We introduce the notion of \( C \)-admissible subspaces and obtain various conditions of \( C \)-admissibility, generalizing well known results of Vu and Schuler. Moreover, we show the uniqueness of solutions for the operator equation \( AX - XB = CD \) with \( A \) generating an analytic \( C \)-semigroup which generalize results of Vu.

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1. Introduction

We introduce the notion of \( C \)-admissible subspaces which is a generalization of the notion of admissible subspaces, and we extend to this situation the results of Vu and Schuler [1].

In [2], Vu proved the following theorem:

**Theorem 1.1.** Let \( A \) be a generator of an analytic semigroup in a Banach space \( E \). Assume that \( B \) is a closed linear operator in a Banach space \( F \) such that \( \Sigma_{\omega,\theta} \subseteq \rho(B) \) and \( \|\lambda(B - \lambda)^{-1}\| \) is uniformly bounded when \( \lambda \) belongs to the sector \( \Sigma_{\omega,\theta} \) (where \( \Sigma_{\omega,\theta} = \{ \lambda \in \mathbb{C} : |\arg(\omega - \lambda)| < \theta \} \cup \{\omega\} \) and \( \sup_{\lambda \in \mathbb{C} \setminus \Sigma_{\omega,\theta}} \|\lambda(A - \lambda)^{-1}\| < \infty \)). Then, the operator equation

\[
AX - XB = D
\]

has a unique solution which is expressed by

\[
X = \frac{1}{2\pi i} \int_{\Gamma} (A - \lambda)^{-1}D(B - \lambda)^{-1}d\lambda.
\]

In Section 4, we generalize such theorem to the case where \( A \) is a generator of an analytic \( C \)-semigroup.

2. \( C \)-admissibility

In this section, we introduce the notion of \( C \)-admissible subspaces which is a generalization of the notion of admissible subspaces, and we extend to this situation the results of Vu [2] and Schuler and Vu [1].

Consider the differential equation

\[
u'(t) = Au(t) + f(t), \quad t \in \mathbb{R}
\]

where \( A \) is a closed linear operator on a Banach space \( E \) and \( f \) is a continuous function from \( \mathbb{R} \) to \( E \).