COMMON FIXED POINT AND INVARIANT APPROXIMATION RESULTS IN CERTAIN METRIZABLE TOPOLOGICAL VECTOR SPACES

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We obtain common fixed point results for generalized I-nonexpansive R-subweakly commuting maps on nonstarshaped domain. As applications, we establish noncommutative versions of various best approximation results for this class of maps in certain metrizable topological vector spaces.

1. Introduction and preliminaries

Let X be a linear space. A p-norm on X is a real-valued function on X with $0 < p \leq 1$, satisfying the following conditions:

(i) $\|x\|_p \geq 0$ and $\|x\|_p = 0 \Leftrightarrow x = 0$,

(ii) $\|\alpha x\|_p = |\alpha|^p \|x\|_p$,

(iii) $\|x+y\|_p \leq \|x\|_p + \|y\|_p$

for all $x, y \in X$ and all scalars α. The pair $(X, \|,\|_p)$ is called a p-normed space. It is a metric linear space with a translation invariant metric d_p defined by $d_p(x,y) = \|x-y\|_p$ for all $x, y \in X$. If $p = 1$, we obtain the concept of the usual normed space. It is well-known that the topology of every Hausdorff locally bounded topological linear space is given by some p-norm, $0 < p \leq 1$ (see [9] and references therein). The spaces l_p and L_p, $0 < p \leq 1$ are p-normed spaces. A p-normed space is not necessarily a locally convex space. Recall that dual space X^* (the dual of X) separates points of X if for each nonzero $x \in X$, there exists $f \in X^*$ such that $f(x) \neq 0$. In this case the weak topology on X is well-defined and is Hausdorff. Notice that if X is not locally convex space, then X^* need not separate the points of X. For example, if $X = L_p[0,1]$, $0 < p < 1$, then $X^* = \{0\}$ ([12, pages 36 and 37]). However, there are some non-locally convex spaces X (such as the p-normed spaces l_p, $0 < p < 1$) whose dual X^* separates the points of X.

Let X be a metric linear space and M a nonempty subset of X. The set $P_M(u) = \{x \in M : d(x,u) = \operatorname{dist}(u,M)\}$ is called the set of best approximants to $u \in X$ out of M, where $\operatorname{dist}(u,M) = \inf\{d(y,u) : y \in M\}$. Let $f : M \to M$ be a mapping. A mapping $T : M \to M$

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