Linking Groundwater Flow and Transport Models, GIS Technology, Satellite Images and Uncertainty Quantification for Decision Making: Buraiman Lake Case Study Jeddah, Saudi Arabia

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Abstract: In this paper, a methodology has been presented to show the combination of groundwater and transport models, satellite images and uncertainty quantification to visualize the flow pattern and the transport mechanisms for decision makers. The methodology has been applied on a wastewater lake, called Buraiman or (Almusk) lake, that has been the dumping site of Jeddah’s sewage wastewater since about ten years. Various flow scenarios has been considered to match the aquifer response under the limited data collected from the literature and satellite images. The Monte-Carlo methodology together with GIS and satellite images are useful tools to account for uncertainty and limitations of the data. The envelope maps that are generated by the Monte-Carlo approach to account for uncertainty are practical tools in the process of decision support system for decision makers and planners.

Key words: Contaminant transport • Sewage lake • Jeddah • Groundwater modeling • GIS applications

INTRODUCTION

Jeddah has more than three million residents. 85% of Jeddah area is not connected to sewerage pipelines. Wastewater accumulates in underground cesspools and later is transported by truck tankers to the sewage lake for the past 10 years. The lake lies in east of Jeddah within the catchment of wadi Bani Malek at about 130m above mean sea level. It contains 9.5 million cubic meters of sewage water spread over an area of 2.88 km².

The sewage lake has caused some wells to become poisoned due to raw sewage leaking into aquifers. The main aquifer that could be identified in the area is the alluvial sand with relatively high permeability. The basement rocks are considered as aquiclude. Investigation of groundwater level shows that some areas east of the highway have very high groundwater level [1]. The main direction of the groundwater flow is towards the sea.

Many studies were performed concerning the lake. Most of the studies focused on the safety of the earth dam [2, 3]. Hydrological analysis of the lake carried out by Ewea [4], who indicated that the sewage influx is reduced from 50000 m³/day to 27500 m³/day in 2009. Polluted groundwater seepage to the sea is estimated to be at a rate of 2.2 million m³/year based on mass balance calculations.

Although most studies have focused on water level variation in the lake as a result of sewage inflow and the impact on dam safety, no one considered the risk of the spreading of the subsurface polluted water on Jeddah city. The purpose of the present paper is to predict the pollution spreading from the sewage lake and its impact on the city of Jeddah via a numerical simulation models. Particle tracking random walk technique together with numerical finite difference groundwater flow model were used. The models are connected with GIS to be able to visualize the flow pattern and the concentration distribution on satellite images in the area under study. Various flow and transport scenarios are demonstrated under the limited data collected from the literature.
Data Availability and Problem Conceptualization:
One of the major issues in arid and semi-arid zones is data availability. In most cases data is scarce and even if the data is available one can hardly get an access to it. In this study we have followed a parsimonious approach to the problem and try to collage as much data as we could to help us to perform the necessary modeling and simulation. Figure 1 (A) shows the location of Buraiman lake on the map of Jeddah. Figure 1 (B) shows the shape of the lake and the protection dams. The large lake on the top of the image is protected by an earth dam, however, because of the seepage from that dam another dam has been built in the downstream of the large lake that has created a reservoir of sewage water in the upstream part of the dam. This recent dam is seeping also water in its downstream side as seen in the figure.

Figure 2 shows the stream network of the wadi where the lake is located. The figure also shows that the stream network passes through the large lake and the downstream protection dam. This map shows the direction where the surface water would go towards the sea. It could also be give an idea regarding the mean underground flow towards the sea.
Figure 3 shows the topographic profile along the main stream from the lake to the red sea that displays almost a uniform gradient of about 0.3%. Data on groundwater level in Jeddah city has been obtained from the groundwater map drawn by Al-Sefry and Sen, [1]. This map was drawn by interpolation of the municipality well data and there was no date mentioned for that map.

Alquhtani and Shehata [5] have studied the groundwater level rise problem in Jeddah. They have drawn a vulnerability map for Jeddah city. They have indicated that the majority of aquifer thickness varies from 10 m to 25 m in the city of Jeddah. These data were derived from drilling logs. They have found that the hydraulic conductivity values vary according to three categories: zones of low permeability (10^{-7} to 10^{-4} cm/sec), zones of intermediate permeability (10^{-4} to 10^{-1} cm/sec) and zones of high permeability (10^{-1} to 1 cm/sec).

From this information, the average value of hydraulic conductivity is estimated to be 120 m/day and its standard deviation is 320 m/day.

In the current study, it is assumed that all the wastes dumped in the lake is domestic wastewater. There is a belief that there is also industrial wastewater dumped in the lake. Since, there is no information available regarding the industrial wastes, the domestic wastewater is the only source of contaminant considered. Liptak, [6] has given the composition of the domestic wastewater together with a representative range of concentration values: Total solids (200-1000 mg/l), total suspended solids (100-500 mg/l), total dissolved (100-500 mg/l), BOD five days 20°C (100-300 mg/l), total Nitrogen (25-90 mg/l), Chlorides (15-200 mg/l), Alkalinity (50-200 mg/l) and fats (0-40 mg/l). Because we do not have data on these concentrations in the wastewater dumped in the lake, we have taken average values of these ranges and sum up to have 1660 mg/l (1.66 Kg/M³) as a lump sum concentration of the wastes. Hatrash, [7] stated that “about 800 tanker trucks dump 50,000 cubic meters of sewage into the lake each day”. Simple computations lead to an average sewage mass rate of about 30,120 Kg/day.

Methodology

Inference of Groundwater Conditions at the Site: A transect across the contour map from the red sea to eastern wadi along with the sewage lake has been drawn. Data along this transect was extracted and displayed in Figure 4 (diamonds). A best fit line was drawn to the data to display a groundwater slope of 0.3%. The best fit equation is given by,

\[ h = 74.78 \cdot 0.003 x \]  

Where, \( h \) is the hydraulic head (groundwater level) at any location \( x \) along the transect.

Two theoretical groundwater profiles in homogenous aquifers were plotted in Figure 4 assuming linear and quadratic (unconfined aquifer) profiles from the lake to the red sea. These profiles are given by the following equations respectively,

\[ h = h_i - \frac{(h_i - h_f)}{L} \cdot x \]  

\[ h = \sqrt{h_i^2 - \frac{(h_i^2 - h_f^2)}{L} \cdot x} \]

Where, \( L \) is the length from the lake to the red sea, \( h_i \) is the hydraulic head boundary at the lake and \( h_f \) is the hydraulic head boundary at the red sea.
The interpolated well data lies below the linear profile in most of the figure. The reason might, in one hand, due to the interpolation scheme used by [1] that seems to be linear and from the other hand, the well data seems to be outdated. AL-Sefry and Sen, [1] stated that: “there was a noticeable variation in the groundwater level from 1996 to 2000 and the groundwater level rise is obvious in the city of Jeddah”. This statement confirms that the data is outdated.

**Modeling Spatial Variability of Hydraulic Conductivity:**
Heterogeneous aquifers are generally modeled as Log-Gaussian random fields characterized by a mean conductivity \( \bar{K} \), a standard deviation \( \sigma_k \) (or variance) and a correlation length \( \lambda \) [8]. These three parameters control the heterogeneities in the aquifer. The correlation length is the distance from a point beyond which there is no physical correlation of the physical property associated with that point. Thus, it controls the size of heterogeneities. When the correlation length is small, the size of heterogeneities is small. The standard deviation controls the hydraulic conductivity variations in the aquifer: for high standard deviation, the conductivity contrast will be significant. Heterogeneous aquifers were artificially generated, using the program TBM2 developed by Elfeki, [9], based on the turning bands method. Aquifers generated with this code are random fields with a specified hydraulic conductivity \( K \) defined as log-normally distributed with an arithmetic mean, a variance and a correlation length. In this study \( K = 120 \) m/day, \( \sigma_k = 302 \) m/day and the correlation length, \( \lambda \) is assumed 500 m from the features appeared in the geological map of the city of Jeddah [10]. Figure 5 shows a simulated aquifer generated using the statistics mentioned above.

**Groundwater Flow and Transport Modeling:** A two dimensional model of steady-state saturated groundwater flow in an isotropic heterogeneous confined (linear response) and unconfined (quadratic response) aquifers is applied on a rectangular domain. The equations to be solved are

\[
\frac{\partial}{\partial x} \left( K \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial h}{\partial y} \right) = 0
\]

\[
\frac{\partial}{\partial x} \left( K \frac{\partial h^2}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial h^2}{\partial y} \right) = 0
\]

(4)

Where \( K \) is the hydraulic conductivity \([L/T]\) and, \( h \) is the hydraulic head \([L]\). A block-centered five-point finite difference method is used to discretize Equation 4. Dirichlet and Neumann boundary conditions are considered. The left boundary is the red sea with zero elevation and the right boundary is the water level at the sewage lake with an elevation of 83 m (obtained from the satellite images) while the rest of cells in the right boundary is assumed to have a hydraulic head of 73 m (i.e. below the water level elevation at the lake by 10 m which is about the average water depth in the lake. Al-Sharif [11] stated that “the water level rose to the critical level of 12 m”. Having 10 m in the calculations is reasonable since the 2 m difference can be accounted for as both a capillary fringe or a groundwater level rise in the vicinity of the lake. The top and bottom boundaries are considered with no flow boundaries since the regional flow is from the up hills to the real sea which is supported by the groundwater map of [1]. The top and bottom boundaries are put far from the lake so that they do not influence the flow pattern in the vicinity of the lake.
The groundwater model used in the current study is an extension of the flow model developed by Elfeki [9] that deals with confined aquifer. It is now ready to handle both confined and unconfined aquifer conditions. The domain of the study is 21800 m in the longitudinal direction and 13785 m in the lateral direction. The domain is divided into cells of 100 × 100 ms. The groundwater head and the flow velocity is computed for every cell. Figure 5 shows the hydraulic head field for the hydraulic conductivity distribution presented in the figure.

The movement of contaminants in the subsurface is represented by the advection-dispersion equation. The contaminant is assumed to be conservative and to have no interaction with the solid matrix. The two-dimensional advection-dispersion equation for this case can be written as [12]:

\[
\frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} - \frac{\partial}{\partial y} \left[ D_{xx} \frac{\partial C}{\partial x} + D_{xy} \frac{\partial C}{\partial y} \right] = 0
\] (5)

Where \( C \) is the concentration of the contaminant at time \( t \) at location \((x,y)\), \( v_x \) and \( v_y \) are the average groundwater flow velocity components in the x- and y-direction respectively and \( D_{xx}, D_{xy}, D_{yy} \) are the components of the hydrodynamic dispersion tensor. Having obtained the velocity field for each realization of the hydraulic conductivity field, the solution of the transport equation and the spatio-temporal evolution of the concentration field are obtained by employing a random walk particle model. It is assumed that \( C(x,y,0) = 0 \) in the domain. The boundary condition \( \cdot C / \cdot y \), for \( t = 0 \) is imposed at top and bottom boundaries of the flow domain. The contaminant source is located at the downstream side of the protection dam with an initial source size of 100 × 100 ms. Figure 5 shows source location in the upstream (the black spot close to the right hand side boundary).

**Random Walk Particle Tracking Model:** The random walk particle tracking model, developed by [9] is used to solve Equation 5. The injected contaminant mass is represented by particles moving in the flow field. Each particle is assigned the same fixed amount of contaminant mass. Dispersion is modeled by superimposing a random movement on the convective particle movement, which has the statistical properties that correspond to the properties of the physical dispersive process. A large number of individual random walks of particles form a dispersing particle cloud characterizing a contaminant mass distribution. Given the analogy between the transport equation and the Fokker-Planck equation [13], the two-dimensional particle tracking equations incorporating dispersion can be written as [14]:

\[
X_x(t+\Delta t) = X_x(t) + v_x \Delta t + \left( \frac{\partial D_{xx}}{\partial x} + \frac{\partial D_{yx}}{\partial y} \right)
\]

\[
\Delta t \frac{v_x}{|v_x|} \frac{Z}{\sqrt{2\alpha_x}} \left( \frac{|v_x| - v_x}{|v_x|} \right) Z \frac{v_y}{|v_y|} \frac{\Delta t}{|\Delta t|}
\] (6)
\[
Y_p(t + \Delta t) = Y_p(t) + v_x \Delta t + \left( \frac{\partial D_{xx}}{\partial x} + \frac{\partial D_{xy}}{\partial y} \right) \\
\Delta t + \frac{v_x}{\sqrt{2 \alpha_x}} Z \sqrt{2 \alpha_x} |v| |\Delta t| \\
\Delta t + \frac{v_y}{\sqrt{2 \alpha_y}} Z \sqrt{2 \alpha_y} |v| |\Delta t|
\]

(7)

Where \( X(t) \), \( Y(t) \) are the \( x \) and \( y \) coordinates of a particle at time \( t \), \( \Delta t \) is the time step used in calculations, \( Z, Z^* \) are two independent random numbers drawn from a normal distribution with mean zero and variance one, \( \alpha_x \) is the longitudinal dispersivity and \( \alpha_y \) is the transverse dispersivity.

On the right hand sides of both Equation 6 and 7, the first terms correspond to the old position of the particle, the second terms correspond to the convective displacement, the third terms are the Fokker-Plank term (a counter-term has to be added to correct the unrealistic accumulation of particles at stagnation zones) and the last two terms are the stochastic dispersive displacements projected in the \( x \) and the \( y \) directions respectively.

The solution of the advection-dispersion transport equation by the random walk method provides the discrete particle displacements and not the concentration values. A discretized grid model, similar to the one used

Fig. 6: Simulation of concentration distribution of the plume in heterogeneous unconfined (quadratic) aquifer: Right column: single realization concentration field and left column is envelope of 10 plumes
Fig. 7: Simulation of concentration distribution of the plume in heterogeneous confined (linear) aquifer: Right column: single realization concentration field and left column is envelope of 10 plumes in the solution of groundwater flow equations, is superimposed to convert the particle displacements into concentrations. The average concentration at time $t$ in a grid cell $(i, j)$ with dimensions ($\Delta x$ and $\Delta y$ in $x$ and $y$ directions respectively), is:

$$C_y(t + \Delta t) = C_y(t) + \frac{M n_y(t + \Delta t)}{N \epsilon H_x \Delta x \Delta y}$$  \hspace{1cm} (8)

Where $C_y(t + \Delta t)$ is the volume averaged concentration in grid cell $(i, j)$ at time $t + \Delta t$, $C_y(t)$ is the volume averaged concentration in grid cell $(i, j)$ at time $t$, $n_y(t + \Delta t)$ is the number of particles in grid cell $(i, j)$ at time $t + \Delta t$, $N$ is the total number of particles released and $H_y$ is the thickness of the grid cell.

In the current study the sewage lake is considered a source of constant strength and can be simulated as a continuous source of pollution by convolution from the solution for instantaneous pulse of contaminants using a relatively small number of particles [14]. Equation 8 shows the convolution process.

The effective porosity is assumed to be an average value of 0.25 since there is no data available. Also the longitudinal and lateral dispersivities are assumed to be $\alpha_x = 2$ and $\alpha_y = 1$ m respectively. These values are supposed to account for the micro-heterogeneities within the 100 ×100 m cell and the time step of the computation is taken to be one month and consequently the injected mass rate is about 903614 kg/month.
Uncertainty Modeling by Monte-Carlo Approach: A Monte-Carlo approach has been followed to estimate the uncertainty due to the variability in hydraulic conductivity. This approach generates many realization of hydraulic conductivity fields that posses the statistical parameters and solves the flow and transport in these fields and calculate the ensemble and variance of the concentration distribution that can be considered as the envelope of all possible plumes. Ten Monte-Carlo simulations are considered and the results are displayed in Figure 6 and 7 under different aquifer conditions.

RESULTS AND DISCUSSION

Figure 8 shows a comparison between the interpolated well data from (AL-Sefry and Sen 2006) and the simulated hydraulic head profiles from the lake to the red sea under linear and quadratic aquifer response conditions. It is obvious from the results that the linear aquifer response condition is closer to the data when compared with quadratic response aquifer condition. This is also supported by the homogeneous analysis presented in Figure 4. This behavior does not mean necessarily that the aquifer is confined but can just be interpreted that the aquifer response is somewhat linear. This point need further investigation by considering other sites.

Figure 7 and 8 show the plume shapes (right column) and the envelope of ensemble plumes (left column) calculated over 10 realizations at 10, 25 and 50 years since the lake was operating. It is obvious that the plume moves faster and spread wider when the linear aquifer response is considered. This is due to the fact that the velocities are faster in comparison with quadratic aquifer response. The expected travel time from the lake to the sea is about 50 years for the linear aquifer response while it is longer for the unconfined aquifer response (55 years). From a simple homogeneous aquifer analysis and under pure advection, a travel time of 36 years and 48 years were estimated for linear and quadratic aquifer responses respectively.

The developed maps in Figure 7 and 8 and be used as decision support system to show the places that are facing problems due to the sewage lake and help decision makers to find ways for preventive measures.

CONCLUSIONS

Groundwater and transport models can be connected with GIS to visualize the flow pattern and the transport mechanisms (in this study advection and dispersion) on satellite images for the area under study. Various flow scenarios is demonstrated (linear and quadratic aquifer responses) under the limited data collected from the literature and satellite images.

The Monte-Carlo methodology together with GIS and satellite images are useful tools to account for uncertainty and limitations of the data. The envelope maps that are generated by Monte-Carlo approach are practical tool in the process of decision support system for decision makers and planners.
The Monte-Carlo methodology together with GIS and satellite images are useful tools to account for uncertainty and limitations of the data. The envelope maps that are generated by Monte-Carlo approach are practical tool in the process of decision support system for decision makers and planners. Both homogeneous and heterogeneous aquifer analyses confirms that the travel time is less in linear aquifer response condition when compared with quadratic aquifer response condition. It is expected the plume reaches the red sea in about 40 years (on average) from now if the dumping rate stays at 50000 m³/day. Field work and reliable data are needed to improve model results and to get more accurate predictions.

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