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Convergence of Ishikawa's iteration method for pseudocontractive mappings

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ABSTRACT

Let C be a nonempty, closed and convex subset of a real Hilbert space H. Let $T_i : C \to C$, $i = 1, 2, \ldots, N$, be a finite family of Lipschitz pseudocontractive mappings. It is our purpose, in this paper, to prove strong convergence of Ishikawa's method to a common fixed point of a finite family of Lipschitz pseudocontractive mappings provided that the interior of the common fixed points is nonempty. No compactness assumption is imposed either on T or on C. Moreover, computation of the closed convex set C_n for each $n \geq 1$ is not required. The results obtained in this paper improve on most of the results that have been proved for this class of nonlinear mappings.

1. Introduction and preliminaries

Let C be a nonempty subset of a real Hilbert space H. The mapping $T : C \to H$ is called *Lipschitz or Lipschitz continuous* if there exists $L \geq 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\| \quad \forall x, y \in C. \tag{1.1}$$

If $L = 1$, then T is called *nonexpansive*; and if $L < 1$ then T is called *a contraction*. It is easy to see from Eq. (1.1) that every contraction mapping is nonexpansive and every nonexpansive mapping is Lipschitz.

A mapping $T : C \to H$ is called *α-strictly pseudocontractive* in the terminology of Browder and Petryshyn [1] if for all $x, y \in C$ there exists $\alpha > 0$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2 - \alpha\|x - y - (Tx - Ty)\|^2. \tag{1.2}$$

Without loss of generality we may assume that $\alpha \in (0, 1)$. If I denotes the identity operator, then (1.2) can be rewritten as

$$\langle (I - T)x - (I - T)y, j(x - y)\rangle \geq \alpha\|(I - T)x - (I - T)y\|^2.$$

A mapping T is called *pseudocontractive* if

$$\langle Tx - Ty, x - y\rangle \leq \|x - y\|^2 \quad \text{for all } x, y \in C. \tag{1.3}$$

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