or $LTI\emptyset$ is not of full rank (see, for example, the problems in subsections 6.1 and 6.2). However, no redundancy appears in the problem of our current subsection, with the dimensional matrix being of full rank. To mitigate the purported redundancy, several authors suggested the use of a dimensional basis of three fundamental quantities only [22, 81]. One such basis uses the three quantities of length, time, and energy as fundamental quantities, and assumes 'voltage' to be dimensionless [22]. This basis shows no redundancy in handling the problems of subsections 6.1 and 6.2, as it produces full-rank matrices. However, it fails to reproduce the solution obtained herein by either the $MLTI$ basis or the $LTI\emptyset$ basis.

We have demonstrated that the $LTI\emptyset$ basis is the basis of choice in the matrix solution of dimensional-analysis problems involving predominantly electromagnetic quantities. Interestingly, the basis of choice in the matrix solution of dimensional-analysis problems involving predominantly mechanical quantities is not the familiar $MLTI$ basis, but seems to be a mechanical basis that is analogous to the $LTI\emptyset$ basis. In 'direct' electromechanical analogy mechanical force F is represented by voltage or potential $\emptyset$ and mechanical velocity v by electric current I, and hence, mass M is represented by inductance $\mathcal{L}$, and length L by electric charge Q, while time T is left intact [82-84]. This means that the $QTvF$ basis can be proposed as an efficient one for predominantly mechanical systems [64]. However, in 'inverse' or 'indirect' electromechanical analogy, mechanical force F is represented by electric current I and mechanical velocity v by voltage or potential $\emptyset$, and hence mass M is represented by admittance $\mathcal{C}$ and length L by magnetic flux Φ, while time T is again left intact [82-84]. This means that the ΦTFv basis can be another efficient one for predominantly mechanical systems. Unfortunately, the use of mass as a fundamental quantity in the familiar $MLTI$ basis is analogous to using inductance or admittance as a fundamental quantity in an electromagnetic dimensional system.

7. Conclusions

This paper proposed a novel approach of Dimensional Analysis, which makes the most of the Gauss-Jordan algorithm through the use of an electromagnetically-oriented basis (the $LTI\emptyset$ basis) for handling EM problems. The paper starts by investigating the issue of selecting fundamental dimensions for electromagnetics. The problem of transformations between the $LTI\emptyset$ basis and the $MLTI$ basis is subsequently explored, first by scalar techniques, and later by a novel application of the Gauss-Jordan algorithm. We list the dimensional exponents for EM and non-EM quantities in both the $LTI\emptyset$ and $MLTI$ bases, and point out certain superior features possessed by the $LTI\emptyset$ basis. Several illustrative examples expose the details of the proposed method, and demonstrate its merits and effectiveness.

Table 10. The Gauss-Jordan procedure for expressing the leakage current of Sec. 6.3 in the $LTI\emptyset$ dimensional basis.

	s	q	p	h	e	i	
	σ	Q	P	H	E	I	
$E_1^{(0)}$	−1	3	−3	−5	−1	0	0
$E_2^{(0)}$	0	−1	1	3	0	0	0
$E_3^{(0)}$	1	0	1	1	0	1	0
$E_4^{(0)}$	−1	0	1	1	1	0	0
$E_1^{(1)} \leftarrow -E_1^{(0)}$	1	−3	3	5	1	0	0
$E_2^{(1)} \leftarrow E_2^{(0)}$	0	−1	1	3	0	0	0
$E_3^{(1)} \leftarrow E_3^{(0)} + E_1^{(0)}$	0	3	−2	−4	−1	1	0
$E_4^{(1)} \leftarrow E_4^{(0)} - E_1^{(0)}$	0	−3	4	6	2	0	0
$E_1^{(2)} \leftarrow E_1^{(1)} - 3\,E_2^{(1)}$	1	0	0	−4	1	0	0
$E_2^{(2)} \leftarrow -E_2^{(1)}$	0	1	−1	−3	0	0	0
$E_3^{(2)} \leftarrow E_3^{(1)} + 3\,E_2^{(1)}$	0	0	1	5	−1	1	0
$E_4^{(2)} \leftarrow E_4^{(1)} - 3\,E_2^{(1)}$	0	0	1	−3	2	0	0
$E_1^{(3)} \leftarrow E_1^{(2)}$	1	0	0	−4	1	0	0
$E_2^{(3)} \leftarrow E_2^{(2)} + E_3^{(2)}$	0	1	0	2	−1	1	0
$E_3^{(3)} \leftarrow E_3^{(2)}$	0	0	1	5	−1	1	0
$E_4^{(3)} \leftarrow E_4^{(2)} - E_3^{(2)}$	0	0	0	−8	3	−1	0
$E_1^{(4)} \leftarrow E_1^{(3)} + 4\,E_4^{(4)}$	1	0	0	0	−4/8	4/8	0
$E_2^{(4)} \leftarrow E_2^{(3)} - 2\,E_4^{(4)}$	0	1	0	0	−2/8	6/8	0
$E_3^{(4)} \leftarrow E_3^{(3)} - 5\,E_4^{(4)}$	0	0	1	0	7/8	3/8	0
$E_4^{(4)} \leftarrow E_4^{(3)}/(-8)$	0	0	0	1	−3/8	1/8	0
π_1	4/8	2/8	−7/8	3/8	1	0	
π_2	−4/8	−6/8	−3/8	−1/8	0	1	

Table 11. The Gauss-Jordan procedure for expressing the leakage current of Sec. 6.3 in the $MLTI$ dimensional basis.

	s	q	p	h	e	i	
	σ	Q	P	H	E	I	
$E_1{}^{(0)}$	-1	0	1	1	1	0	0
$E_2{}^{(0)}$	-3	3	-1	-3	1	0	0
$E_3{}^{(0)}$	3	-1	-2	0	-3	0	0
$E_4{}^{(0)}$	2	0	0	0	-1	1	0
$E_1{}^{(1)} \leftarrow -E_1{}^{(0)}$	1	0	-1	-1	-1	0	0
$E_2{}^{(1)} \leftarrow E_2{}^{(0)} - 3\,E_1{}^{(0)}$	0	3	-4	-6	-2	0	0
$E_3{}^{(1)} \leftarrow E_3{}^{(0)} + 3\,E_1{}^{(0)}$	0	-1	1	3	0	0	0
$E_4{}^{(1)} \leftarrow E_4{}^{(0)} + 2\,E_1{}^{(0)}$	0	0	2	2	1	1	0
$E_1{}^{(2)} \leftarrow E_1{}^{(1)} + E_3{}^{(1)}$	1	-1	0	2	-1	0	0
$E_2{}^{(2)} \leftarrow E_2{}^{(1)} + 4\,E_3{}^{(1)}$	0	-1	0	6	-2	0	0
$E_3{}^{(2)} \leftarrow E_3{}^{(1)}$	0	-1	1	3	0	0	0
$E_4{}^{(2)} \leftarrow E_4{}^{(1)} - 2\,E_3{}^{(1)}$	0	2	0	-4	1	1	0
$E_1{}^{(3)} \leftarrow E_1{}^{(2)} - E_2{}^{(2)}$	1	0	0	-4	1	0	0
$E_2{}^{(3)} \leftarrow -\,E_2{}^{(2)}$	0	1	0	-6	2	0	0
$E_3{}^{(3)} \leftarrow E_3{}^{(2)} - E_2{}^{(2)}$	0	0	1	-3	2	0	0
$E_4{}^{(3)} \leftarrow E_4{}^{(2)} + 2\,E_2{}^{(2)}$	0	0	0	8	-3	1	0
$E_1{}^{(4)} \leftarrow E_1{}^{(3)} + 4\,E_4{}^{(4)}$	1	0	0	0	$-4/8$	$4/8$	0
$E_2{}^{(4)} \leftarrow E_2{}^{(3)} + 6\,E_4{}^{(4)}$	0	1	0	0	$-2/8$	$6/8$	0
$E_3{}^{(4)} \leftarrow E_3{}^{(3)} + 3\,E_4{}^{(4)}$	0	0	1	0	$7/8$	$3/8$	0
$E_4{}^{(4)} \leftarrow E_4{}^{(3)}/(8)$	0	0	0	1	$-3/8$	$1/8$	0
π_1	$4/8$	$2/8$	$-7/8$	$3/8$	1	0	
π_2	$-4/8$	$-6/8$	$-3/8$	$-1/8$	0	1	

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تحليل الأبعاد بواسطة مجموعة للأبعاد الأساسية معنية بالكهرومغناطيسيات

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المستخلص. تصف ورقة البحث هذه قاعدة الأبعاد *ط ز ت ج* المعنية بالكهرومغناطيسيات والتي تستخدم الأبعاد الإسنادية للطول (*ط*)، والزمن (*ز*)، والتيار الكهربائي (*ت*)، والجهد الكهربائي (*ج*). ننتفع بهذه القاعدة في الحل المصفوفي لمسائل تحليل الأبعاد (ح ب) التي يغلب عليها استعمال كميات كهرومغناطيسية . إن تمثيلات الكميات الكهرومغناطيسية في قاعدة الأبعاد الجديدة *ط ز ت ج* (مقارنة مع قاعدة الأبعاد المعيارية *ك ط ز ت* (التي تستخدم الكتلة (*ك*) بدلا من الجهد (*ج*)) تكسبنا معلومات أوفر وتتسم بسهولة أوضح ولها خصائص مزاوجة بارزة. وفضلا عن ذلك، فإن حسابات تحليل الأبعاد لمسائل الكهرومغناطيسيات باستخدام خوارزمية غاوس-جوردان في قاعدة الأبعاد *ط ز ت ج* تعد أكثر كفاية وأقل عرضة للأخطاء وأسرع في اكتشاف الاعتماد الخطي بين معادلات الأبعاد. يتم استكشاف كلا من تفصيلات ومزايا الطريقة المقترحة من خلال أمثلة توضيحية، كلها تتمتع بأهمية واضحة في تعلم وتعليم الكهرومغناطيسيات.

الكلمات الدالة : تحليل الأبعاد، المضروبات عديمة الأبعاد، متغيرات الدخل والخرج، حذف غاوس-جوردان، قاعدة أبعاد معنية بالكهرومغناطيسيات، تعلم وتعليم الكهرومغناطيسيات.