SLANT HELICES IN MINKOWSKI SPACE $E^3_1$

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Abstract. We consider a curve $\alpha = \alpha(s)$ in Minkowski 3-space $E^3_1$ and denote by $\{T, N, B\}$ the Frenet frame of $\alpha$. We say that $\alpha$ is a slant helix if there exists a fixed direction $U$ of $E^3_1$ such that the function $\langle N(s), U \rangle$ is constant. In this work we give characterizations of slant helices in terms of the curvature and torsion of $\alpha$. Finally, we discuss the tangent and binormal indicatrices of slant curves, proving that they are helices in $E^3_1$.

1. Introduction and statement of results

Let $E^3_1$ be the Minkowski 3-space, that is, $E^3_1$ is the real vector space $\mathbb{R}^3$ endowed with the standard flat metric

$$\langle , \rangle = dx_1^2 + dx_2^2 - dx_3^2,$$

where $(x_1, x_2, x_3)$ is a rectangular coordinate system of $E^3_1$. An arbitrary vector $v \in E^3_1$ is said spacelike if $\langle v, v \rangle > 0$ or $v = 0$, timelike if $\langle v, v \rangle < 0$, and lightlike (or null) if $\langle v, v \rangle = 0$ and $v \neq 0$. The norm (length) of a vector $v$ is given by $|v| = \sqrt{\langle v, v \rangle}$.

Given a regular (smooth) curve $\alpha : I \subset \mathbb{R} \rightarrow E^3_1$, we say that $\alpha$ is spacelike (resp. timelike, lightlike) if $\alpha'(t)$ is spacelike (resp. timelike, lightlike) at any $t \in I$. If $\alpha$ is spacelike or timelike we say that $\alpha$ is a non-null curve. In such case, we can reparametrize $\alpha$ by the arc-length $s = s(t)$, that is, $|\alpha'(s)| = 1$. We say then that $\alpha$ is arc-length parametrized. If the curve $\alpha$ is lightlike, the acceleration vector $\alpha''(t)$ must be spacelike for all $t$. We change the parameter $t$ by $s = s(t)$ in such way that $|\alpha''(s)| = 1$ and we say that $\alpha$ is pseudo arc-length parametrized. In any of the above cases, we say that $\alpha$ is a unit speed curve.

Given a unit speed curve $\alpha$ in Minkowski space $E^3_1$ it is possible to define a Frenet frame $\{T(s), N(s), B(s)\}$ associated for each point $s$ [5, 7, 10]. Here $T$, $N$ and $B$ are the tangent, normal and binormal vector field, respectively. The