Zegeye, H., Shahzad, N.
Convergence theorems for strongly continuous semi-groups of
asymptotically nonexpansive mappings

a Bahir Dar University, P.O.Box. 859, Bahir Dar, Ethiopia
b Department of Mathematics, King Abdul Aziz University, P. O. B. 80203, Jeddah,
21589, Saudi Arabia

Abstract
Let K be a nonempty closed convex subset of a real Banach space E. Let \( T \{colon equals\} \{T (t) : t \in \mathbb{R}^+\} \) be a strongly continuous semi-group of asymptotically nonexpansive mappings from K into K with a sequence \( \{L_t\} \subset [1, \infty) \). Suppose \( F(T) \neq \emptyset \{combining long solidus overlay\} \). Then, for a given \( u_0 \in K \) and \( t_n > 0 \) there exists a sequence \( \{u_n\} \subset K \) such that \( u_n = (1 - \alpha_n) T(t_n) u_n + \alpha_n u_0 \), for \( n \in \mathbb{N} \) such that \( \{\alpha_n\} \subset (0, 1) \) and \( L_{t_n} - 1 < \alpha_n \), where \( t_n \in \mathbb{R}^+ \). Suppose, in addition, that E is reflexive strictly convex with a uniformly Gâteaux differentiable norm and that \( \lim_{n \to \infty} t_n = \infty \), \( \lim_{n \to \infty} \alpha_n = \lim_{n \to \infty} \frac{L_{t_n} - 1}{\alpha_n} = 0 \). Then the sequence \( \{u_n\} \) converges strongly to a point of \( F(T) \). Moreover, it is proved that an explicit sequence \( \{x_n\} \) generated from \( x_1 \in K \) by \( x_{n+1} = \alpha_n u + (1 - \alpha_n) T(t_n) x_n \), \( n \geq 1 \), converges to a fixed point of \( T \). © 2009 Elsevier Ltd. All rights reserved.

Author Keywords
Asymptotically nonexpansive mappings; Fixed points; Nonexpansive mappings;
Strongly continuous semi-groups of asymptotically nonexpansive mappings;
Strongly continuous semi-groups of nonexpansive mappings

ISSN: 0362546X