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Viscosity methods of approximation for a common fixed point of a family of quasi-nonexpansive mappings

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Abstract

Let K be a nonempty closed convex subset of a real reflexive Banach space E that has weakly continuous duality mapping J_φ for some gauge φ. Let $T_i : K \to K, i = 1, 2, \ldots,$ be a family of quasi-nonexpansive mappings with $F := \cap_{i\geq 1} F(T_i) \neq \emptyset$ which is a sunny nonexpansive retract of K with Q a nonexpansive retraction. For given $x_0 \in K$, let $\{x_n\}$ be generated by the algorithm $x_{n+1} := \alpha_n f(x_n) + (1-\alpha_n)T_n(x_n), n \geq 0$, where $f : K \to K$ is a contraction mapping and $\{\alpha_n\} \subseteq (0, 1)$ a sequence satisfying certain conditions. Suppose that $\{x_n\}$ *satisfies condition* (A). Then it is proved that $\{x_n\}$ converges strongly to a common fixed point $\bar{x} = Qf(\bar{x})$ of a family $T_i, i = 1, 2, \ldots$. Moreover, $\bar{x}$ is the unique solution in F to a certain variational inequality.

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1. Introduction

Let E be a real Banach space with dual E^*. A gauge function is a continuous strictly increasing function $\varphi : \mathbf{R}^+ \to \mathbf{R}^+$ such that $\varphi(0) = 0$ and $\lim_{t\to\infty} \varphi(t) = \infty$. The duality mapping $J_\varphi : E \to E^*$ associated with a gauge function φ is defined by $J_\varphi(x) := \{u^* : \langle x, u^*\rangle = \|x\|.\|u^*\|, \|u^*\| = \varphi(\|x\|)\}, x \in E$, where $\langle ., .\rangle$ denotes the generalized duality pairing. In the particular case $\varphi(t) = t$, the duality map $J = J_\varphi$ is called the *normalized duality map*. We note that $J_\varphi(x) = \frac{\varphi(\|x\|)}{\|x\|} J(x)$. It is known that if E is smooth then J_φ is single valued and norm to w^* continuous (see, e.g., [6]).

Following Browder [3], we say that a Banach space E has the *weakly continuous duality mapping* if there exists a gauge function φ for which the duality map J_φ is single valued and weak to weak* sequentially continuous (i.e. if $\{x_n\}$ is a sequence in E weakly convergent to a point x, then the sequence $\{J_\varphi(x_n)\}$ converges weak* to $J_\varphi(x)$).

It is known that $l^p (1 < p < \infty)$ spaces have a weakly continuous duality mapping J_φ with a gauge $\varphi(t) = t^{p-1}$.

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