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Common best proximity points: Global optimization of multi-objective functions

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ABSTRACT

Assume that A and B are non-void subsets of a metric space, and that $S : A \longrightarrow B$ and $T : A \longrightarrow B$ are given non-self-mappings. In light of the fact that S and T are non-self-mappings, it may happen that the equations $Sx = x$ and $Tx = x$ have no common solution, named a common fixed point of the mappings S and T. Subsequently, in the event that there is no common solution of the preceding equations, one speculates about finding an element x that is in close proximity to Sx and Tx in the sense that $d(x, Sx)$ and $d(x, Tx)$ are minimum. Indeed, a common best proximity point theorem investigates the existence of such an optimal approximate solution, named a common best proximity point of the mappings S and T, to the equations $Sx = x$ and $Tx = x$ when there is no common solution. Moreover, it is emphasized that the real valued functions $x \longrightarrow d(x, Sx)$ and $x \longrightarrow d(x, Tx)$ evaluate the degree of the error involved for any common approximate solution of the equations $Sx = x$ and $Tx = x$. Owing to the fact that the distance between x and Sx, and the distance between x and Tx are at least the distance between A and B for all x in A, a common best proximity point theorem accomplishes the global minimum of both functions $x \longrightarrow d(x, Sx)$ and $x \longrightarrow d(x, Tx)$ by postulating a common approximate solution of the equations $Sx = x$ and $Tx = x$ for meeting the condition that $d(x, Sx) = d(x, Tx) = d(A, B)$. This work is devoted to an interesting common best proximity point theorem for pairs of non-self-mappings satisfying a contraction-like condition, thereby producing common optimal approximate solutions of certain simultaneous fixed point equations.

1. Introduction

Fixed point theory sheds light on the methodologies for finding a solution to non-linear equations of the type $Tx = x$ where T is a self-mapping defined on a subset of a metric space, a normed linear space, a topological vector space or some appropriate space. But, the equation $Tx = x$ is unlikely to have a solution when T is not a self-mapping. Therefore, one deals with the problem of finding an element x that is in some sense in close proximity to Tx. In fact, best approximation theorems and best proximity point theorems are applicable for solving such problems. If K is a non-empty compact convex subset of a Hausdorff locally convex topological vector space E and $T : K \longrightarrow E$ is a non-self-continuous map, then a classical best approximation theorem, due to Fan [1], asserts that there is an element x satisfying the condition that $d(x, Tx) = d(Tx, K)$. Later, this result was extended in several directions by many authors, including Prolla [2], Reich [3] and Sehgal and Singh [4,5]. A unification of all such best approximation theorems has been accomplished by Vetrivel et al. [6].

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