Coincidence point theorems for multivalued $f$-weak contraction mappings and applications

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Abstract We prove the existence of coincidence points and common fixed points for multivalued $f$-weak contraction mappings which assume closed values only. As an application, related common fixed point, invariant approximation, random coincidence point and random invariant approximation results are also obtained. Our results provide extensions as well as substantial improvements of several well known results in the existing literature.

Keywords Coincidence point · Random fixed point · Multivalued $f$-weak contraction map · Random approximation · Measurable space

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1 Introduction and preliminaries

Let $(X, d)$ be a metric space. We denote by $CB(X)$ and $CL(X)$, the families of all nonempty closed bounded and nonempty closed subsets of $X$, respectively. For $A, B \in CL(X)$. Set, $E_{A,B} = \{ \varepsilon > 0 : A \subseteq N_{\varepsilon}(B), B \subseteq N_{\varepsilon}(A) \}$. We define a generalized Hausdorff metric $H$ on $CL(X)$ by

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