Refined chloride mass-balance method and its application in Saudi Arabia

Ali Subyani¹* and Zekai Şen²

¹ King Abdulaziz University, Faculty of Earth Sciences, Hydrogeology Department, PO Box 80206, Jeddah 21589, Saudi Arabia
² Saudi Geological Survey, Hydrogeology Division, PO Box 51141, Jeddah 21514, Saudi Arabia

Abstract:

The rainfall and infiltration elements of the hydrological cycle in arid regions are characterized by temporal and spatial variations that are random and sporadic. Consequently, the chloride concentration in rainfall has a similar behaviour. Despite this, the classical chloride mass balance (CMB) approach only employs arithmetic and weighted averages for recharge estimation. In this paper, the classical CMB method is modified by taking into account some perceived deficiencies in the methodology. The modified CMB method takes into consideration additional statistical parameters, namely variances and the correlation coefficient between variables concerned based on the application of the perturbation method. Strategic aquifer planning in the Kingdom of Saudi Arabia requires a quick method for estimating groundwater recharge in order to determine the temporal management of available water resources. To demonstrate the difference between the classical and the refined CMB methods, both were applied to a representative basin, i.e. Wadi Yalamlam, in the western part of Saudi Arabia. Based on the refined calculations, recharge to groundwater is found to be 11% of the effective annual rainfall. This refined method provides higher recharge rates because it takes into account the actual variability in the variables concerned and can, thus, improve the accuracy of future groundwater recharge estimation studies. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS arid region; chloride; rainfall; recharge; Saudi Arabia

INTRODUCTION

In arid and semi-arid regions, groundwater is a significant part of the total water resources. Although evaluating the recharge component is an essential part of any groundwater balance assessment, acquiring field measurements to perform such an evaluation is often not an easy task. This is particularly true upstream, due to inaccessibility, climatic heterogeneity, variability of geological characteristics, etc. In dry climates, the recharge component of the hydrologic cycle becomes the most significant element after rainfall occurrences; its direct calculation is not possible, however. The recharge rates in arid and semi-arid regions are small and must, therefore, be carefully and accurately estimated. Since groundwater storage rates in the alluvial aquifers of the middle and lower wadis depend on the recharge amount, an accurate determination of the latter is essential for planning and programming vital human activities, domestic and agricultural, that are dependent on short- and long-term assessments of storage volumes.

In the Kingdom of Saudi Arabia, there are now strategic plans for the exploitation and use of groundwater resources, especially in the western regions along some potential wadi courses. Accurate recharge estimation is, therefore, vital in such arid and semi-arid regions of the country. Consideration of these difficulties, especially during the last decade, has led many researchers to use the simple method of the chloride (Cl⁻) mass balance (CMB) approach. The Cl⁻ ion is used in chemical recharge studies because of its conservative nature; it is neither leached from nor absorbed by sediment particles. This is based on the assumption that the

* Correspondence to: Ali Subyani, King Abdulaziz University, Faculty of Earth Sciences, Hydrogeology Department, PO Box 80206, Jeddah 21589, Saudi Arabia. E-mail: asubyani@hotmail.com

Received 16 August 2004
Accepted 10 August 2005
\( \text{Cl}^- \) concentrations in the rainfall and the recharge areas are in a steady-state balance. That is, input is equal to output without \( \text{Cl}^- \) storage change during a specific time period, often taken as 1 month or a year. The CMB method underestimates recharge rates in non-irrigated areas compared with other methods. It agrees with these methods, however, in irrigated areas (Grismer et al., 2000; Flint et al., 2002; Russo et al., 2003).

It has been shown by Wood and Sanford (1995) and Wood and Imes (1995) that the CMB method can yield regional rates of recharge under certain conditions and assumptions. On the other hand, Bazuhair and Wood (1996) stated that the CMB method yields groundwater recharge rates that are integrated spatially over the watershed over tens of thousands of years. Various applications of the CMB method have been presented for different parts of the world (Eriksson and Khunakasem, 1969; Eriksson, 1976; Allison and Hughes, 1978; Grismer et al., 2000; Edmunds et al., 2002; Harrington et al., 2002; Scanlon et al., 2002). However, it is not clear how temporal and spatial variations are integrated into the recharge calculations. In the classical CMB equation, only averages are used without quantitatively considering spatial or temporal variations, by considering additional statistical variables in the calculations.

The main purpose of this paper is to present a formal CMB equation derivation on the basis of the perturbation methodology (Hinch, 1991). A refined CMB equation is thus obtained, which is both mathematically and statistically sound; no arbitrary averages, such as the weighted average, are incorporated, as is the case in the classical CMB approach. The perturbation methodology dictates that additional statistical parameters, such as the standard deviations and correlation coefficient of the basic variables, appear in the final formulation of the average recharge estimation by the CMB method. The refined CMB formulation is then applied to the case of the upstream of Wadi Yalamlam basin in the western part of Saudi Arabia along the Red Sea coast. Although, Allison and Hughes (1978) explicitly explained the uses of averages, they were concerned with averaging of the measured \( \text{Cl}^- \) concentrations in the soil or groundwater, i.e. spatial averaging.

**CLASSICAL CMB METHOD**

\( \text{Cl}^- \) is used for recharge estimation because of its conservative nature and its relative abundance in precipitation. The application is based on comparison of the \( \text{Cl}^- \) deposition rate at the soil surface with the concentration in the soil water or groundwater. The \( \text{Cl}^- \) concentration increases relative to the concentration of rainwater as a result of interception, soil evaporation, and/or root water uptake by the vegetation. The total (wet and dry) \( \text{Cl}^- \) deposition and the total precipitation depth determine the \( \text{Cl}^- \) concentration of the rainwater at the surface. Subsequent evaporation can then be estimated from the increase in concentration, provided that no other major sources of \( \text{Cl}^- \) exist. The same assumption is valid for groundwater recharge estimation. The method is usually termed the \( \text{Cl}^- \) concentration method or, more frequently, the CMB method.

The application of the CMB method is simple, with no sophisticated instrumental dependence. It is based on the knowledge of annual precipitation and \( \text{Cl}^- \) concentrations in rainfall and groundwater storage. In arid regions, however, recharge is sporadic, depending on rare rainfall events during a year. Moreover, rainfall occurs at the upstream portion of wadis, where topographically controlled orographic rainfall often takes place over locally concentrated areas. This leads to small recharge areas where rainfall reaches the water table through infiltration processes. Under these conditions, the extent of spatial averaging will depend to some extent on the method of groundwater sampling, particularly the length of screened intervals of piezometers. Some spatial averaging will occur within the aquifer due to dispersion processes. Recently, Harrington et al. (2002) attempted to quantify spatial averaging inherent in the CMB method. The \( \text{Cl}^- \) concentration is homogeneously distributed within the aquifer. Rainfall duration is comparatively very short in arid regions. After its occurrence, owing to the rather intensive solar irradiation, evapotranspiration takes place from the moist and unconfined soil surfaces, increasing the \( \text{Cl}^- \) concentration. Generally, the application of the CMB method implicitly incorporates a set of assumptions that should be taken into consideration in interpreting the results. The assumptions in the CMB approach for recharge calculations are that:
1. There is no Cl\textsuperscript{−} source in groundwater storage prior to the rainfall. This is the reason why the application of the method is valid for the upstream portions of the catchments.

2. There are no additional sources or sinks for Cl\textsuperscript{−} concentration in the area of application. This is mostly a valid assumption in the upstream portions of the wadis in the Kingdom of Saudi Arabia. In thousands of geochemical studies within the wadis, Cl\textsuperscript{−} is found only in negligible amounts, except after a rainfall event (Basmaci and Al-Kabir, 1988; Subyani and Bayumi, 2001; Bazuhair et al., 2002).

3. The rainfall either evaporates or infiltrates in the region without any runoff, which is a rather unrealistic assumption and can only be valid for low-intensity rainfall events. However, most often, rainfalls are intense, especially at the upstream portions of the wadi system, due to orographic conditions.

4. Long-term rainfall and its Cl\textsuperscript{−} concentration amounts have a balanced situation, i.e. they are in a steady-state condition. This implies stable and long-term averages, as the classical CMB method requires. The standard deviations around the averages are completely ignored in this assumption, however. This is implicitly equivalent to assuming that the fluctuations around the average rainfall and Cl\textsuperscript{−} records are negligibly small and are hence ignored in the classical CMB equation.

On the basis of these assumptions, the fundamental equation applicable to recharge calculations is presented by Wood and Sanford (1995) as

\[
q = R \frac{C_l \text{war}}{C_l \text{gw}}
\]

in which \( q \) is the recharge flux, \( R \) is average annual rainfall, \( C_l \text{war} \) is the weighted average Cl\textsuperscript{−} concentration in rainfall, and \( C_l \text{gw} \) is the average Cl\textsuperscript{−} concentration in groundwater. The inspection of this classical CMB equation raises several significant questions pointing to the inconsistency and arbitrariness in the use of averages and, therefore, the subsequent interpretation of its results. Among the arguments regarding this equation are:

1. It is useful for long-term-average rainfall and Cl\textsuperscript{−} concentration estimation both in groundwater and rainfall, and it is more accurate for less variability; the equation is more valid under steady-state conditions. Otherwise, the equation must be viewed as a gross simplification.

2. More important than the first point is the use of averages in the equation. For instance, it is stated that \( q \) is the recharge flux, which may mean the amount of water reaching the groundwater storage per time per area. It is not specified, however, whether it is an average and, if so, whether it is arithmetic or weighted?

3. Classical CMB equation does not have homogeneity in ‘averages’, similar to ‘dimensional’ homogeneity in scientific methodologies. The question is: is it on an arbitrary basis that one selects an arithmetic or a weighted average for each variable in the equation?

4. What about the deviations around the averages? Equation (1) does not account at all for such variations.

**REFINED CMB METHOD**

After these criticisms, the view taken in this study is to present a refined CMB approach with a sound mathematical and statistical treatment. It is certain that, during a short duration, Equation (1) is valid without a description of averages for its terms. In this form, it is derived from the physical mass conservation principle with no change in the storage, as simple as input is equal to output, i.e. the steady-state condition.

In practice, however, it is necessary to consider rather long time intervals, such as a month or a year, and the application area will, at least, be several square kilometres. These temporal and spatial scales give rise to variations in each term of the CMB equation, both temporally and spatially. Since there are many sampling measurements over such time and space scales, the questions are which one of these measurements and what types of average must be inserted into Equation (1). The simplest answer will be to consider using the arithmetic average of each term, which leads to an expression similar to Equation (1) with explicit averages:

\[
\bar{q} = \bar{R} \frac{C_l \text{ir}}{C_l \text{gw}}
\]
where the overbars indicate arithmetic averages. The subscript in the Cl$^-$/NUL concentration is indicated by ‘r’ and not by ‘war’ as in Equation (1). This indicates that one cannot decide arbitrarily on the weighted-average concept. Equation (2) has homogeneity in ‘averages’ and is, in this sense, better than Equation (1), because there is no arbitrary use of averages. Equation (2) is written without a mathematical or statistical basis, but on the basis of common sense, which is not quite valid in practical applications, as will be shown in the following presentation.

In the case of random variations in each term in Equation (1) during a certain time period, hydrogeologists will have to sample the phenomena several times. Practically, none of these measurements will be equal or even close to one another and, therefore, there are deviations around the averages. Hence, the basic variables should be considered in conjunction with their averages and their deviations as follows:

$$q = \bar{q} + q'$$
$$R = \bar{R} + R'$$
$$Cl_r = \overline{Cl_r} + Cl'_r$$

and

$$Cl_{gw} = \overline{Cl_{gw}} + Cl'_{gw}$$

Each one of these equations is referred to as the perturbation of the variable concerned with deviations (the primed variables) added to the arithmetic average.

In order to simplify the mathematical derivations, it is assumed herein that Cl$^-$/NUL concentration deviations within the aquifer are homogeneous due to long-term mixture; therefore, its deviations will be ignored, which means that $Cl_{gw} = \overline{Cl_{gw}}$. The substitution of Equations (3)–(6) with this simplification into Equation (1) leads to

$$\bar{q} + q' = (\bar{R} + R')(\overline{Cl_r} + Cl'_r)/\overline{Cl_{gw}}$$

The expansion of the right-hand side parentheses gives

$$\bar{q} + q' = (\bar{R} \overline{Cl_r} + \bar{R}Cl'_r + R'\overline{Cl_r} + R'Cl'_r)/\overline{Cl_{gw}}$$

If both sides of this expression are averaged and, keeping in mind that, by definition, the perturbation terms have zero arithmetic averages, then one can simplify this to

$$\bar{q} = (\bar{R} \overline{Cl_r} + \bar{R} \overline{Cl'_r})/\overline{Cl_{gw}}$$

The second term in the parentheses on the right-hand side is the average of rainfall perturbation multiplied by the Cl$^-$/NUL concentration perturbation. This multiplication term might be thought of as similar to the weighted average as used in previous publications; it does not lead to the same result, however. In statistics literature, the arithmetic average of the product of two variables is defined as the covariance. By definition, covariance is equal to the product of the correlation coefficient between two variables divided by their standard deviations. Hence, it is possible to rewrite the final equation as follows:

$$\bar{q} = (\bar{R} \overline{Cl_r} + \hat{\rho}_{RCl} \hat{\sigma}_R \hat{\sigma}_{Cl_r})/\overline{Cl_{gw}}$$

where $\hat{\rho}_{RCl}$ is the correlation coefficient between the rainfall and its Cl$^-$/NUL concentration, $\hat{\sigma}_R$ and $\hat{\sigma}_{Cl_r}$ are respectively the standard deviations of rainfall and its Cl$^-$/NUL concentration. This last expression is the refined CMB equation, which reduces to Equation (2) either

1. when the correlation coefficient between the rainfall and its Cl$^-$/NUL concentration is equal to zero, implying independence of Cl$^-$/NUL concentration of rainfall amount, or
Table I. Rainfall and Cl\textsuperscript{−} measurements in Taiwan (Ting \textit{et al}., 1998)

<table>
<thead>
<tr>
<th>Month</th>
<th>Site I Rainfall (mm)</th>
<th>Site I Cl\textsuperscript{−} (mg l\textsuperscript{−1})</th>
<th>Site II Rainfall (mm)</th>
<th>Site II Cl\textsuperscript{−} (mg l\textsuperscript{−1})</th>
<th>Site III Rainfall (mm)</th>
<th>Site III Cl\textsuperscript{−} (mg l\textsuperscript{−1})</th>
<th>Site IV Rainfall (mm)</th>
<th>Site IV Cl\textsuperscript{−} (mg l\textsuperscript{−1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>May</td>
<td>393</td>
<td>1.88</td>
<td>210</td>
<td>0.99</td>
<td>200</td>
<td>1.07</td>
<td>175</td>
<td>0.82</td>
</tr>
<tr>
<td>June</td>
<td>247</td>
<td>0.39</td>
<td>292</td>
<td>0.47</td>
<td>328</td>
<td>1.16</td>
<td>336</td>
<td>1.22</td>
</tr>
<tr>
<td>July</td>
<td>307</td>
<td>0.95</td>
<td>338</td>
<td>1.32</td>
<td>372</td>
<td>1.04</td>
<td>354</td>
<td>0.74</td>
</tr>
<tr>
<td>August</td>
<td>915</td>
<td>0.57</td>
<td>943</td>
<td>1.16</td>
<td>1073</td>
<td>3.06</td>
<td>1090</td>
<td>3.71</td>
</tr>
<tr>
<td>September</td>
<td>186</td>
<td>0.71</td>
<td>193</td>
<td>0.83</td>
<td>145</td>
<td>1.28</td>
<td>138</td>
<td>4.46</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>−0.092</td>
<td>0.39</td>
<td></td>
<td></td>
<td>0.95</td>
<td></td>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>

2. If one of the standard deviations is equal to zero. In this case, the variable with a zero standard deviation implies homogeneity, i.e. temporal and/or spatial constancy.

In practice, the correlation coefficient between the rainfall and its Cl\textsuperscript{−} concentration could be a positive value depending on the sources and seasonal rainfall. This is perhaps physically plausible, because the greater the rainfall, the higher the Cl\textsuperscript{−} concentration. Table I lists the rainfall and corresponding Cl\textsuperscript{−} concentrations of four sites during a 5 month period in Taiwan as presented by Ting \textit{et al}. (1998). It is obvious that there are some sites exhibiting very significant correlation coefficients between the rainfall amounts and their Cl\textsuperscript{−} concentrations. This is the main reason why, in the classical CMB method, a weighted average is used for the rainfall and its Cl\textsuperscript{−} concentration. However, Equation (10) is expected to yield comparatively higher results than Equation (2). It is further possible to write the classical CMB equation within the refined expression by considering Equations (2) and (10) as

\[
\bar{q} = \bar{q}_c + \rho_{RCI} \delta_R \delta_{Cl} / Cl_{gw}
\]

where \(\bar{q}_c\) is the average recharge rate according to the classical CMB method. It is obvious that the additional term depends extensively on the statistical parameters, including standard deviations and correlation coefficients.

APPLICATION

In order to show the performance of the refined CMB method, data from Wadi Yalamlam in the western part of Saudi Arabia are considered. Wadi Yalamlam is one of the important wadis in the western part of Saudi Arabia. It lies about 125 km southeast of the city of Jeddah and 70 km south of the city of Makkah (Figure 1). It flows into the Red Sea coastal plain, locally called Tihamah. It is bound by latitudes 20°30’ and 21°10’N and longitudes 39°45’ and 40°30’E. This wadi is a part of the Scarp–Hijaz Mountains of the Arabian Shield, which extends north–south parallel to the Red Sea. This escarpment is one of the outstanding landscape features of Saudi Arabia. Three physiographic units, namely the Red Sea coastal plain, the hills, and the Scarp–Hijaz Mountains, characterize the area. This basin drains a wide catchment area of about 1600 km\textsupersquare which starts from the Scarp Mountains and is characterized by a high amount of annual rainfall of more than 200 mm. Towards the drainage opening into the plain, the wadi loses its defined course into wide spans of sheet wash. Further downstream, it is integrated as part of the Red Sea coastal plain (El-Khatib, 1980; Noory, 1983). The hydrology and precipitation features of the region are extensively identified by Şen (1983), Alehaideb (1985) and Al-Yamani and Şen (1992).

In order to apply the refined CMB method as presented in Equation (10), mean monthly rainfall records (1981–2000) and rainfall Cl\textsuperscript{−} concentration data are collected from Ashafa station, which lies near the Red
Sea escarpment at the upstream region of Wadi Yalamlam. This station is the most representative of rainfall and subsequent runoff event evaluations near the study area. The months June and July are not considered in this study, as they are characterized by insignificant rainfall amounts (Table II). Groundwater samples for average Cl⁻ concentration are also taken from the wells in the upstream region of Wadi Yalamlam basin.
Table II. Statistical summary of mean monthly rainfall and rainfall Cl\(^-\) concentration at Ashafa station (1981–2000)

<table>
<thead>
<tr>
<th>Month</th>
<th>Mean monthly rainfall (mm)</th>
<th>Cl(^-) concentration (mg l(^-1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>19.8</td>
<td>7.5</td>
</tr>
<tr>
<td>February</td>
<td>13.4</td>
<td>7</td>
</tr>
<tr>
<td>March</td>
<td>21.9</td>
<td>8</td>
</tr>
<tr>
<td>April</td>
<td>30.6</td>
<td>9.5</td>
</tr>
<tr>
<td>May</td>
<td>28</td>
<td>9.9</td>
</tr>
<tr>
<td>August</td>
<td>20.4</td>
<td>8.5</td>
</tr>
<tr>
<td>September</td>
<td>12.3</td>
<td>7.8</td>
</tr>
<tr>
<td>October</td>
<td>14</td>
<td>8.1</td>
</tr>
<tr>
<td>November</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>December</td>
<td>15.4</td>
<td>7.3</td>
</tr>
<tr>
<td>Mean</td>
<td>19.28</td>
<td>8.26</td>
</tr>
<tr>
<td>Harmonic mean</td>
<td>17.71</td>
<td>8.16</td>
</tr>
<tr>
<td>Geometric mean</td>
<td>18.45</td>
<td>8.21</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>6.18</td>
<td>0.96</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.77</td>
<td></td>
</tr>
</tbody>
</table>

In addition, there are no irrigation activities in this area, nor is there active dry deposition of Cl\(^-\). All the data needed for the application of the refined CMB method are given in Table II, where it is obvious that the harmonic mean concentration of chlorides in the rainfall is Cl\(^-\)\(_{gw}\) = 80 mg l\(^-1\) and that the arithmetic, harmonic, and weighted means are not significantly different. In this study, the harmonic means are used because their amounts fall between the other two averages, which makes them more representative than the others. The average concentration of Cl\(^-\) during the same period is calculated to be Cl\(^-\)\(_g\) = 8.16 mg l\(^-1\). The rainfall Cl\(^-\) concentration standard deviation is 0.96 mg l\(^-1\) and the standard deviation of monthly rainfall is 6.2 mm, in addition to the monthly harmonic mean of rainfall, which equals 17.7 mm. The estimated correlation between monthly rainfall and its Cl\(^-\) concentration is \(\rho_{\text{RCl}} = 0.77\), at the 0.05 level of significance. This indicates that there is a significant correlation between the two variables, which would not have been taken into consideration at all in any classical CMB approach. Substitution of all the relevant values into Equation (10) gives 1.9 mm month\(^-1\). In the case of classical calculations using Equation (2), a value of 1.8 mm month\(^-1\) is obtained. It is possible to conclude, then, that the refined and classical approaches result in recharge rates of 11% and 10% respectively. As expected, the recharge estimate using the refined approach in Equation (10) is slightly higher than that obtained using Equation (2). However, the difference between the two approaches will increase, especially with the increase in the standard deviations of monthly rainfall and/or rainfall concentration.

CONCLUSIONS

The perturbation methodology is used to derive a refined CMB equation with homogeneity in ‘averages’. It takes into consideration, in addition to the averages of variables, the correlation coefficient between the rainfall, its Cl\(^-\) concentration and the standard deviations. It reduces to the classical CMB equation if there are no deviations in the measurements about their respective averages. The new formulation takes into account the temporal and spatial variations in the calculations. Both the refined methodology and the classical approach were applied to the case of Wadi Yalamlam in the western part of the Kingdom of Saudi Arabia. The refined method yielded a higher recharge value due to the positive correlation coefficient between the rainfall and its Cl\(^-\) values, in addition to the standard deviations.
REFERENCES


