Common fixed points of almost generalized contractive mappings in ordered metric spaces

Ljubomir Ćirić, Mujahid Abbas, Reza Saadati, Nawab Hussain

1. Introduction and preliminaries

In 1968, Kannan [15] proved a fixed point theorem for a map satisfying a contractive condition that did not require continuity at each point. This paper was a genesis for a multitude of fixed point papers over the next two decades. On the other hand, Sessa [19] introduced the notion of weakly commuting maps, which are generalization of commuting maps, while Jungck [13] generalized the notion of weak commutativity by introducing compatible maps and then weakly compatible maps [14].

Definition 1.1. Let \((X,d)\) be a metric space. A map \(g : X \rightarrow X\) is called an almost contraction with respect to a mapping \(f : X \rightarrow X\) if there exists a constant \(\delta \in [0,1)\) and some \(L \geq 0\) such that

\[
d(gx, gy) \leq \delta d(fx, fy) + Ld(fy, gx),
\]

for all \(x, y \in X\).

If we choose \(f = I_X, I_X\) is the identity map on \(X\), we obtain the definition of almost contraction, the concept introduced by Berinde ([4,5]). This concept by Berinde in [4] was called as ‘weak contraction’, but in [5], Berinde renamed it as ‘almost contraction’ which is appropriate. Berinde [4] proved some fixed point theorems for almost contractions in complete metric spaces. Then many authors have studied this problematic and obtained significant results ([3,6–12,16,17]).

It was shown in [4] that any strict contraction, the Kannan [15] and Zamfirescu [20] mappings, as well as a large class of quasi-contractions, are all almost contractions.

Let \(g\) and \(f\) be two selfmaps of a metric space \((X,d)\). \(g\) is said to be \(f\)-contraction if there exists \(k \in [0,1)\) such that

\[
d(gx, gy) \leq kd(fx, fy)
\]

for all \(x, y \in E\).

In 2006, Al-Thagafi and Shahzad [1] proved the following theorem which is a generalization of many known results.