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Mathematica Module for Frozen Orbits Characteristics

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Abstract: In this paper, *Mathematica* module frozen orbits characteristics is developed and illustrated numerically.

Key words: Frozen orbits, artificial satellites, remote sensing.

1. Introduction

Most satellites experience noticeable variations in orbital eccentricity and argument of perigee caused by the higher order potential harmonics (shape terms) of the Earth's gravitational field[1-4]. Fortunately, the distorting effects of these higher order harmonics can be induced to cancel one another provided the mission planners are free to choose the proper combination of *initial orbital altitude*, *inclination*, *eccentricity* and *argument of perigee.* When choices of this type are possible, skilled experts can, for example, achieve nearly constant and persistent values for the *mean eccentricity* and the *argument of perigee* of the of a satellite's orbit.

A satellite whose orbital parameters are controlled by this technique is said to be in a *Frozen Orbit*. Practical examples of satellite that have been launched into frozen orbits include:

● The Topex/Poseidon oceanographic satellite jointly sponsored by Jet Propulsion Laboratory and French government

● The Earth Radiation Budget Satellite used in studying the overall thermal properties of the Earth.

● The Landsat 4 and 5 Earth Resources Satellites

- Landsat 7 which is scheduled for launch in the second quarter of 1998.

Depending on the mission requirements, specific Frozen Orbits can be selected to achieve a variety of practical results. These include satellites whose orbital altitude decays uniformly with atmospheric drag, satellites with essentially no ground –trace variations due to perigee (apsidal) rotation, and satellites with extremely small altitude variations over the Northern Hemisphere(or conversely over the Southern Hemisphere)

In the present paper, *Mathematica* module of the frozen orbits characteristics is developed and illustrated numerically.

2. Basic Formulations

As mentioned, the frozen orbit technique is used to design orbits that minimize global variations in altitude by nulling the long-periodic variations in the eccentricity e and argument of perigee ω [1], Consider the simplified example of an Earth satellite perturbed only by the J_2 and J_3 zonal harmonics[5] The long-periodic argument of perigee ω rate obtained after averaging the short- periodic variations is:

$$\frac{d\omega}{dt}=\frac{3n}{(1-e^2)^2}\,J_2\left(\frac{R}{a}\right)^2\left(1-\frac{5}{4}\sin^2 i\right)\theta$$

$$\mu = n^2 a^3$$

$$\theta = 1+\frac{J_3}{J_2}\left(\frac{R}{a}\right)\left(\frac{1}{1-e^2}\right)\left(\frac{\sin^2 i - e\cos^2 i}{\sin i}\right)\frac{\sin\omega}{e}$$

The long-periodic eccentricity rate is:

$$\frac{de}{dt}=-\frac{3}{2}\,\frac{n}{(1-e^2)^2}\,J_3\left(\frac{R}{a}\right)^3\sin i\times\left(1-\frac{5}{4}\sin^2 i\right)\cos\omega$$

where, R is the Earth's equatorial radius, while, i, n, and a are the satellite's orbital inclination, mean motion, and the semi-major axis respectively, finally μ is the gravitational parameter. Note that $d\omega/dt$ vanishes at critical i_ω or when $\theta = 0$. For given i and a ,values of $e = e_0$ and $\omega = \omega_0$ exist which satisfy the condition $\theta = 0$. However, a nonvanishing eccentricity rate will cause the eccentricity value to change so the condition $\theta = 0$ is no longer satisfied. Hence, $d\omega/dt$ becomes nonzero and ω drifts from the value selected to help satisfy the condition $\theta = 0$.

The eccentricity rate vanishes if $\mathrm{i}=0, \mathrm{i}=\mathrm{i}_{\omega}$, or $\omega_0 = 90^{\circ}, 270^{\circ}$. Any of these conditions results in a frozen eccentricity. The argument of perigee is frozen for $\mathrm{i}=\mathrm{i}_0, \mathrm{i}_{\omega}$, and for specific values of e, ω that satisfy $\theta = 0$ These frozen–orbit elements are essentially single –averaged, mean elements resulting from eliminating the short –periodic variations.

3. Mathematica Module

FrozenOrbits [a , ω , e0 , i0 , eps , nmax] : =

Module

$\{\mu = 398600.64 \ , \ \mathrm{R} = 6378.14 \ , \ \mathrm{J2} = 0.001082627 \ , \ \mathrm{J3} = -0.0000025364 14\}$,

$x = \{e, i\}; x0 = \{e0, i0\}; n = sqrt[\mu / a\wedge 3]; p = a*(1-e\wedge 2);$

$\mathrm{f1} = 1.5*\mathrm{J3}*(\mathrm{R/a})\wedge 3*\mathrm{n}*\mathrm{Sin[i]}*\mathrm{Cos}[\omega]*(1.25*\mathrm{Sin[i]}\wedge 2-1)/(1-\mathrm{e}\wedge 2)\wedge 2;$

$\mathrm{g2} = 3*\mathrm{n}*\mathrm{J2}*(R/a)\wedge 2*(1-1.25*\mathrm{Sin[i]}\wedge 2)/(1-\mathrm{e}\wedge 2)\wedge 2;$

$\mathrm{g4} = 1 + (J3/J2)*(R/a)*(1/(1-e\wedge 2))*((Sin[i]\wedge 2 - e*Cos[i]\wedge 2)/Sin[i])*(Sin[\omega]/e)$

f= {f1 , g2*g4} ;

With

[{jac = N[outer [D, f, x]]}, FixedPoint List [(#+LinearSolv e [jac/Threa d [x → N[#]],-f/.Thr ead [x → N[#]]])δ,

$N[x0], n\max, SameTest \to (Sqrt[(\#1-\#2).(\#1-\#2)] \prec eps\,\delta)]]]$

The stopping criterion is the 2-norm (or the square root of the sum of the squares)of the difference between the last two iterations: we can give the eps for this criterion (users of Mathematica 5 can use $(Norm[\#1-\#2] \prec eps\,\delta)$ in the stopping test). The new point could be calculated from $x_{n+1} = x_n - j_n^{-1} f_n$ where j_n is the Jacobian of the system (jac in the program) . However . we can avoid calculating the inverse of j_n by solving the linear system $j_n \delta_x = -f_n$ and then calculating $x_{n+1} = x_n + \delta_x$. This method is faster.

4. Numerical Example

Consider the given values :

$$a = 7711.94km; c0 = 0.0001; i0 = {}^{180}\!/_{\pi}; \omega = {}^{\pi}\!/_{3}; n\max = 10; eps = 10^{-6}$$

The out put of the program is listed as :

e	$d\omega / dt$
0.0001	57.2958
0.000105094	57.8311
0.000104674	57.6259
0.000104793	57.6557
0.000104794	57.6558
0.000104794	57.6558

5. Conclusion

This method is faster and we can avoid calculating the inverse of j_n .

References

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نموذج باستخدام برنامج الـ Mathematica لخصائص المدارات المجمدة

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المستخلص: في هذا البحث تم عمل نموذج ببرنامج الـ *Mathematica* لخصائص المدارات المجمدة، وبتطوير هذا النموذج، تم الحصول على نتائج باستخدام الطرق العددية، وبهذه الطريقة أمكن الاستغناء عن حل معكوسة جاكوبين بحل النظام الخطي لجاكوبين دلتاx.