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Convergence of Mann's type iteration method for generalized asymptotically nonexpansive mappings

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ABSTRACT

Let C be a nonempty, closed and convex subset of a real Hilbert space H. Let $T_i : C \to H,\ i = 1, 2, \ldots, N$, be a finite family of generalized asymptotically nonexpansive mappings. It is our purpose, in this paper to prove strong convergence of Mann's type method to a common fixed point of $\{T_i : i = 1, 2, \ldots, N\}$ provided that the interior of common fixed points is nonempty. No compactness assumption is imposed either on T or on C. As a consequence, it is proved that Mann's method converges for a fixed point of nonexpansive mapping provided that interior of $F(T) \neq \emptyset$. The results obtained in this paper improve most of the results that have been proved for this class of nonlinear mappings.

1. Introduction and preliminaries

Let C be a nonempty subset of a real Hilbert space H; a mapping $T : C \to C$ is a contraction if there exists $k \in [0, 1)$ such that for all $x, y \in C$ we have $\|Tx - Ty\| \leq k\|x - y\|$. It is said to be *nonexpansive* if for all $x, y \in C$ we have $\|Tx-Ty\| \leq \|x-y\|$. T is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ such that $\|T^n x-T^n y\| \leq k_n\|x-y\|$ for all integers $n \geq 1$ and all $x, y \in C$. Clearly, every contraction mapping is nonexpansive and every nonexpansive mapping is *asymptotically nonexpansive* with sequence $k_n = 1,\ \forall n \geq 1$. There are however, asymptotically nonexpansive mappings which are not nonexpansive (see e.g., [1]).

As a generalization of the class of nonexpansive mappings, the class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [2] in 1972 and has been studied by several authors (see e.g., [3–6]). Goebel and Kirk proved that if C is a nonempty closed convex and bounded subset of a uniformly convex Banach space (more general than a Hilbert space) then every asymptotically nonexpansive self-mapping of C has a fixed point.

The weak and strong convergence problems to a fixed points of nonexpansive and asymptotically nonexpansive mappings have been studied by many authors (for example, see [7,8,2,3,9–11] and the references therein).

Let C be a closed subset of a Hilbert space H and T be a self-mapping contraction, the classical *Picard iteration method*,

$$x_0 \in C, \qquad x_{n+1} = Tx_n, \quad n \geq 1 \tag{1.1}$$

converges to the unique fixed point of T. Unfortunately, the Picard iteration method does not always converge to a fixed point of nonexpansive mappings. It suffices to take, for example, T to be the anticlockwise rotation of the unit disk in $\mathbb{R}^2$ (with the usual Euclidean norm) about the origin of coordinate of an angle, say, θ.

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