FULLY ADAPTIVE HIGH PASS FILTER AND ADAPTIVE LAPLACIAN OPERATOR USING THE TDLMS ALGORITHM

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ABSTRACT. This paper presents a modified TDLMS adaptive algorithm, which can be implemented to perform as an adaptive HP Filter or as an adaptive Laplacian edge detector. This adaptive filter combines both processes involved in edge enhancement or in edge detection. The pre-filtering for noise smoothing (regularisation) and applying the HP or the Laplacian mask, into only one adaptive filtering process. The filter changes coefficients depending on the image local statistics. The proposed technique is simple, stable and produces efficient results and reduces the sensitivity of the HP filter and the Laplacian mask to the noise. Results are presented to illustrate these new edge enhancement and edge detection techniques and to illustrate the noise suppression effects of the proposed method.

1. INTRODUCTION

High Pass filters (HPF masks) are used in image processing for the purpose of edge enhancement (image sharpening) and Laplacian masks for edge detection. These techniques are widely used and well documented [1,4]. Number of disadvantages are associated with these methods. First, they are designed independently of the image data. Second, they are very sensitive to the noise, and noise amplification may occur. Third, they are typically designed locally in a square shape to isolate edges within the operator kernel. One of the most widely used operator of this class is the fixed coefficient Laplacian ($\nabla^2$).

In [1] they proved that the problem of image edge detection is ill-posed, and the differentiation process involved is not robust against noise. Also they illustrated that filtering of the image prior to differentiation is necessary for regularising the problem and making it well-posed. In order to increase the noise immunity, the operator masks can be extended using larger neighbourhood.

In this paper we propose the use of Two-Dimensional Adaptive LMS filter (TDLMS) for image HPF and edge detection. A modified version of this adaptive filter combines both processes involved in edge detection, pre-filtering for regularisation and applying the HP or the Laplacian operator into only one adaptive filtering process. The filter changes coefficients and characteristics during the adaptation process depending on the image local statistics. The proposed technique is simple, stable and produces efficient results. Also it reduces the sensitivity of the HP filter and the Laplacian operator to noise.

The organisation of the paper is as follows: Section 2 reviews the problems of high-pass filtering and edge detection, section 3 presents the TDLMS algorithm. In section 4 we present the proposed adaptive HPF and adaptive Laplacian. In section 5 an all adaptive self regularising HPF and Laplacian mask are presented. Results and Comparisons with some other HPF and Laplacian techniques are given in section 6.
2. HIGH PASS FILTERING AND EDGE DETECTION.

Image sharpening is normally achieved by using enhancement operators which perform HP filtering. Most of these operators are based on digital approximations of analogue differentiation operators. The difference operator is the simplest digital approximation to differentiation. In two dimensions, the digital Laplacian operator can be taken as:

\[ y(n_1, n_2) = x(n_1 + 1, n_2) - 2x(n_1, n_2) + x(n_1 - 1, n_2) + \]
\[ x(n_1, n_2 + 1) - 2x(n_1, n_2) + x(n_1, n_2 - 1) \quad (1) \]

The mask of this Laplacian and its corresponding HPF mask are:

\[ L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -5 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (2) \]

Other Laplacian and HPF masks can be found in literature. The Laplacian can provide several image processing enhancements, for example, image sharpening and image deblurring.

3. THE TWO-DIMENSIONAL LEAST MEAN SQUARE (TDLMS) ALGORITHM.

The TDLMS adaptive algorithm illustrated in Fig. (1) is based on the method of steepest decent. In this method, the next weight matrix is equal to the present weight matrix plus a change proportional to the negative gradient of the error power. It is a method to find an approximate solution for the optimum Wiener-Hopf equation without the calculation of any correlation matrices. The filter output is given by

\[ y(m, n) = \sum_{l}^{l} \sum_{k}^{k} W_{j}(l, k) X(m - l, n - k) \quad (3) \]

where \( W_{j} \) is the filter coefficient matrix and \( X \) is the delayed input image (called the reference input). The error signal \( e_{j} \) at the \( (m,n) \)th position is then given as:

\[ e_{j} = D(m, n) - \sum_{l}^{l} \sum_{k}^{k} W_{j}(l, k) X(m - l, n - k) \quad (4) \]

where \( D \) is the input image (primary input). According to the TDLMS technique, the two dimensional weight update algorithm is given as:

\[ W_{j+1} = W_{j} - \mu_{j} G_{j} \quad (5) \]

where : \( W_{j+1} \) is the weight matrix after updating, \( W_{j} \) is the weight matrix before updating, \( \mu_{j} \) is a scalar multiplier controlling the rate of convergence and the algorithm stability, \( G_{j} \) is an estimate for the two dimensional instantaneous gradient of \( E[e_{j}^{2}] \) with respect to \( W_{j} \), where

\[ G_{j} = \frac{\partial E[e_{j}^{2}]}{\partial W} \quad (6) \]

and \( j \) is the one dimensional running position index \( j = mM + n \) on the \( M \times N \) two dimensional array. In the TDLMS algorithm, an estimate of the instantaneous gradient is given by:

\[ \hat{\sigma}_{j} = \frac{\partial E[e_{j}^{2}]}{\partial W}_{j = \sigma_{j}, k = \sigma_{k}} = \nabla X(m - l, n - k) - x_{j}(l, k) \quad (7) \]

and

\[ -L \leq l \leq L, \quad -K \leq k \leq K \]

Substituting from (7) into equation (5) we get

\[ W_{j+1} = W_{j} + 2 \mu_{j} e_{j} x_{j} \quad (8) \]

which can be rewritten as

\[ W_{j+1}(l, k) = W_{j}(l, k) + 2 \mu_{j} e_{j} X(m - l, n - k) \quad (9) \]
Equations (9) give the two dimensional weight updating algorithm for the TDLMS adaptive filter. It is worth mentioning that the scalar multiplier \( \mu \) can be optimised and updated according to the changes in the image local statistics according to one of the techniques given in [7,8]. As shown through the applications [4,5], this adaptive technique changes characteristics according to the changes in the image local statistics. Also, it filters the noise in the image while keeping the edges and fine details with a very small distortion.

![Diagram of the Two-Dimensional LMS adaptive filter](image1)

![Diagram of the Modified Two-Dimensional LMS adaptive HP and Laplacian filter](image2)

**Figure 1** The Two-Dimensional LMS adaptive filter

**Figure 2** The Modified Two-Dimensional LMS adaptive HP and Laplacian filter

### 4- ADAPTIVE HPF AND ADAPTIVE LAPLACIAN.

In this paper, by imposing some simple constraints on the coefficients of the adaptive filter in Fig.1, the filter changes characteristics to an adaptive HPF or to the adaptive Laplacian characteristics. Figure 2 shows the modified Two-Dimensional adaptive filter, where the input image is used as the primary input of the adaptive filter. This image is delayed to obtain the reference input of the adaptive filter. The output of the filter is an estimate of the signal in the primary input. Subtracting the filter output from the primary input produces an error signal which is fed back to the filter to be used for updating the coefficients before next iteration according to equations (8 or 9). The adaptive filter has the coefficients corresponding to the elements in a specified window surrounding the pixel under calculation. Keeping in mind that the method is applicable to any matrix dimensions or to the one dimensional case, consider the following 3x3 Two-Dimensional filter coefficient matrix as an example:

\[
W_j = \begin{bmatrix}
W_j(-1,-1) & W_j(-1,0) & W_j(-1,1) \\
W_j(0,-1) & W_j(0,0) & W_j(0,1) \\
W_j(1,-1) & W_j(1,0) & W_j(1,1)
\end{bmatrix}
\]  

(10)

The output of the filter is given by equation (5). Where in case of 3x3 matrix both \( L = 1 \), and \( K = 1 \). By imposing the following constraint after each iteration:

\[
W_j(0,0) = 1 + \left( \sum_{l=-1}^{1} \sum_{k=-1}^{1} W_j(l,k) \right)
\]

(11)

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we obtain an adaptive filter with HPF characteristics. (The summation is over all the values of \( W_j(l,k) \) other than \((l,k) = (0,0)\)). The value of the centre element \( W_j(0,0) \) can assume any suitable value. This value depends on the amount of sharpening required by the filter. If the value of \( W_j(0,0) \) assumes a positive value > 1, the sum of the other elements in the matrix is negative and is less in magnitude than \( W_j(0,0) \) by 1. This form gives a matrix with adaptive coefficients and similar in form to the fixed coefficients HPF masks. If \( W_j(0,0) \) is taken as a negative value and the sum of the other elements in the matrix will assume positive value less the magnitude of \( W_j(0,0) \) by 1. The filter output in this latter case is phase reversed by 180° than the output of the first case. Similarly, by changing the constraint in equation (11) to be:

\[
W_j(0,0) = 0 + (\sum_{i=-i}^{i} \sum_{k=-k}^{k} W_j(l,k))
\]

\((l,k) \neq (0,0)\)

an adaptive Laplacian is obtained.

An advantage of these adaptive masks is that the coefficients are estimated and optimised adaptively without any pre-processing or any calculations of correlation functions. The second advantage is that the amount of sharpening is governed by the value chosen for \( W_j(0,0) \). The larger the amount of sharpening is required the larger the magnitude of \( W_j(0,0) \) is chosen. The third advantage is that the other coefficients are scaled automatically, through the updating process, according to the selection of \( W_j(0,0) \) and the changes in image statistics.

A common limitation of the HP filters and the Laplacian masks is the amplification of the high frequency noise in the image. This is a result of the inherent differencing operation involved. The Two-Dimensional adaptive HP filter and the Laplacian masks obtained are still sensitive to the noise as the case of their fixed counterparts. In the next section we propose an adaptive regulation method which results in adaptive HP filter and Laplacian masks less sensitive to the noise while they are filtering the image data.

5. SELF REGULISED ALL ADAPTIVE HPF AND ADAPTIVE LAPLACIAN

In this section, we propose a modification to the method proposed in the previous section in order to reduce the filter sensitivity and amplification of the noise. A regularisation [1,3] process is implemented during the HP filtering. The adaptive coefficients of the regularising filter are impeded inside the HPF or the Laplacian masks. This all adaptive self-regulating method will be explained in the following 3x3 simple example. The method is equally applicable for any matrix of a any other dimensions. Consider the adaptive coefficient matrix

\[
W_j = \begin{bmatrix}
H_j(-1,-1) & W_j(-1,0) & H_j(-1,1) \\
W_j(0,-1) & W_j(0,0) & W_j(0,1) \\
H_j(1,-1) & W_j(1,0) & H_j(1,1)
\end{bmatrix}
\]

(13)

Imposing the modified constraint

\[
\sum_{p,q} W_j(p,q) = 1 + \sum_{i,k} H_j(i,k)
\]

(14)

where the values of \( l, k, p, q \) are as shown in the matrix (13) and both \( H_j(l,k) \) and \( W_j(p,q) \) are the adaptive coefficients of the modified adaptive HPF and both are updated independently using the TD-LMS algorithm according to equations (8 or 9). The elements \( W_j(p,q) \) of the matrix in (13) partially represents an adaptive smoothing LPF implemented on top of the inside elements the adaptive HPF mask.
Similarly the following constraint is applied to obtain an adaptive Laplacian mask:
\[ \sum_{p,q} W_j(p,q) = 0 + \sum_{l,k} H_j(l,k) \]  
(15)

The value of the sum \( \sum_{p,q} W_j(p,q) \) is chosen in a similar way of choosing the values of \( W_j(0,0) \) in the previous section. The sum \( \sum_{l,k} H_j(l,k) \) is also adaptively adjusted to satisfy equations (14 or 15) according to the choice of the sum \( \sum_{p,q} W_j(p,q) \) and according to the image statistics.

6. RESULTS.
In this section we presents the results of applying the adaptive method proposed in this paper. The results are also compared with the methods of fixed HPF and Laplacian masks.

6.1 HPF filtering:
The original image used is shown in Fig. 3-a, the image in Fig.3-b is the noisy image with noise variance of 400.0 which gives a S/N ratio of 9 dB.

![Figure 3 Input images](image1)

(a)  
(b)

Figure: 3 Input images, (a) original image, and (b) noisy image with S/N ratio of 9 dB.

Figures 4-a and 4-b show the results of using fixed 3x3 HPF on the images in Fig. 3-a and in Fig. 3-b respectively. The result in Fig. 4-a is sharp, and the result in Fig. 4-b shows that the fixed filter amplifies the noise in the image. Figure 4-c shows the result of using adaptive 3x3 HPF on the original image in Fig. 3-a. The result is comparable to that in Fig. 4-a using fixed filter. Figure 4-d displays the result of using 3x3 adaptive HPF in section 4 on the noisy image in Fig. 3-b. The result is sharp and noise amplifications controlled. Fig. 5-a illustrates the results of using adaptive 5x5 self regularising HPF on the noisy image in Fig. 3-b. The result shows that increasing the size of the adaptive self regularising HPF reduces the noise while HP filtering and shows very small amount of noise as compared to the fixed filter result in Fig. 4-a. Figure 5-b shows improved result for adaptive HP filtering the noisy image where the shape of the inside LP filter window is changed to a 3x3 filter. This shows a great suppression of the noise as compared to Fig. 4-a of the fixed filter.
Figure: 4 Fixed and adaptive High Pass filters results, (a) Output of a 3x3 fixed HPF mask from original image in Fig. 3-a, (b) Output of a 3x3 fixed HPF mask from noisy image in Fig. 3-b, (c) The output of a 3x3 adaptive HPF presented in section (4) from the original image in Fig. 3-a, and (d) is the output of a 3x3 adaptive HPF presented in section (4) from noisy image in Fig. 3-b.

Figure: 5 Modified self regularising adaptive HP filter in section (5) results, (a) Output of a 5x5 adaptive HPF filter from noisy image in Fig. 3-b, (b) Output of a 5x5 adaptive HPF filter (type 2, i.e. with a 3x3 LPF in the centre) from noisy image in Fig. 3-b.
6.2 Laplacian:
Figures 6-a and b show the results of using fixed 3x3 Laplacian on the images in Fig. 3-a and in Fig. 3-b respectively. The result in Fig. 6-a shows a clear and thin edges but the result in Fig. 6-b shows that the fixed Laplacian filter amplifies the noise in the image. Figure 6-c shows the result of using adaptive 3x3 self regularising Laplacian on the original image in Fig. 3-a. The result is comparable to that in Fig. 6-a using fixed Laplacian filter. Figure 6-d shows the results of using an adaptive 3x3 self regularising Laplacian on the noisy image in Fig. 3-b. The result shows that the adaptive self regularising Laplacian reduces the noise while filtering and shows smaller amount of noise as compared to the fixed Laplacian filter result in Fig. 6-b.

Figure: 6 Fixed and adaptive Laplacian masks results, (a) Output of a 3x3 fixed Laplacian mask from original image in Fig. 3-a, (b) Output of a 3x3 fixed Laplacian from noisy image in Fig. 3-b, (c) The output of a 3x3 adaptive Laplacian presented in section (4) from the original image in Fig. 3-a, and (d) is the output of a 3x3 adaptive Laplacian presented in section (4) from noisy image in Fig. 3-b.

Improved results can be obtained by increasing the size of the Laplacian masks. Fig. 7-a shows the result of using 5x5 Laplacian mask on the image in Fig. 3-b. The result in Fig. 7-a shows the improvement obtained when using adaptive self regularising filter and when increasing the filter size. Further improvement in noise reduction can be obtained by changing the shape of the Low pass filter window inside the Laplacian mask. A 3x3 LPF window is used inside the 5x5 Laplacian. This is called type 2 filter, which produced the result in Fig. 7-b, it shows a great reduction in noise.
Figure: 7 Self regularising adaptive Laplacian in section (5) results, (a) Output of a 5x5 adaptive Laplacian filter from noisy image in Fig. 3-b, (b) Output of a 5x5 adaptive Laplacian filter (Type 2, i.e. with 3x3 LPF in the centre) from noisy image in Fig. 3-b.

7. CONCLUSIONS

This paper presented a modified TDLMS adaptive algorithm, which can be implemented to perform as adaptive HP filter or as adaptive Laplacian edge detector. This adaptive filter combines both processes involved in edge enhancement or in edge detection, the pre-filtering for noise smoothing (regularisation) and applying the HP or the Laplacian mask, into only one adaptive filtering process. The filter changes coefficients depending on the image local statistics. The proposed technique is simple, stable and produces efficient results and reduces the sensitivity of the HP filter and the Laplacian mask to the noise. Results are presented to illustrate these new edge enhancement and edge detection techniques and to illustrate the noise suppression effects of the proposed method.

REFERENCES