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Viscosity approximation methods for pseudocontractive mappings in Banach spaces

Habtu Zegeye [a], Naseer Shahzad [b,*], Tefera Mekonen [a]

[a] Bahir Dar University, P.O. Box. 859 Bahir Dar, Ethiopia
[b] Department of Mathematics, King Abdul Aziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

Abstract

Let K be a closed convex subset of a Banach space E and let $T : K \to E$ be a continuous weakly inward pseudocontractive mapping. Then for $t \in (0,1)$, there exists a sequence $\{y_t\} \subset K$ satisfying $y_t = (1 - t)f(y_t) + tT(y_t)$, where $f \in \Pi_K := \{f : K \to K$, a contraction with a suitable contractive constant$\}$. Suppose further that $F(T) \neq \emptyset$ and E is reflexive and strictly convex which has uniformly Gâteaux differentiable norm. Then it is proved that $\{y_t\}$ converges strongly to a fixed point of T which is also a solution of certain variational inequality. Moreover, an explicit iteration process which converges strongly to a fixed point of T and hence to a solution of certain variational inequality is constructed provided that T is Lipschitzian.

Keywords: Nonexpansive mappings; Pseudocontractive mappings; Strongly pseudocontractive mappings; Uniform Gâteaux differentiable norms

1. Introduction

Let E be a real Banach space with dual E^*. We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$Jx := \left\{f^* \in E^* : \langle x, f^* \rangle = \|x\|^2 = \|f^*\|^2\right\},$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. It is well known that if E^* is strictly convex, then J is single-valued and norm to weak* continuous (see e.g., [7]). In the sequel, we shall denote the single-valued normalized duality map by j. A mapping T with domain $D(T)$ and range $R(T)$ in E is called *pseudocontractive* if the inequality

$$\|x - y\| \leqslant \|x - y + t((I - T)x - (I - T)y)\| \tag{1.1}$$

holds for each $x, y \in D(T)$ and for all $t > 0$. As a result of Kato [11], it follows from inequality (1.1) that T is *pseudocontractive* if and only if there exists $j(x - y) \in J(x - y)$ such that $\langle Tx - Ty, j(x - y)\rangle \leqslant \|x - y\|^2$ for

* Corresponding author.
E-mail addresses: habtuzh@yahoo.com (H. Zegeye), nshahzad@kaau.edu.sa, naseer_shahzad@hotmail.com (N. Shahzad).

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