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Geodetic Applications of Satellite Data

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Abstract. The global positioning *system* (GPS) provides an accurate data for locations on the Earth's surface. In this paper, data for longitudes and latitudes of more than seven thousand places in Saudi Arabia is considered. A database for qibla direction and geodesic distance to Makkah for these places is established. Spherical trigonometry is applied to perform geodetic computations. The database is illustrated and a sample of it for some main cities is presented.

Keywords: Qibla direction, Geodesic distance, Spherical Trigonometry, GPS.

Introduction

Qibla is the direction of the Kaaba (the sacred building at Makkah, Saudi Arabia) to which Muslims turn at prayer. The qibla is not only important for the prayers, but also relevant to everyday ceremonies in Islam. Therefore, determining the precise direction of the Kaaba has been always the concern of all Muslims around the World.

The qibla for a place on the Earth's surface is the angle between the North direction and the direction of the Kaaba with respect to this place. This angle is measured clockwise and ranged from 0 to 360 degrees [1].

On the other hand, geodesic distance is the shortest possible line between two points on a sphere or other curved surface.

On a sphere, geodesics are great circles. For instance, the shortest route from the north pole P to the south pole Q of the Earth is given by the shorter arc of the great circle passing through P and Q .

In this paper, more than seven thousand places in Saudi Arabia are treated. Qibla direction for these places is determined, and geodesic distance between Makkah and these places is computed. Basic spherical trigonometric formula [2, 3] is applied to determine the qibla direction, while Andoyer's formula [4] is used to compute the geodesic distance.

Data for longitudes and latitudes used in this paper are collected from [5], and they are expressed in degrees and minutes.

Qibla Direction

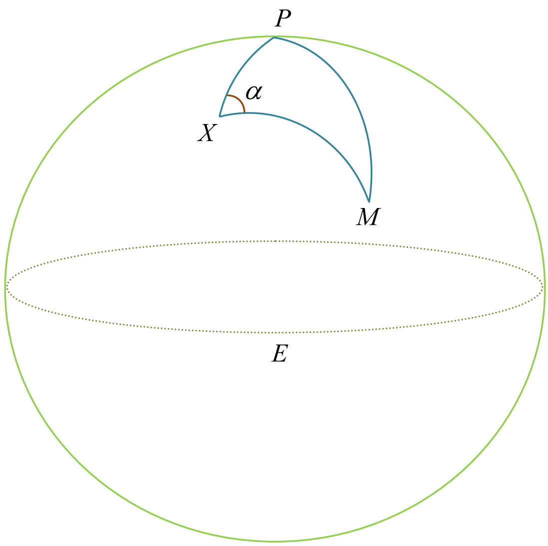

Fig. 1. Definition of qibla.

The problem of qibla involves spherical trigonometry, since it considers the angles between the shortest lines on a sphere. The qibla direction can be determined using more than one method such as; direct solar observation and shadow method. However, basic spherical trigonometric formula [2, 3] seems to be accurate enough for calculating the qibla direction.

Figure 1 shows a spherical triangle PXM, where

E : Equatorial plane

P : North pole

M : Makkah

α : Qibla

λ_M : Longitude of Makkah

ϕ_M : Latitude of Makkah

and

$\lambda_M = +39^\circ 50',$

$\phi_M = +21^\circ 25',$

$XP = 90 - \phi,$

$PM = 90 - \phi_M,$

$XPM = \lambda_M - \lambda.$

Consider a place X on the Earth's surface whose longitude λ and latitude ϕ, respectively, then the qibla direction is given by:

$$\alpha = tan^{-1}\left[\frac{sin(\lambda_M - \lambda)}{cos(\phi)tan(\phi_M) - sin(\phi)cos(\lambda_M - \lambda)}\right]. \qquad (1)$$

In the above equation, the quadrant where the angle α is located should be taken into account.

Geodesic Distance

Consider two places X_1 and X_2 on the Earth's surface. Let λ_1 and ϕ_1 be the longitude and latitude of X_1, respectively. Let λ_2 and ϕ_2 be the longitude and latitude of X_2, respectively. If we suppose that X_1 and X_2 are at sea level. Then the geodesic distance can be obtained using a high accuracy formula, by Andoyer [4], which considers the Earth's flattening:

$$\beta = D\left[1 + fA_1 sin^2(B)cos^2(C) - fA_2 cos^2(B)sin^2(C)\right], \qquad (2)$$

where

$$B = \tfrac{1}{2}(\phi_1 + \phi_2),$$

$$C = \tfrac{1}{2}(\phi_1 - \phi_2),$$

$$\lambda = \tfrac{1}{2}(\lambda_1 - \lambda_2),$$

$$F = sin^2(C)cos^2(\lambda) + sin^2(\lambda)cos^2(B),$$

$$G = cos^2(C)cos^2(\lambda) + sin^2(\lambda)sin^2(B),$$

$$H = tan^{-1}(\sqrt{F/G}),$$

$$J = \sqrt{FG}/H,$$

$$D = 2aH,$$

$$A_1 = (3J - 1)/2G,$$

$$A_2 = (3J + 1)/2F,$$

$$f = (a - b)/a,$$

and a (6,378.1 km) is the Earth's equatorial radius, b (6,356.8 km) is the polar radius and f is the Earth's flattening.

Example

Calculate the qibla direction and geodesic distance to Makkah, for the capital of Saudi Arabia, Riyadh.

The longitudes and latitudes of Makkah and Riyadh, as stated in [5], are:

$$\lambda_M = +39^\circ 50',$$

$$\phi_M = +21^\circ 25',$$

$$\lambda_R = +46^\circ 47',$$

$$\phi_R = +24^\circ 41',$$

Using *Mathematica* and applying equations (1) and (2), we get:

$$\alpha = 244^{\circ}.398,$$

$$\beta = 798.682 \text{ km}.$$

Summary

We have established a database which contains longitude, latitude, qibla direction and geodesic distance to Makkah for more than seven thousand places in Saudi Arabia. In this paper, we only present a sample of this huge database for some main cities, see Table 1.

In Fig. 2, we illustrate the longitudes and latitudes of more than seven thousand places in Saudi Arabia. Figures 3 and 4 show contour plots for qibla direction and geodesic distance. Three-dimensional plots of qibla direction and geodesic distance as functions of longitudes and latitudes are illustrated in Fig. 5 and 6.

Table. Qibla direction and geodesic distance to Makkah for some main cities.

City	λ	ϕ	α	β
Riyadh	$46^{\circ}\ 47'$	$24^{\circ}\ 41'$	$244^{\circ}.336$	798.682 km
Dammam	$50^{\circ}\ 05'$	$26^{\circ}\ 26'$	$243^{\circ}.990$	1181.590 km
Jeddah	$39^{\circ}\ 12'$	$21^{\circ}\ 29'$	$096^{\circ}.337$	066.062 km
Abha	$42^{\circ}\ 31'$	$18^{\circ}\ 13'$	$322^{\circ}.177$	452.207 km
Bahah	$40^{\circ}\ 33'$	$26^{\circ}\ 03'$	$188^{\circ}.206$	518.336 km
Arar	$41^{\circ}\ 01'$	$30^{\circ}\ 59'$	$186^{\circ}.603$	1066.470 km
Sakakah	$40^{\circ}\ 13'$	$29^{\circ}\ 58'$	$182^{\circ}.399$	947.989 km
Medina	$39^{\circ}\ 38'$	$24^{\circ}\ 27'$	$176^{\circ}.486$	336.547 km
Buraydah	$43^{\circ}\ 58'$	$26^{\circ}\ 20'$	$218^{\circ}.409$	688.168 km

Ha'il	41° 42′	27° 31′	195°.960	701.634 km
Jizan	42° 33′	16° 54′	330°.832	575.824 km
Najran	44° 12′	17° 32′	314°.037	628.371 km
Tabuk	36° 32′	28° 24′	156°.085	842.204 km

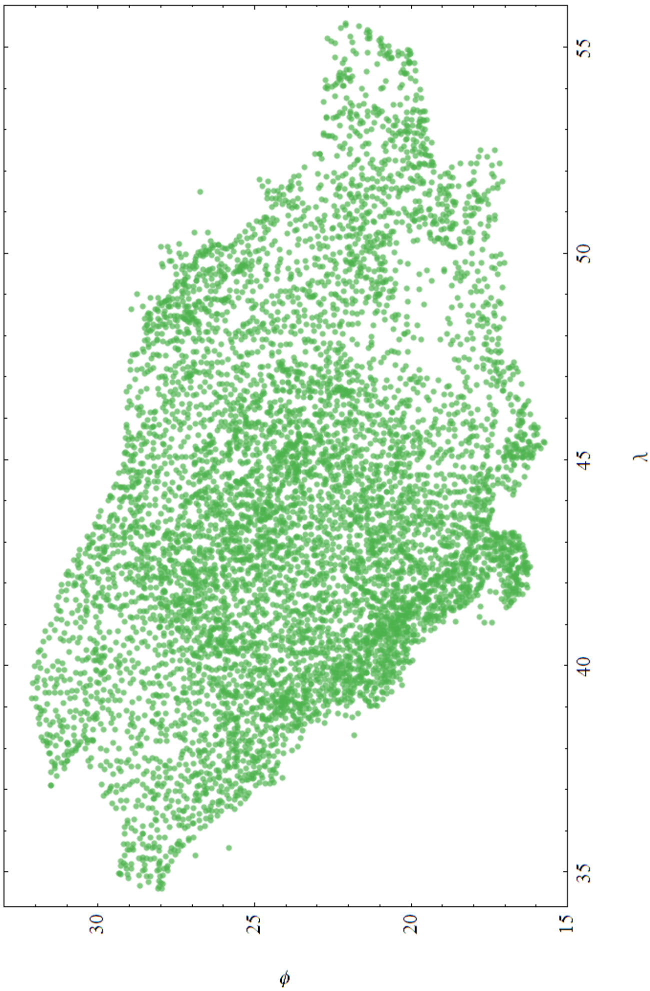

Fig. 2. Longitudes and latitudes of more than seven thousand places.

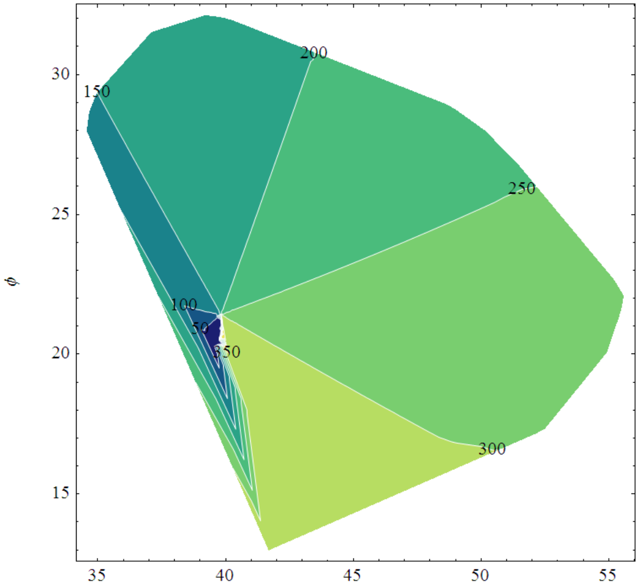

Fig. 3. A contour plot for the qibla direction.

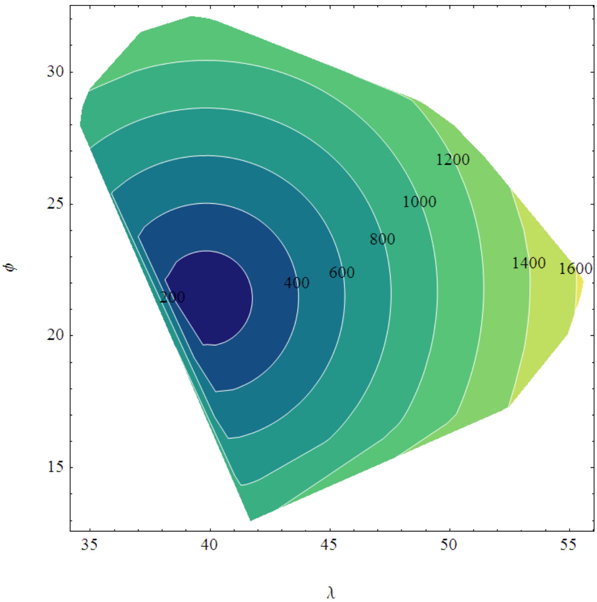

Fig. 4. : A contour plot for the geodesic distance to Makkah.

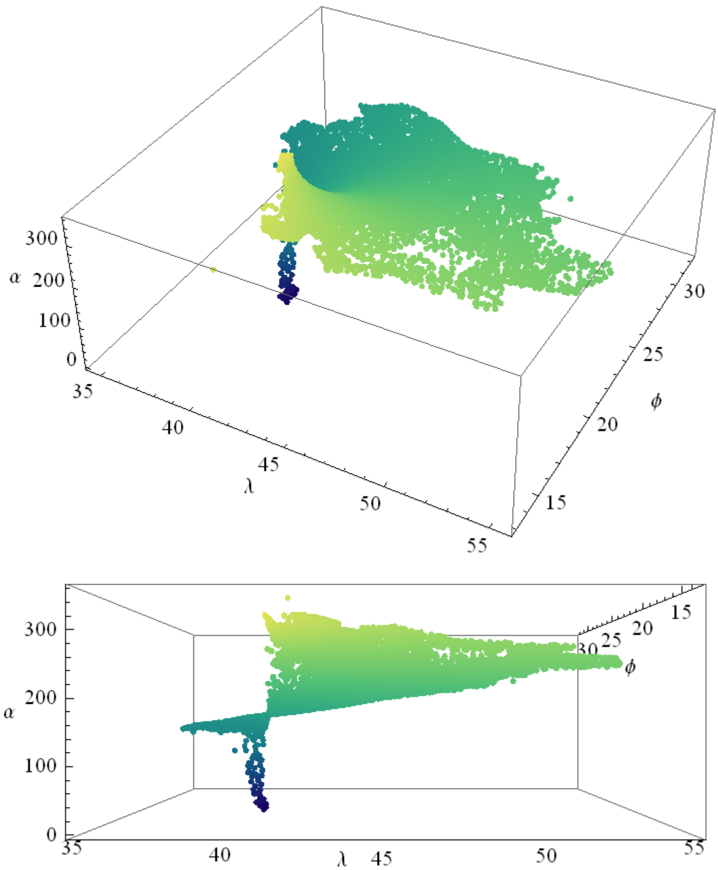

Fig. 5. Qibla direction for more than seven thousand places.

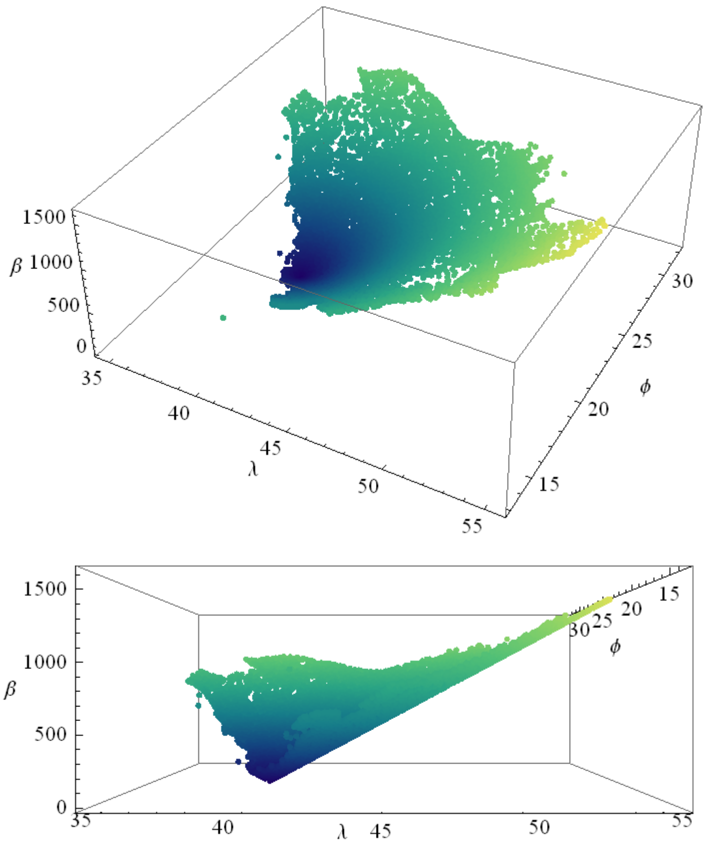

Fig. 6. Geodesic distance to Makkah for more than seven thousand places.

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تطبيقات جيوديسية لبيانات الأقمار الصناعية

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المستخلص: النظام العالمي لتحديد المواقع يوفر بيانات دقيقة عن المواقع على سطح الكرة الأرضية. في هذا البحث، نقوم بدراسة بيانات خطوط الطول ودوائر العرض لأكثر من سبعة آلاف مكان في المملكة العربية السعودية. حيث نؤسس قاعدة بيانات لاتجاه القبلة والمسافة الجيوديسية إلى مكة المكرمة لهذه الأماكن. نستخدم المثلثات الكروية لاجراء الحسابات الجيوديسية. أخيراً نمثل قاعدة البيانات بيانياً، بينما نقدم عينة منها تشتمل على بعض المدن الرئيسية.