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Common Best Proximity Points: Global Optimal Solutions

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Abstract Let $S : A \to B$ and $T : A \to B$ be given non-self mappings, where A and B are non-empty subsets of a metric space. As S and T are non-self mappings, the equations $Sx = x$ and $Tx = x$ do not necessarily have a common solution, called a common fixed point of the mappings S and T. Therefore, in such cases of non-existence of a common solution, it is attempted to find an element x that is closest to both Sx and Tx in some sense. Indeed, common best proximity point theorems explore the existence of such optimal solutions, known as common best proximity points, to the equations $Sx = x$ and $Tx = x$ when there is no common solution. It is remarked that the functions $x \to d(x, Sx)$ and $x \to d(x, Tx)$ gauge the error involved for an approximate solution of the equations $Sx = x$ and $Tx = x$. In view of the fact that, for any element x in A, the distance between x and Sx, and the distance between x and Tx are at least the distance between the sets A and B, a common best proximity point theorem achieves global minimum of both functions $x \to d(x, Sx)$ and $x \to d(x, Tx)$ by stipulating a common approximate solution of the equations $Sx = x$ and $Tx = x$ to fulfill the condition that $d(x, Sx) = d(x, Tx) = d(A, B)$. The purpose of this article is to elicit common best proximity point theorems for pairs of *contractive* non-self mappings and for pairs of *contraction* non-self mappings, yield-

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