

CHAPTER

2

PROPERTIES OF FLUIDS

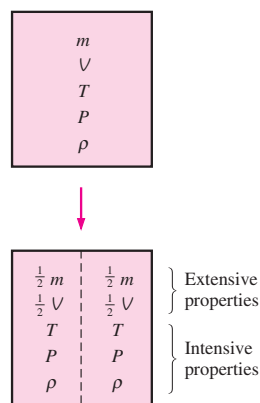
In this chapter, we discuss properties that are encountered in the analysis of fluid flow. First we discuss *intensive* and *extensive properties* and define *density* and *specific gravity*. This is followed by a discussion of the properties *vapor pressure*, *energy* and its various forms, the *specific heats* of ideal gases and incompressible substances, and the *coefficient of compressibility*. Then we discuss the property *viscosity*, which plays a dominant role in most aspects of fluid flow. Finally, we present the property *surface tension* and determine the *capillary rise* from static equilibrium conditions. The property *pressure* is discussed in Chap. 3 together with fluid statics.



OBJECTIVES

When you finish reading this chapter, you should be able to

- Have a working knowledge of the basic properties of fluids and understand the continuum approximation
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow
- Calculate the capillary rises and drops due to the surface tension effect

**FIGURE 2-1**

Criteria to differentiate intensive and extensive properties.

2-1 ■ INTRODUCTION

Any characteristic of a system is called a **property**. Some familiar properties are pressure P , temperature T , volume V , and mass m . The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.

Properties are considered to be either *intensive* or *extensive*. **Intensive properties** are those that are independent of the mass of a system, such as temperature, pressure, and density. **Extensive properties** are those whose values depend on the size—or extent—of the system. Total mass, total volume V , and total momentum are some examples of extensive properties. An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition, as shown in Fig. 2-1. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.

Generally, uppercase letters are used to denote extensive properties (with mass m being a major exception), and lowercase letters are used for intensive properties (with pressure P and temperature T being the obvious exceptions).

Extensive properties per unit mass are called **specific properties**. Some examples of specific properties are specific volume ($\nu = V/m$) and specific total energy ($e = E/m$).

The state of a system is described by its properties. But we know from experience that we do not need to specify all the properties in order to fix a state. Once the values of a sufficient number of properties are specified, the rest of the properties assume certain values. That is, specifying a certain number of properties is sufficient to fix a state. The number of properties required to fix the state of a system is given by the **state postulate**: *The state of a simple compressible system is completely specified by two independent, intensive properties.*

Two properties are independent if one property can be varied while the other one is held constant. Not all properties are independent, and some are defined in terms of others, as explained in Section 2-2.

Continuum

Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a **continuum**. The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules. This is the case in practically all problems, except some specialized ones. The continuum idealization is implicit in many statements we make, such as “the density of water in a glass is the same at any point.”

To have a sense of the distances involved at the molecular level, consider a container filled with oxygen at atmospheric conditions. The diameter of the oxygen molecule is about 3×10^{-10} m and its mass is 5.3×10^{-26} kg. Also, the *mean free path* of oxygen at 1 atm pressure and 20°C is 6.3×10^{-8} m. That is, an oxygen molecule travels, on average, a distance of 6.3×10^{-8} m (about 200 times its diameter) before it collides with another molecule.

Also, there are about 2.5×10^{16} molecules of oxygen in the tiny volume of 1 mm^3 at 1 atm pressure and 20°C (Fig. 2–2). The continuum model is applicable as long as the characteristic length of the system (such as its diameter) is much larger than the mean free path of the molecules. At very high vacuums or very high elevations, the mean free path may become large (for example, it is about 0.1 m for atmospheric air at an elevation of 100 km). For such cases the **rarefied gas flow theory** should be used, and the impact of individual molecules should be considered. In this text we limit our consideration to substances that can be modeled as a continuum.

2–2 ■ DENSITY AND SPECIFIC GRAVITY

Density is defined as *mass per unit volume* (Fig. 2–3). That is,

$$\text{Density:} \quad \rho = \frac{m}{V} \quad (\text{kg/m}^3) \quad (2-1)$$

The reciprocal of density is the **specific volume** ν , which is defined as *volume per unit mass*. That is, $\nu = V/m = 1/\rho$. For a differential volume element of mass δm and volume δV , density can be expressed as $\rho = \delta m/\delta V$.

The density of a substance, in general, depends on temperature and pressure. The density of most gases is proportional to pressure and inversely proportional to temperature. Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible. At 20°C , for example, the density of water changes from 998 kg/m^3 at 1 atm to 1003 kg/m^3 at 100 atm, a change of just 0.5 percent. The density of liquids and solids depends more strongly on temperature than it does on pressure. At 1 atm, for example, the density of water changes from 998 kg/m^3 at 20°C to 975 kg/m^3 at 75°C , a change of 2.3 percent, which can still be neglected in many engineering analyses.

Sometimes the density of a substance is given relative to the density of a well-known substance. Then it is called **specific gravity**, or **relative density**, and is defined as *the ratio of the density of a substance to the density of some standard substance at a specified temperature* (usually water at 4°C , for which $\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$). That is,

$$\text{Specific gravity:} \quad \text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}} \quad (2-2)$$

Note that the specific gravity of a substance is a dimensionless quantity. However, in SI units, the numerical value of the specific gravity of a substance is exactly equal to its density in g/cm^3 or kg/L (or 0.001 times the density in kg/m^3) since the density of water at 4°C is $1 \text{ g/cm}^3 = 1 \text{ kg/L} = 1000 \text{ kg/m}^3$. The specific gravity of mercury at 0°C , for example, is 13.6. Therefore, its density at 0°C is $13.6 \text{ g/cm}^3 = 13.6 \text{ kg/L} = 13,600 \text{ kg/m}^3$. The specific gravities of some substances at 0°C are given in Table 2–1. Note that substances with specific gravities less than 1 are lighter than water, and thus they would float on water.

The weight of a unit volume of a substance is called **specific weight** and is expressed as

$$\text{Specific weight:} \quad \gamma_s = \rho g \quad (\text{N/m}^3) \quad (2-3)$$

where g is the gravitational acceleration.

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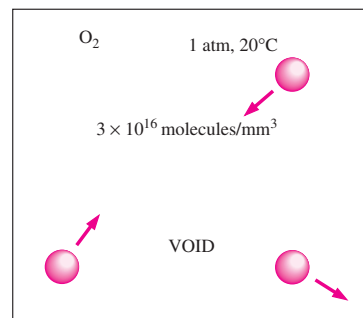


FIGURE 2–2

Despite the large gaps between molecules, a substance can be treated as a continuum because of the very large number of molecules even in an extremely small volume.

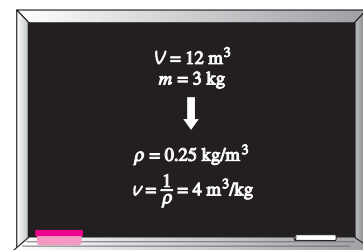


FIGURE 2–3

Density is mass per unit volume; specific volume is volume per unit mass.

TABLE 2–1

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Recall from Chap. 1 that the densities of liquids are essentially constant, and thus they can often be approximated as being incompressible substances during most processes without sacrificing much in accuracy.

Density of Ideal Gases

Property tables provide very accurate and precise information about the properties, but sometimes it is convenient to have some simple relations among the properties that are sufficiently general and accurate. Any equation that relates the pressure, temperature, and density (or specific volume) of a substance is called an **equation of state**. The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state, expressed as

$$Pv = RT \quad \text{or} \quad P = \rho RT \quad (2-4)$$

where P is the absolute pressure, v is the specific volume, T is the thermodynamic (absolute) temperature, ρ is the density, and R is the gas constant. The gas constant R is different for each gas and is determined from $R = R_u/M$, where R_u is the **universal gas constant** whose value is $R_u = 8.314 \text{ kJ/kmol} \cdot \text{K} = 1.986 \text{ Btu/lbmol} \cdot \text{R}$, and M is the molar mass (also called *molecular weight*) of the gas. The values of R and M for several substances are given in Table A-1.

The thermodynamic temperature scale in the SI is the **Kelvin scale**, and the temperature unit on this scale is the **kelvin**, designated by K. In the English system, it is the **Rankine scale**, and the temperature unit on this scale is the **rankine**, R. Various temperature scales are related to each other by

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15 \quad (2-5)$$

$$T(\text{R}) = T(^{\circ}\text{F}) + 459.67 \quad (2-6)$$

It is common practice to round the constants 273.15 and 459.67 to 273 and 460, respectively.

Equation 2-4 is called the **ideal-gas equation of state**, or simply the **ideal-gas relation**, and a gas that obeys this relation is called an **ideal gas**. For an ideal gas of volume V , mass m , and number of moles $N = m/M$, the ideal-gas equation of state can also be written as $PV = mRT$ or $PV = NR_uT$. For a fixed mass m , writing the ideal-gas relation twice and simplifying, the properties of an ideal gas at two different states are related to each other by $P_1V_1/T_1 = P_2V_2/T_2$.

An ideal gas is a hypothetical substance that obeys the relation $Pv = RT$. It has been experimentally observed that the ideal-gas relation closely approximates the P - v - T behavior of real gases at low densities. At low pressures and high temperatures, the density of a gas decreases and the gas behaves like an ideal gas. In the range of practical interest, many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, and krypton and even heavier gases such as carbon dioxide can be treated as ideal gases with negligible error (often less than 1 percent). Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not be treated as ideal gases since they usually exist at a state near saturation.

EXAMPLE 2-1 Density, Specific Gravity, and Mass of Air in a Room

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C (Fig. 2-4).

Solution The density, specific gravity, and mass of the air in a room are to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas.

Properties The gas constant of air is $R = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$.

Analysis The density of air is determined from the ideal-gas relation $P = \rho RT$ to be

$$\rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273) \text{ K}} = 1.17 \text{ kg/m}^3$$

Then the specific gravity of air becomes

$$\text{SG} = \frac{\rho}{\rho_{\text{H}_2\text{O}}} = \frac{1.17 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 0.00117$$

Finally, the volume and the mass of air in the room are

$$V = (4 \text{ m})(5 \text{ m})(6 \text{ m}) = 120 \text{ m}^3$$

$$m = \rho V = (1.17 \text{ kg/m}^3)(120 \text{ m}^3) = 140 \text{ kg}$$

Discussion Note that we converted the temperature to the unit K from °C before using it in the ideal-gas relation.

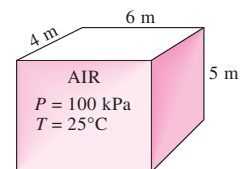


FIGURE 2-4
Schematic for Example 2-1.

2-3 ■ VAPOR PRESSURE AND CAVITATION

It is well-established that temperature and pressure are dependent properties for pure substances during phase-change processes, and there is one-to-one correspondence between temperatures and pressures. At a given pressure, the temperature at which a pure substance changes phase is called the **saturation temperature** T_{sat} . Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the **saturation pressure** P_{sat} . At an absolute pressure of 1 standard atmosphere (1 atm or 101.325 kPa), for example, the saturation temperature of water is 100°C. Conversely, at a temperature of 100°C, the saturation pressure of water is 1 atm.

The **vapor pressure** P_v of a pure substance is defined as *the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature*. P_v is a property of the pure substance, and turns out to be identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{\text{sat}}$). We must be careful not to confuse vapor pressure with *partial pressure*. **Partial pressure** is defined as *the pressure of a gas or vapor in a mixture with other gases*. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor. The partial pressure of water vapor constitutes a small fraction (usually under 3 percent) of the atmospheric pressure since air is mostly nitrogen and oxygen. The partial pressure of a vapor must be less than or equal to the vapor pressure if there is no liquid present. However, when both vapor and liquid are present and the system is in phase equilibrium, the partial pressure of the vapor must equal the vapor pressure, and the system is said to be *saturated*. The rate of evaporation from open water bodies such as

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TABLE 2-2

Saturation (or vapor) pressure of water at various temperatures

Temperature $T, ^\circ\text{C}$	Saturation Pressure $P_{\text{sat}}, \text{kPa}$
-10	0.260
-5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

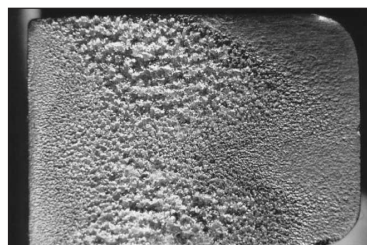


FIGURE 2-5

Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 h. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.

Photograph by David Stinebring,
ARL/Pennsylvania State University.
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lakes is controlled by the difference between the vapor pressure and the partial pressure. For example, the vapor pressure of water at 20°C is 2.34 kPa. Therefore, a bucket of water at 20°C left in a room with dry air at 1 atm will continue evaporating until one of two things happens: the water evaporates away (there is not enough water to establish phase equilibrium in the room), or the evaporation stops when the partial pressure of the water vapor in the room rises to 2.34 kPa at which point phase equilibrium is established.

For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure and the vapor pressure are equivalent since the vapor is pure. Note that the pressure value would be the same whether it is measured in the vapor or liquid phase (provided that it is measured at a location close to the liquid–vapor interface to avoid the hydrostatic effects). Vapor pressure increases with temperature. Thus, a substance at higher temperatures boils at higher pressures. For example, water boils at 134°C in a pressure cooker operating at 3 atm absolute pressure, but it boils at 93°C in an ordinary pan at a 2000-m elevation, where the atmospheric pressure is 0.8 atm. The saturation (or vapor) pressures are given in Appendices 1 and 2 for various substances. A mini table for water is given in Table 2-2 for easy reference.

The reason for our interest in vapor pressure is the possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization. For example, water at 10°C will flash into vapor and form bubbles at locations (such as the tip regions of impellers or suction sides of pumps) where the pressure drops below 1.23 kPa. The vapor bubbles (called **cavitation bubbles** since they form “cavities” in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves. This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called **cavitation**, and it is an important consideration in the design of hydraulic turbines and pumps (Fig. 2-5).

Cavitation must be avoided (or at least minimized) in flow systems since it reduces performance, generates annoying vibrations and noise, and causes damage to equipment. The pressure spikes resulting from the large number of bubbles collapsing near a solid surface over a long period of time may cause erosion, surface pitting, fatigue failure, and the eventual destruction of the components or machinery. The presence of cavitation in a flow system can be sensed by its characteristic tumbling sound.

EXAMPLE 2-2 Minimum Pressure to Avoid Cavitation

In a water distribution system, the temperature of water is observed to be as high as 30°C . Determine the minimum pressure allowed in the system to avoid cavitation.

SOLUTION The minimum pressure in a water distribution system to avoid cavitation is to be determined.

Properties The vapor pressure of water at 30°C is 4.25 kPa.

Analysis To avoid cavitation, the pressure anywhere in flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature. That is,

$$P_{\text{min}} = P_{\text{sat}@30^\circ\text{C}} = 4.25 \text{ kPa}$$

Therefore, the pressure should be maintained above 4.25 kPa everywhere in the flow.

Discussion Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

2-4 ■ ENERGY AND SPECIFIC HEATS

Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, and nuclear, and their sum constitutes the **total energy** E (or e on a unit mass basis) of a system. The forms of energy related to the molecular structure of a system and the degree of the molecular activity are referred to as the *microscopic energy*. The sum of all microscopic forms of energy is called the **internal energy** of a system, and is denoted by U (or u on a unit mass basis).

The *macroscopic* energy of a system is related to motion and the influence of some external effects such as gravity, magnetism, electricity, and surface tension. The energy that a system possesses as a result of its motion relative to some reference frame is called **kinetic energy**. When all parts of a system move with the same velocity, the kinetic energy per unit mass is expressed as $ke = V^2/2$ where V denotes the velocity of the system relative to some fixed reference frame. The energy that a system possesses as a result of its elevation in a gravitational field is called **potential energy** and is expressed on a per-unit mass basis as $pe = gz$ where g is the gravitational acceleration and z is the elevation of the center of gravity of a system relative to some arbitrarily selected reference plane.

In daily life, we frequently refer to the sensible and latent forms of internal energy as **heat**, and we talk about the heat content of bodies. In engineering, however, those forms of energy are usually referred to as **thermal energy** to prevent any confusion with *heat transfer*.

The international unit of energy is the *joule* (J) or *kilojoule* (1 kJ = 1000 J). In the English system, the unit of energy is the *British thermal unit* (Btu), which is defined as the energy needed to raise the temperature of 1 lbm of water at 68°F by 1°F. The magnitudes of kJ and Btu are almost identical (1 Btu = 1.0551 kJ). Another well-known unit of energy is the *calorie* (1 cal = 4.1868 J), which is defined as the energy needed to raise the temperature of 1 g of water at 14.5°C by 1°C.

In the analysis of systems that involve fluid flow, we frequently encounter the combination of properties u and Pv . For convenience, this combination is called **enthalpy** h . That is,

$$\text{Enthalpy:} \quad h = u + Pv = u + \frac{P}{\rho} \quad (2-7)$$

where P/ρ is the *flow energy*, also called the *flow work*, which is the energy per unit mass needed to move the fluid and maintain flow. In the energy analysis of flowing fluids, it is convenient to treat the flow energy as part of the energy of the fluid and to represent the microscopic energy of a fluid stream by enthalpy h (Fig. 2-6). Note that enthalpy is a quantity per unit mass, and thus it is a *specific* property.

In the absence of such effects as magnetic, electric, and surface tension, a system is called a simple compressible system. The total energy of a simple

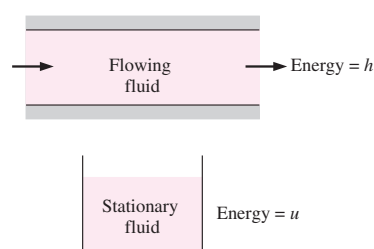


FIGURE 2-6

The *internal energy* u represents the microscopic energy of a nonflowing fluid per unit mass, whereas *enthalpy* h represents the microscopic energy of a flowing fluid per unit mass.

compressible system consists of three parts: internal, kinetic, and potential energies. On a unit-mass basis, it is expressed as $e = u + ke + pe$. The fluid entering or leaving a control volume possesses an additional form of energy—the *flow energy* P/ρ . Then the total energy of a **flowing fluid** on a unit-mass basis becomes

$$e_{\text{flowing}} = P/\rho + e = h + ke + pe = h + \frac{V^2}{2} + gz \quad (\text{kJ/kg}) \quad (2-8)$$

where $h = P/\rho + u$ is the enthalpy, V is the velocity, and z is the elevation of the system relative to some external reference point.

By using the enthalpy instead of the internal energy to represent the energy of a flowing fluid, one does not need to be concerned about the flow work. The energy associated with pushing the fluid is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.

The differential and finite changes in the internal energy and enthalpy of an *ideal gas* can be expressed in terms of the specific heats as

$$du = c_v dT \quad \text{and} \quad dh = c_p dT \quad (2-9)$$

where c_v and c_p are the constant-volume and constant-pressure specific heats of the ideal gas. Using specific heat values at the average temperature, the finite changes in internal energy and enthalpy can be expressed approximately as

$$\Delta u = c_{v,\text{ave}} \Delta T \quad \text{and} \quad \Delta h = c_{p,\text{ave}} \Delta T \quad (2-10)$$

For *incompressible substances*, the constant-volume and constant-pressure specific heats are identical. Therefore, $c_p \cong c_v \cong c$ for liquids, and the change in the internal energy of liquids can be expressed as $\Delta u = c_{\text{ave}} \Delta T$.

Noting that $\rho = \text{constant}$ for incompressible substances, the differentiation of enthalpy $h = u + P/\rho$ gives $dh = du + dP/\rho$. Integrating, the enthalpy change becomes

$$\Delta h = \Delta u + \Delta P/\rho \cong c_{\text{ave}} \Delta T + \Delta P/\rho \quad (2-11)$$

Therefore, $\Delta h = \Delta u \cong c_{\text{ave}} \Delta T$ for constant-pressure processes, and $\Delta h = \Delta P/\rho$ for constant-temperature processes of liquids.

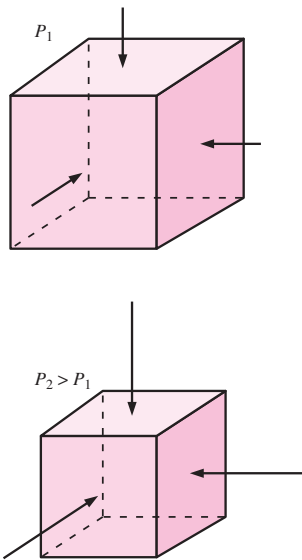


FIGURE 2-7
Fluids, like solids, compress when the applied pressure is increased from P_1 to P_2 .

2-5 ■ COEFFICIENT OF COMPRESSIBILITY

We know from experience that the volume (or density) of a fluid changes with a change in its temperature or pressure. Fluids usually expand as they are heated or depressurized and contract as they are cooled or pressurized. But the amount of volume change is different for different fluids, and we need to define properties that relate volume changes to the changes in pressure and temperature. Two such properties are the bulk modulus of elasticity κ and the coefficient of volume expansion β .

It is a common observation that a fluid contracts when more pressure is applied on it and expands when the pressure acting on it is reduced (Fig. 2-7). That is, fluids act like elastic solids with respect to pressure. Therefore, in an analogous manner to Young's modulus of elasticity for solids, it is appropriate to define a **coefficient of compressibility** κ (also called the **bulk modulus of compressibility** or **bulk modulus of elasticity**) for fluids as

$$\kappa = -v \left(\frac{\partial P}{\partial v} \right)_T = \rho \left(\frac{\partial P}{\partial \rho} \right)_T \quad (\text{Pa}) \quad (2-12)$$

It can also be expressed approximately in terms of finite changes as

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \quad (T = \text{constant}) \quad (2-13)$$

Noting that $\Delta v/v$ or $\Delta \rho/\rho$ is dimensionless, κ must have the dimension of pressure (Pa or psi). Also, the coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant. Then it follows that the coefficient of compressibility of a truly incompressible substance ($v = \text{constant}$) is infinity.

A large value of κ indicates that a large change in pressure is needed to cause a small fractional change in volume, and thus a fluid with a large κ is essentially incompressible. This is typical for liquids, and explains why liquids are usually considered to be *incompressible*. For example, the pressure of water at normal atmospheric conditions must be raised to 210 atm to compress it 1 percent, corresponding to a coefficient of compressibility value of $\kappa = 21,000$ atm.

Small density changes in liquids can still cause interesting phenomena in piping systems such as the *water hammer*—characterized by a sound that resembles the sound produced when a pipe is “hammered.” This occurs when a liquid in a piping network encounters an abrupt flow restriction (such as a closing valve) and is locally compressed. The acoustic waves produced strike the pipe surfaces, bends, and valves as they propagate and reflect along the pipe, causing the pipe to vibrate and produce the familiar sound.

Note that volume and pressure are inversely proportional (volume decreases as pressure is increased and thus $\partial P/\partial v$ is a negative quantity), and the negative sign in the definition (Eq. 2-12) ensures that κ is a positive quantity. Also, differentiating $\rho = 1/v$ gives $d\rho = -dv/v^2$, which can be rearranged as

$$\frac{d\rho}{\rho} = -\frac{dv}{v} \quad (2-14)$$

That is, the fractional changes in the specific volume and the density of a fluid are equal in magnitude but opposite in sign.

For an ideal gas, $P = \rho RT$ and $(\partial P/\partial \rho)_T = RT = P/\rho$, and thus

$$\kappa_{\text{ideal gas}} = P \quad (\text{Pa}) \quad (2-15)$$

Therefore, the coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure. Substituting $\kappa = P$ into the definition of the coefficient of compressibility and rearranging gives

$$\text{Ideal gas:} \quad \frac{\Delta \rho}{\rho} = \frac{\Delta P}{P} \quad (T = \text{constant}) \quad (2-16)$$

Therefore, the percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

For air at 1 atm pressure, $\kappa = P = 1$ atm and a decrease of 1 percent in volume ($\Delta v/v = -0.01$) corresponds to an increase of $\Delta P = 0.01$ atm in pressure. But for air at 1000 atm, $\kappa = 1000$ atm and a decrease of 1 percent in volume corresponds to an increase of $\Delta P = 10$ atm in pressure. Therefore,

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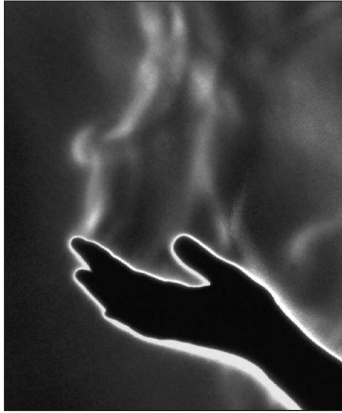
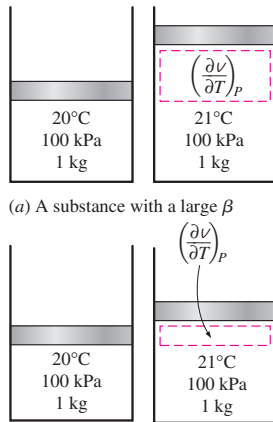


FIGURE 2-8
Natural convection over a woman's hand.

G. S. Settles, Gas Dynamics Lab,
Penn State University. Used by permission.



(b) A substance with a small β

FIGURE 2-9
The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

a small fractional change in the volume of a gas can cause a large change in pressure at very high pressures.

The inverse of the coefficient of compressibility is called the **isothermal compressibility** α and is expressed as

$$\alpha = \frac{1}{\kappa} = -\frac{1}{v} \left(\frac{\partial v}{\partial P} \right)_T = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial P} \right)_T \quad (1/\text{Pa}) \quad (2-17)$$

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

Coefficient of Volume Expansion

The density of a fluid, in general, depends more strongly on temperature than it does on pressure, and the variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, rise of plumes in chimneys, the operation of hot-air balloons, heat transfer by natural convection, and even the rise of hot air and thus the phrase “heat rises” (Fig. 2–8). To quantify these effects, we need a property that represents the *variation of the density of a fluid with temperature at constant pressure*.

The property that provides that information is the **coefficient of volume expansion** (or volume expansivity) β , defined as (Fig. 2–9)

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_P = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P \quad (1/\text{K}) \quad (2-18)$$

It can also be expressed approximately in terms of finite changes as

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T} \quad (\text{at constant } P) \quad (2-19)$$

A large value of β for a fluid means a large change in density with temperature, and the product $\beta \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change of ΔT at constant pressure.

It can be shown easily that the volume expansion coefficient of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \quad (1/\text{K}) \quad (2-20)$$

where T is the *absolute* temperature.

In the study of natural convection currents, the condition of the main fluid body that surrounds the finite hot or cold regions is indicated by the subscript “infinity” to serve as a reminder that this is the value at a distance where the presence of the hot or cold region is not felt. In such cases, the volume expansion coefficient can be expressed approximately as

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T} \quad \text{or} \quad \rho_\infty - \rho = \rho\beta(T - T_\infty) \quad (2-21)$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the confined hot or cold fluid pocket.

We will see in Chap. 3 that natural convection currents are initiated by the *buoyancy force*, which is proportional to the *density difference*, which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the hot or cold fluid pocket and the surrounding main fluid body, the *larger* the buoyancy force and thus the *stronger* the natural convection currents.

The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of T and P . Differentiating $v = v(T, P)$ and using the definitions of the compression and expansion coefficients α and β give

$$dv = \left(\frac{\partial v}{\partial T} \right)_P dT + \left(\frac{\partial v}{\partial P} \right)_T dP = (\beta dT - \alpha dP)v \quad (2-22)$$

Then the fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

$$\frac{\Delta v}{v} = -\frac{\Delta \rho}{\rho} \cong \beta \Delta T - \alpha \Delta P \quad (2-23)$$

EXAMPLE 2-3 Variation of Density with Temperature and Pressure

Consider water initially at 20°C and 1 atm. Determine the final density of water (a) if it is heated to 50°C at a constant pressure of 1 atm, and (b) if it is compressed to 100-atm pressure at a constant temperature of 20°C. Take the isothermal compressibility of water to be $\alpha = 4.80 \times 10^{-5} \text{ atm}^{-1}$.

SOLUTION Water at a given temperature and pressure is considered. The densities of water after it is heated and after it is compressed are to be determined.

Assumptions 1 The coefficient of volume expansion and the isothermal compressibility of water are constant in the given temperature range. 2 An approximate analysis is performed by replacing differential changes in quantities by finite changes.

Properties The density of water at 20°C and 1 atm pressure is $\rho_1 = 998.0 \text{ kg/m}^3$. The coefficient of volume expansion at the average temperature of $(20 + 50)/2 = 35^\circ\text{C}$ is $\beta = 0.337 \times 10^{-3} \text{ K}^{-1}$. The isothermal compressibility of water is given to be $\alpha = 4.80 \times 10^{-5} \text{ atm}^{-1}$.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as (Eq. 2-23)

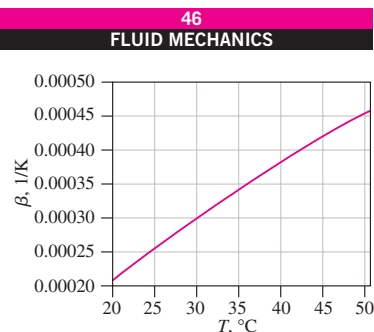
$$\Delta \rho = \alpha \rho \Delta P - \beta \rho \Delta T$$

(a) The change in density due to the change of temperature from 20°C to 50°C at constant pressure is

$$\begin{aligned} \Delta \rho &= -\beta \rho \Delta T = -(0.337 \times 10^{-3} \text{ K}^{-1})(998 \text{ kg/m}^3)(50 - 20) \text{ K} \\ &= -10.0 \text{ kg/m}^3 \end{aligned}$$

Noting that $\Delta \rho = \rho_2 - \rho_1$, the density of water at 50°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta \rho = 998.0 + (-10.0) = \mathbf{988.0 \text{ kg/m}^3}$$

**FIGURE 2-10**

The variation of the coefficient of volume expansion of water β with temperature in the range of 20°C to 50°C.

Data were generated and plotted using EES.

which is almost identical to the listed value of 988.1 kg/m³ at 50°C in Table A-3. This is mostly due to β varying with temperature almost linearly, as shown in Fig. 2-10.

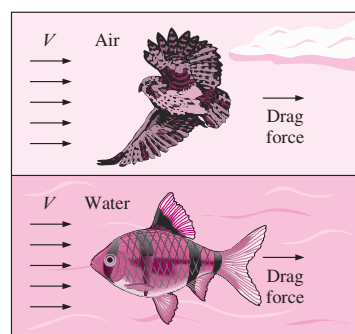
(b) The change in density due to a change of pressure from 1 atm to 100 atm at constant temperature is

$$\Delta\rho = \alpha\rho\Delta P = (4.80 \times 10^{-5} \text{ atm}^{-1})(998 \text{ kg/m}^3)(100 - 1) \text{ atm} = 4.7 \text{ kg/m}^3$$

Then the density of water at 100 atm and 20°C becomes

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + 4.7 = \mathbf{1002.7 \text{ kg/m}^3}$$

Discussion Note that the density of water decreases while being heated and increases while being compressed, as expected. This problem can be solved more accurately using differential analysis when functional forms of properties are available.

**FIGURE 2-11**

A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.

2-6 ■ VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the *friction coefficient* between the table and the floor.

The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the “fluidity,” and that property is the **viscosity**. The force a flowing fluid exerts on a body in the flow direction is called the **drag force**, and the magnitude of this force depends, in part, on viscosity (Fig. 2-11).

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance ℓ (Fig. 2-12). Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant velocity V . The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same velocity, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A} \quad (2-24)$$

where A is the contact area between the plate and the fluid. Note that the fluid layer deforms continuously under the influence of shear stress.

The fluid in contact with the lower plate assumes the velocity of that plate, which is zero (again because of the no-slip condition). In steady laminar

flow, the fluid velocity between the plates varies linearly between 0 and V , and thus the *velocity profile* and the *velocity gradient* are

$$u(y) = \frac{y}{\ell} V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell} \quad (2-25)$$

where y is the vertical distance from the lower plate.

During a differential time interval dt , the sides of fluid particles along a vertical line MN rotate through a differential angle $d\beta$ while the upper plate moves a differential distance $da = V dt$. The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan \beta = \frac{da}{\ell} = \frac{V dt}{\ell} = \frac{du}{dy} dt \quad (2-26)$$

Rearranging, the rate of deformation under the influence of shear stress τ becomes

$$\frac{d\beta}{dt} = \frac{du}{dy} \quad (2-27)$$

Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient du/dy . Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress τ ,

$$\tau \propto \frac{d\beta}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy} \quad (2-28)$$

Fluids for which the rate of deformation is proportional to the shear stress are called **Newtonian fluids** after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids.

In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

$$\text{Shear stress:} \quad \tau = \mu \frac{du}{dy} \quad (\text{N/m}^2) \quad (2-29)$$

where the constant of proportionality μ is called the **coefficient of viscosity** or the **dynamic** (or **absolute**) **viscosity** of the fluid, whose unit is $\text{kg/m} \cdot \text{s}$, or equivalently, $\text{N} \cdot \text{s/m}^2$ (or $\text{Pa} \cdot \text{s}$ where Pa is the pressure unit pascal). A common viscosity unit is **poise**, which is equivalent to $0.1 \text{ Pa} \cdot \text{s}$ (or **centipoise**, which is one-hundredth of a poise). The viscosity of water at 20°C is 1 centipoise, and thus the unit centipoise serves as a useful reference. A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Fig. 2-13. Note that viscosity is independent of the rate of deformation.

The **shear force** acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

$$\text{Shear force:} \quad F = \tau A = \mu A \frac{du}{dy} \quad (\text{N}) \quad (2-30)$$

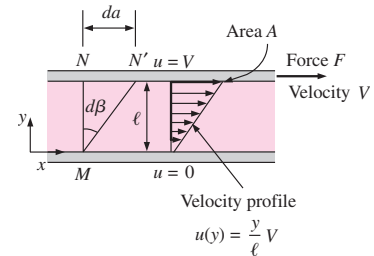


FIGURE 2-12

The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

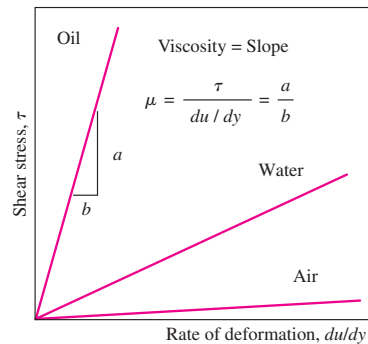


FIGURE 2-13

The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

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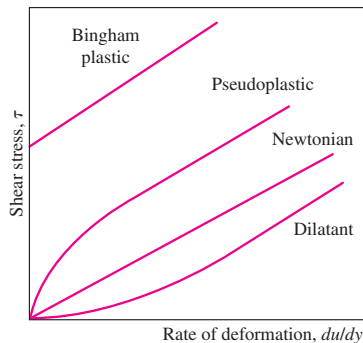


FIGURE 2-14

Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

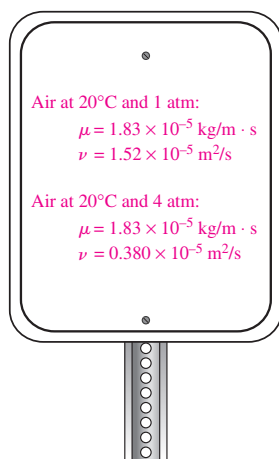


FIGURE 2-15

Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.

where again A is the contact area between the plate and the fluid. Then the force F required to move the upper plate in Fig. 2-12 at a constant velocity of V while the lower plate remains stationary is

$$F = \mu A \frac{V}{\ell} \quad (\text{N}) \quad (2-31)$$

This relation can alternately be used to calculate μ when the force F is measured. Therefore, the experimental setup just described can be used to measure the viscosity of fluids. Note that under identical conditions, the force F will be very different for different fluids.

For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, as shown in Fig. 2-14. The slope of the curve on the τ versus du/dy chart is referred to as the *apparent viscosity* of the fluid. Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as *dilatant* or *shear thickening fluids*, and those that exhibit the opposite behavior (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as *pseudoplastic* or *shear thinning fluids*. Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and thus behave as a fluid. Such materials are referred to as Bingham plastics after E. C. Bingham, who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century.

In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name **kinematic viscosity** ν and is expressed as $\nu = \mu/\rho$. Two common units of kinematic viscosity are m^2/s and **stoke** ($1 \text{ stoke} = 1 \text{ cm}^2/\text{s} = 0.0001 \text{ m}^2/\text{s}$).

In general, the viscosity of a fluid depends on both temperature and pressure, although the dependence on pressure is rather weak. For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure, and any small variation with pressure is usually disregarded, except at extremely high pressures. For *gases*, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure (Fig. 2-15).

The viscosity of a fluid is a measure of its “resistance to deformation.” Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature. The viscosity of liquids decreases with temperature, whereas the viscosity of gases increases with temperature (Fig. 2-16). This is because in a liquid the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, on the other hand, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow. The viscosity of a fluid is directly

related to the pumping power needed to transport a fluid in a pipe or to move a body (such as a car in air or a submarine in the sea) through a fluid.

The kinetic theory of gases predicts the viscosity of gases to be proportional to the square root of temperature. That is, $\mu_{\text{gas}} \propto \sqrt{T}$. This prediction is confirmed by practical observations, but deviations for different gases need to be accounted for by incorporating some correction factors. The viscosity of *gases* is expressed as a function of temperature by the Sutherland correlation (from The U.S. Standard Atmosphere) as

$$\mu = \frac{aT^{1/2}}{1 + b/T} \quad (2-32)$$

where T is absolute temperature and a and b are experimentally determined constants. Note that measuring viscosities at two different temperatures is sufficient to determine these constants. For air, the values of these constants are $a = 1.458 \times 10^{-6} \text{ kg/(m} \cdot \text{s} \cdot \text{K}^{1/2})$ and $b = 110.4 \text{ K}$ at atmospheric conditions. The viscosity of gases is independent of pressure at low to moderate pressures (from a few percent of 1 atm to several atm). But viscosity increases at high pressures due to the increase in density.

For *liquids*, the viscosity is approximated as

$$\mu = a10^{b/(T-c)} \quad (2-33)$$

where again T is absolute temperature and a , b , and c are experimentally determined constants. For water, using the values $a = 2.414 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$, $b = 247.8 \text{ K}$, and $c = 140 \text{ K}$ results in less than 2.5 percent error in viscosity in the temperature range of 0°C to 370°C (Touloukian et al., 1975).

Consider a fluid layer of thickness ℓ within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap between the cylinders can be modeled as two parallel flat plates separated by a fluid. Noting that torque is $T = FR$ (force times the moment arm, which is the radius R of the inner cylinder in this case), the tangential velocity is $V = \omega R$ (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be $A = 2\pi RL$ by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

$$T = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell} \quad (2-34)$$

where L is the length of the cylinder and \dot{n} is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute). Note that the angular distance traveled during one rotation is 2π rad, and thus the relation between the angular velocity in rad/min and the rpm is $\omega = 2\pi\dot{n}$. Equation 2-34 can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity. Therefore, two concentric cylinders can be used as a *viscometer*, a device that measures viscosity.

The viscosities of some fluids at room temperature are listed in Table 2-3. They are plotted against temperature in Fig. 2-17. Note that the viscosities of different fluids differ by several orders of magnitude. Also note that it is more difficult to move an object in a higher-viscosity fluid such as engine oil than it is in a lower-viscosity fluid such as water. Liquids, in general, are much more viscous than gases.

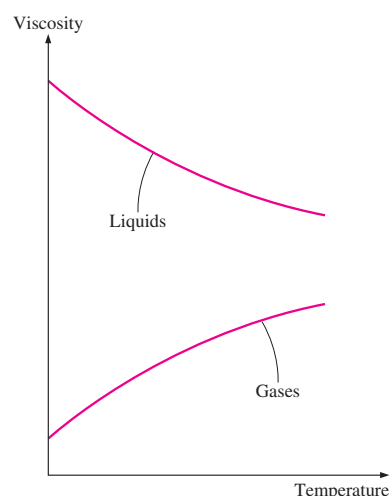


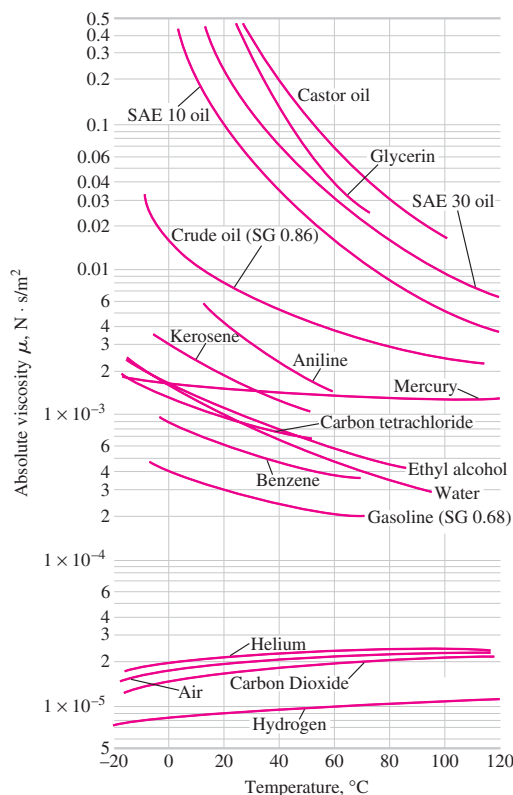
FIGURE 2-16

The viscosity of liquids decreases and the viscosity of gases increases with temperature.

TABLE 2-3

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic Viscosity μ , $\text{kg/m} \cdot \text{s}$
Glycerin:	
-20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

**FIGURE 2-17**

The variation of dynamic (absolute) viscosities of common fluids with temperature at 1 atm ($1 \text{ N} \cdot \text{s}/\text{m}^2 = 1 \text{ kg}/\text{m} \cdot \text{s} = 0.020886 \text{ lbf} \cdot \text{s}/\text{ft}^2$).

*F. M. White, Fluid Mechanics 4e.
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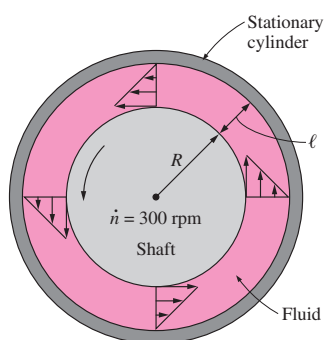


FIGURE 2-18
Schematic for Example 2-4.

EXAMPLE 2-4 Determining the Viscosity of a Fluid

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders (Fig. 2-18). The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be $1.8 \text{ N} \cdot \text{m}$. Determine the viscosity of the fluid.

SOLUTION The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions 1 The inner cylinder is completely submerged in oil. 2 The viscous effects on the two ends of the inner cylinder are negligible.

Analysis The velocity profile is linear only when the curvature effects are negligible, and the profile can be approximated as being linear in this case since $\ell/R \ll 1$. Solving Eq. 2-34 for viscosity and substituting the given values, the viscosity of the fluid is determined to be

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n}L} = \frac{(1.8 \text{ N} \cdot \text{m})(0.0015 \text{ m})}{4\pi^2 (0.06 \text{ m})^3 (300/60 \text{ 1/s})(0.4 \text{ m})} = \mathbf{0.158 \text{ N} \cdot \text{s}/\text{m}^2}$$

Discussion Viscosity is a strong function of temperature, and a viscosity value without a corresponding temperature is of little value. Therefore, the temperature of the fluid should have also been measured during this experiment, and reported with this calculation.

2-7 ■ SURFACE TENSION AND CAPILLARY EFFECT

It is often observed that a drop of blood forms a hump on a horizontal glass; a drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface; water droplets from rain or dew hang from branches or leaves of trees; a liquid fuel injected into an engine forms a mist of spherical droplets; water dripping from a leaky faucet falls as spherical droplets; a soap bubble released into the air forms a spherical shape; and water beads up into small drops on flower petals (Fig. 2-19).

In these and other observances, liquid droplets behave like small spherical balloons filled with the liquid, and the surface of the liquid acts like a stretched elastic membrane under tension. The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid. The magnitude of this force per unit length is called **surface tension** σ_s , and is usually expressed in the unit N/m (or lbf/ft in English units). This effect is also called *surface energy* and is expressed in the equivalent unit of $\text{N} \cdot \text{m}/\text{m}^2$ or J/m^2 . In this case, σ_s represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount.

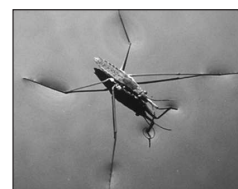
To visualize how surface tension arises, we present a microscopic view in Fig. 2-20 by considering two liquid molecules, one at the surface and one deep within the liquid body. The attractive forces applied on the interior molecule by the surrounding molecules balance each other because of symmetry. But the attractive forces acting on the surface molecule are not symmetric, and the attractive forces applied by the gas molecules above are usually very small. Therefore, there is a net attractive force acting on the molecule at the surface of the liquid, which tends to pull the molecules on the surface toward the interior of the liquid. This force is balanced by the repulsive forces from the molecules below the surface that are being compressed. The resulting compression effect causes the liquid to minimize its surface area. This is the reason for the tendency of the liquid droplets to attain a spherical shape, which has the minimum surface area for a given volume.

You also may have observed, with amusement, that some insects can land on water or even walk on water (Fig. 2-19b) and that small steel needles can float on water. These phenomena are again made possible by surface tension that balances the weights of these objects.

To understand the surface tension effect better, consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side (Fig. 2-21). Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area. A force F needs to be applied on the movable wire in the opposite direction to balance this pulling effect. The thin film in the device has two surfaces (the top and bottom



(a)



(b)

FIGURE 2-19

Some consequences of surface tension.

(a) © Pegasus/Visuals Unlimited.

(b) © Dennis Drenner/Visuals Unlimited.

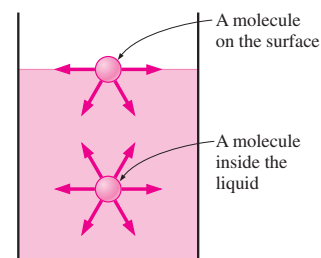


FIGURE 2-20

Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.

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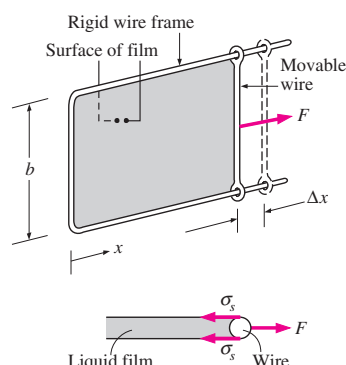


FIGURE 2-21

Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length b .

surfaces) exposed to air, and thus the length along which the tension acts in this case is $2b$. Then a force balance on the movable wire gives $F = 2b\sigma_s$, and thus the surface tension can be expressed as

$$\sigma_s = \frac{F}{2b} \quad (2-35)$$

Note that for $b = 0.5$ m, the force F measured (in N) is simply the surface tension in N/m. An apparatus of this kind with sufficient precision can be used to measure the surface tension of various fluids.

In the U-shaped wire, the force F remains constant as the movable wire is pulled to stretch the film and increase its surface area. When the movable wire is pulled a distance Δx , the surface area increases by $\Delta A = 2b \Delta x$, and the work done W during this stretching process is

$$W = \text{Force} \times \text{Distance} = F \Delta x = 2b\sigma_s \Delta x = \sigma_s \Delta A$$

since the force remains constant in this case. This result can also be interpreted as *the surface energy of the film is increased by an amount $\sigma_s \Delta A$ during this stretching process*, which is consistent with the alternative interpretation of σ_s as surface energy. This is similar to a rubber band having more potential (elastic) energy after it is stretched further. In the case of liquid film, the work is used to move liquid molecules from the interior parts to the surface against the attraction forces of other molecules. Therefore, surface tension also can be defined as *the work done per unit increase in the surface area of the liquid*.

The surface tension varies greatly from substance to substance, and with temperature for a given substance, as shown in Table 2-4. At 20°C, for example, the surface tension is 0.073 N/m for water and 0.440 N/m for mercury surrounded by atmospheric air. Mercury droplets form spherical balls that can be rolled like a solid ball on a surface without wetting the surface. The surface tension of a liquid, in general, decreases with temperature and becomes zero at the critical point (and thus there is no distinct liquid–vapor interface at temperatures above the critical point). The effect of pressure on surface tension is usually negligible.

The surface tension of a substance can be changed considerably by *impurities*. Therefore, certain chemicals, called *surfactants*, can be added to a liquid to decrease its surface tension. For example, soaps and detergents lower the surface tension of water and enable it to penetrate through the small openings between fibers for more effective washing. But this also means that devices whose operation depends on surface tension (such as heat pipes) can be destroyed by the presence of impurities due to poor workmanship.

We speak of surface tension for liquids only at liquid–liquid or liquid–gas interfaces. Therefore, it is important to specify the adjacent liquid or gas when specifying surface tension. Also, surface tension determines the size of the liquid droplets that form. A droplet that keeps growing by the addition of more mass will break down when the surface tension can no longer hold it together. This is like a balloon that will burst while being inflated when the pressure inside rises above the strength of the balloon material.

A curved interface indicates a pressure difference (or “pressure jump”) across the interface with pressure being higher on the concave side. The

TABLE 2-4

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s , N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

* Multiply by 0.06852 to convert to lbf/ft.

excess pressure ΔP inside a droplet or bubble above the atmospheric pressure, for example, can be determined by considering the free-body diagram of half a droplet or bubble (Fig. 2–22). Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet and the bubble give

$$\text{Droplet:} \quad (2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R} \quad (2-36)$$

$$\text{Bubble:} \quad 2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R} \quad (2-37)$$

where P_i and P_o are the pressures inside and outside the droplet or bubble, respectively. When the droplet or bubble is in the atmosphere, P_o is simply atmospheric pressure. The factor 2 in the force balance for the bubble is due to the bubble consisting of a film with *two* surfaces (inner and outer surfaces) and thus two circumferences in the cross section.

The excess pressure in a droplet (or bubble) also can be determined by considering a differential increase in the radius of the droplet due to the addition of a differential amount of mass and interpreting the surface tension as the increase in the surface energy per unit area. Then the increase in the surface energy of the droplet during this differential expansion process becomes

$$\delta W_{\text{surface}} = \sigma_s dA = \sigma_s d(4\pi R^2) = 8\pi R\sigma_s dR$$

The expansion work done during this differential process is determined by multiplying the force by distance to obtain

$$\delta W_{\text{expansion}} = \text{Force} \times \text{Distance} = F dR = (\Delta P A) dR = 4\pi R^2 \Delta P dR$$

Equating the two expressions above gives $\Delta P_{\text{droplet}} = 2\sigma_s/R$, which is the same relation obtained before and given in Eq. 2–36. Note that the excess pressure in a droplet or bubble is inversely proportional to the radius.

Capillary Effect

Another interesting consequence of surface tension is the **capillary effect**, which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid. Such narrow tubes or confined flow channels are called **capillaries**. The rise of kerosene through a cotton wick inserted into the reservoir of a kerosene lamp is due to this effect. The capillary effect is also partially responsible for the rise of water to the top of tall trees. The curved free surface of a liquid in a capillary tube is called the **meniscus**.

It is commonly observed that water in a glass container curves up slightly at the edges where it touches the glass surface; but the opposite occurs for mercury: it curves down at the edges (Fig. 2–23). This effect is usually expressed by saying that water *wets* the glass (by sticking to it) while mercury does not. The strength of the capillary effect is quantified by the **contact** (or **wetting**) **angle** ϕ , defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact*. The surface tension force acts along this tangent line toward the solid surface. A liquid is said to wet the surface when $\phi < 90^\circ$ and not to wet the surface when $\phi > 90^\circ$. In atmospheric air, the contact angle of water (and most other organic

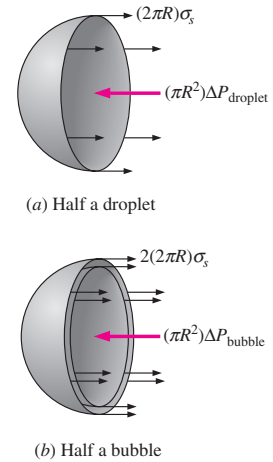


FIGURE 2–22

The free-body diagram of half a droplet and half a bubble.

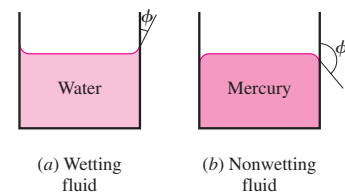


FIGURE 2–23

The contact angle for wetting and nonwetting fluids.

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FLUID MECHANICS

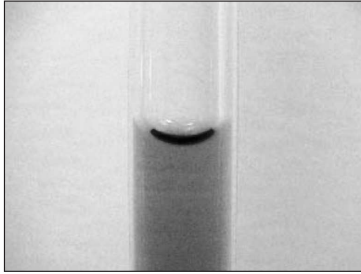


FIGURE 2-24

The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.

Photo by Gabrielle Trembley, Pennsylvania State University. Used by permission.

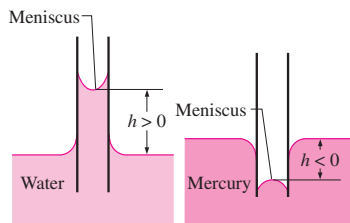


FIGURE 2-25

The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

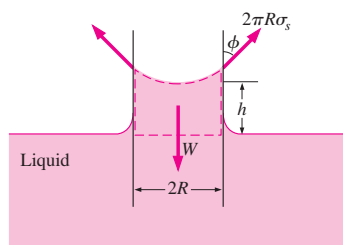


FIGURE 2-26

The forces acting on a liquid column that has risen in a tube due to the capillary effect.

liquids) with glass is nearly zero, $\phi \approx 0^\circ$ (Fig. 2-24). Therefore, the surface tension force acts upward on water in a glass tube along the circumference, tending to pull the water up. As a result, water rises in the tube until the weight of the liquid in the tube above the liquid level of the reservoir balances the surface tension force. The contact angle is 130° for mercury–glass and 26° for kerosene–glass in air. Note that the contact angle, in general, is different in different environments (such as another gas or liquid in place of air).

The phenomenon of capillary effect can be explained microscopically by considering *cohesive forces* (the forces between like molecules, such as water and water) and *adhesive forces* (the forces between unlike molecules, such as water and glass). The liquid molecules at the solid–liquid interface are subjected to both cohesive forces by other liquid molecules and adhesive forces by the molecules of the solid. The relative magnitudes of these forces determine whether a liquid wets a solid surface or not. Obviously, the water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed (Fig. 2-25).

The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height h in the tube (Fig. 2-26). The bottom of the liquid column is at the same level as the free surface of the reservoir, and thus the pressure there must be atmospheric pressure. This balances the atmospheric pressure acting at the top surface, and thus these two effects cancel each other. The weight of the liquid column is approximately

$$W = mg = \rho Vg = \rho g(\pi R^2 h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{\text{surface}} \rightarrow \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos \phi$$

Solving for h gives the capillary rise to be

$$\text{Capillary rise: } h = \frac{2\sigma_s}{\rho g R} \cos \phi \quad (R = \text{constant}) \quad (2-38)$$

This relation is also valid for nonwetting liquids (such as mercury in glass) and gives the capillary drop. In this case $\phi > 90^\circ$ and thus $\cos \phi < 0$, which makes h negative. Therefore, a negative value of capillary rise corresponds to a capillary drop (Fig. 2-25).

Note that the capillary rise is inversely proportional to the radius of the tube. Therefore, the thinner the tube is, the greater the rise (or fall) of the liquid in the tube. In practice, the capillary effect is usually negligible in tubes whose diameter is greater than 1 cm. When pressure measurements are made using manometers and barometers, it is important to use sufficiently large tubes to minimize the capillary effect. The capillary rise is also inversely proportional to the density of the liquid, as expected. Therefore, lighter liquids experience greater capillary rises. Finally, it should be kept in mind that Eq. 2-38 is derived for constant-diameter tubes and should not be used for tubes of variable cross section.

EXAMPLE 2-5 The Capillary Rise of Water in a Tube

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig. 2-27).

SOLUTION The rise of water in a slender tube as a result of the capillary effect is to be determined.

Assumptions 1 There are no impurities in the water and no contamination on the surfaces of the glass tube. 2 The experiment is conducted in atmospheric air.

Properties The surface tension of water at 20°C is 0.073 N/m (Table 2-3). The contact angle of water with glass is 0° (from preceding text). We take the density of liquid water to be 1000 kg/m³.

Analysis The capillary rise is determined directly from Eq. 2-15 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi = \frac{2(0.073 \text{ N/m})}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.3 \times 10^{-3} \text{ m})} (\cos 0^\circ) \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) \\ = 0.050 \text{ m} = \mathbf{5.0 \text{ cm}}$$

Therefore, water rises in the tube 5 cm above the liquid level in the cup.

Discussion Note that if the tube diameter were 1 cm, the capillary rise would be 0.3 mm, which is hardly noticeable to the eye. Actually, the capillary rise in a large-diameter tube occurs only at the rim. The center does not rise at all. Therefore, the capillary effect can be ignored for large-diameter tubes.

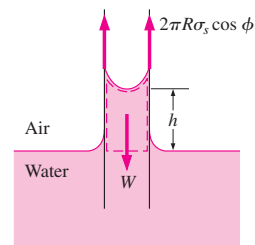


FIGURE 2-27
Schematic for Example 2-5.

SUMMARY

In this chapter various properties commonly used in fluid mechanics are discussed. The mass-dependent properties of a system are called *extensive properties* and the others, *intensive properties*. *Density* is mass per unit volume, and *specific volume* is volume per unit mass. The *specific gravity* is defined as the ratio of the density of a substance to the density of water at 4°C,

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}}}$$

The ideal-gas equation of state is expressed as

$$P = \rho RT$$

where P is the absolute pressure, T is the thermodynamic temperature, ρ is the density, and R is the gas constant.

At a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure*. For phase-change processes between the liquid and vapor phases of a pure substance, the saturation pressure is commonly called the *vapor pressure* P_v . Vapor bubbles that form in the

low-pressure regions in a liquid (a phenomenon called *cavitation*) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.

Energy can exist in numerous forms, and their sum constitutes the *total energy* E (or e on a unit-mass basis) of a system. The sum of all microscopic forms of energy is called the *internal energy* U of a system. The energy that a system possesses as a result of its motion relative to some reference frame is called *kinetic energy* expressed per unit mass as $ke = V^2/2$, and the energy that a system possesses as a result of its elevation in a gravitational field is called *potential energy* expressed per unit mass as $pe = gz$.

The compressibility effects in a fluid are represented by the **coefficient of compressibility** κ (also called **bulk modulus of elasticity**) defined as

$$\kappa = -v \left(\frac{\partial P}{\partial v} \right)_T = \rho \left(\frac{\partial P}{\partial \rho} \right)_T \cong - \frac{\Delta P}{\Delta v/v}$$

The property that represents the variation of the density of a fluid with temperature at constant pressure is the **volume expansion coefficient** (or volume expansivity) β , defined as

$$\beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p = - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \cong - \frac{\Delta \rho / \rho}{\Delta T}$$

The *viscosity* of a fluid is a measure of its resistance to deformation. The tangential force per unit area is called *shear stress* and is expressed for simple shear flow between plates (one-dimensional flow) as

$$\tau = \mu \frac{du}{dy}$$

where μ is the coefficient of viscosity or the *dynamic* (or *absolute*) viscosity of the fluid, u is the velocity component in the flow direction, and y is the direction normal to the flow direction. The fluids that obey this linear relationship are called *Newtonian fluids*. The ratio of dynamic viscosity to density is called the *kinematic viscosity* ν .

The pulling effect on the liquid molecules at an interface caused by the attractive forces of molecules per unit length is called *surface tension* σ_s . The excess pressure ΔP inside a spherical droplet or bubble is given by

$$\Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R} \quad \text{and} \quad \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

where P_i and P_o are the pressures inside and outside the droplet or bubble. The rise or fall of a liquid in a small-diameter tube inserted into the liquid due to surface tension is called the *capillary effect*. The capillary rise or drop is given by

$$h = \frac{2\sigma_s}{\rho g R} \cos \phi$$

where ϕ is the *contact angle*. The capillary rise is inversely proportional to the radius of the tube and is negligible for tubes whose diameter is larger than about 1 cm.

Density and viscosity are two of the most fundamental properties of fluids, and they are used extensively in the chapters that follow. In Chap. 3, the effect of density on the variation of pressure in a fluid is considered, and the hydrostatic forces acting on surfaces are determined. In Chap. 8, the pressure drop caused by viscous effects during flow is calculated and used in the determination of the pumping power requirements. Viscosity is also used as a key property in the formulation and solutions of the equations of fluid motion in Chaps. 9 and 10.

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APPLICATION SPOTLIGHT ■ Cavitation

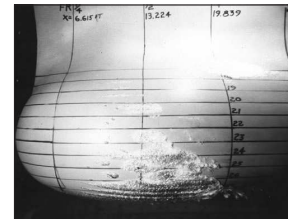
Guest Authors: G. C. Lauchle and M. L. Billet,
Penn State University

Cavitation is the rupture of a liquid, or of a fluid–solid interface, caused by a reduction of the local static pressure produced by the dynamic action of the fluid in the interior and/or boundaries of a liquid system. The rupture is the formation of a visible bubble. Liquids, such as water, contain many microscopic voids that act as *cavitation nuclei*. Cavitation occurs when these nuclei grow to a significant, visible size. Although boiling is also the formation of voids in a liquid, we usually separate this phenomenon from cavitation because it is caused by an increase in temperature, rather than by a reduction in pressure. Cavitation can be used in beneficial ways, such as in ultrasonic cleaners, etchers, and cutters. But more often than not, cavitation is to be avoided in fluid flow applications because it spoils hydrodynamic performance, it causes extremely loud noise and high vibration levels, and it damages (erodes) the surfaces that support it. When cavitation bubbles enter regions of high pressure and collapse, the underwater shock waves sometimes create minute amounts of light. This phenomenon is called *sonoluminescence*.

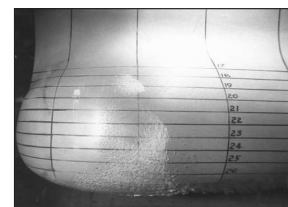
Body cavitation is illustrated in Fig. 2–28. The body is a model of the underwater bulbous bow region of a surface ship. It is shaped this way because located within it is a *sound navigation and ranging* (sonar) system that is spherical in shape. This part of the surface ship is thus called a *sonar dome*. As ship speeds get faster and faster some of these domes start to cavitate and the noise created by the cavitation renders the sonar system useless. Naval architects and fluid dynamicists attempt to design these domes so that they will not cavitate. Model-scale testing allows the engineer to see first hand whether a given design provides improved cavitation performance. Because such tests are conducted in water tunnels, the conditions of the test water should have sufficient nuclei to model those conditions in which the prototype operates. This assures that the effect of liquid tension (nuclei distribution) is minimized. Important variables are the gas content level (nuclei distribution) of the water, the temperature, and the hydrostatic pressure at which the body operates. Cavitation first appears—as either the speed V is increased, or as the submergence depth h is decreased—at the minimum pressure point $C_{p_{\min}}$ of the body. Thus, good hydrodynamic design requires $2(P_{\infty} - P_v)/\rho V^2 > C_{p_{\min}}$, where ρ is density, $P_{\infty} = \rho gh$ is the reference to static pressure, C_p is the pressure coefficient (Chap. 7), and P_v is the vapor pressure of water.

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(a)



(b)

FIGURE 2–28

(a) *Vaporous cavitation* occurs in water that has very little entrained gas, such as that found very deep in a body of water. Cavitation bubbles are formed when the speed of the body—in this case the bulbous bow region of a surface ship sonar dome—increases to the point where the local static pressure falls below the vapor pressure of the water. The cavitation bubbles are filled essentially with water vapor. This type of cavitation is very violent and noisy. (b) On the other hand, in shallow water, there is much more entrained gas in the water to act as cavitation nuclei. That's because of the proximity of the dome to the atmosphere at the free surface. The cavitation bubbles first appear at a slower speed, and hence at a higher local static pressure. They are predominantly filled with the gases that are entrained in the water, so this is known as *gaseous cavitation*.

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PROBLEMS*

Density and Specific Gravity


2-1C What is the difference between intensive and extensive properties?

2-2C What is specific gravity? How is it related to density?

2-3C Under what conditions is the ideal-gas assumption suitable for real gases?

2-4C What is the difference between R and R_u ? How are these two related?



2-5 A spherical balloon with a diameter of 6 m is filled with helium at 20°C and 200 kPa. Determine the mole number and the mass of the helium in the balloon. *Answers: 9.28 kmol, 37.2 kg*

2-6  Reconsider Prob. 2-5. Using EES (or other) software, investigate the effect of the balloon diameter on the mass of helium contained in the balloon for the pressures of (a) 100 kPa and (b) 200 kPa. Let the diameter vary from 5 m to 15 m. Plot the mass of helium against the diameter for both cases.

2-7 The pressure in an automobile tire depends on the temperature of the air in the tire. When the air temperature is 25°C, the pressure gage reads 210 kPa. If the volume of the tire is 0.025 m³, determine the pressure rise in the tire when the air temperature in the tire rises to 50°C. Also, determine the amount of air that must be bled off to restore pressure to its original value at this temperature. Assume the atmospheric pressure to be 100 kPa.




FIGURE P2-7

* Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the  icon are solved using EES, and complete solutions together with parametric studies are included on the enclosed DVD Problems with the  icon are comprehensive in nature and are intended to be solved with a computer, preferably using the EES software that accompanies this text.

2-8E The air in an automobile tire with a volume of 0.53 ft³ is at 90°F and 20 psig. Determine the amount of air that must be added to raise the pressure to the recommended value of 30 psig. Assume the atmospheric pressure to be 14.6 psia and the temperature and the volume to remain constant. *Answer: 0.0260 lbm*

2-9E A rigid tank contains 20 lbm of air at 20 psia and 70°F. More air is added to the tank until the pressure and temperature rise to 35 psia and 90°F, respectively. Determine the amount of air added to the tank. *Answer: 13.7 lbm*

2-10  The density of atmospheric air varies with elevation, decreasing with increasing altitude. (a) Using the data given in the table, obtain a relation for the variation of density with elevation, and calculate the density at an elevation of 7000 m. (b) Calculate the mass of the atmosphere using the correlation you obtained. Assume the earth to be a perfect sphere with a radius of 6377 km, and take the thickness of the atmosphere to be 25 km.

z , km	ρ , kg/m ³
6377	1.225
6378	1.112
6379	1.007
6380	0.9093
6381	0.8194
6382	0.7364
6383	0.6601
6385	0.5258
6387	0.4135
6392	0.1948
6397	0.08891
6402	0.04008

Vapor Pressure and Cavitation

2-11C What is vapor pressure? How is it related to saturation pressure?

2-12C Does water boil at higher temperatures at higher pressures? Explain.

2-13C If the pressure of a substance is increased during a boiling process, will the temperature also increase or will it remain constant? Why?

2-14C What is cavitation? What causes it?

2-15 In a piping system, the water temperature remains under 40°C. Determine the minimum pressure allowed in the system to avoid cavitation.

2-16 The analysis of a propeller that operates in water at 20°C shows that the pressure at the tips of the propeller drops to 2 kPa at high speeds. Determine if there is a danger of cavitation for this propeller.

2-17E The analysis of a propeller that operates in water at 70°F shows that the pressure at the tips of the propeller drops to 0.1 psia at high speeds. Determine if there is a danger of cavitation for this propeller.

2-18 A pump is used to transport water to a higher reservoir. If the water temperature is 25°C, determine the lowest pressure that can exist in the pump without cavitation.

Energy and Specific Heats

2-19C What is the difference between the macroscopic and microscopic forms of energy?

2-20C What is total energy? Identify the different forms of energy that constitute the total energy.

2-21C List the forms of energy that contribute to the internal energy of a system.

2-22C How are heat, internal energy, and thermal energy related to each other?

2-23C What is flow energy? Do fluids at rest possess any flow energy?

2-24C How do the energies of a flowing fluid and a fluid at rest compare? Name the specific forms of energy associated with each case.

2-25C Using average specific heats, explain how internal energy changes of ideal gases and incompressible substances can be determined.

2-26C Using average specific heats, explain how enthalpy changes of ideal gases and incompressible substances can be determined.

Coefficient of Compressibility

2-27C What does the coefficient of compressibility of a fluid represent? How does it differ from isothermal compressibility?

2-28C What does the coefficient of volume expansion of a fluid represent? How does it differ from the coefficient of compressibility?

2-29C Can the coefficient of compressibility of a fluid be negative? How about the coefficient of volume expansion?

2-30 It is observed that the density of an ideal gas decreases by 10 percent when compressed isothermally from 10 atm to 11 atm. Determine the percent decrease in density of the gas if it is compressed isothermally from 100 atm to 101 atm.

2-31 Using the definition of the coefficient of volume expansion and the expression $\beta_{\text{ideal gas}} = 1/T$, show that the percent increase in the specific volume of an ideal gas during isobaric expansion is equal to the percent increase in absolute temperature.

2-32 Water at 1 atm pressure is compressed to 800 atm pressure isothermally. Determine the increase in the density of water. Take the isothermal compressibility of water to be $4.80 \times 10^{-5} \text{ atm}^{-1}$.

2-33 Water at 15°C and 1 atm pressure is heated to 100°C at constant pressure. Using coefficient of volume expansion data, determine the change in the density of water.

Answer: -38.7 kg/m^3

2-34 Saturated refrigerant-134a liquid at 10°C is cooled to 0°C at constant pressure. Using coefficient of volume expansion data, determine the change in the density of the refrigerant.

2-35 A water tank is completely filled with liquid water at 20°C. The tank material is such that it can withstand tension caused by a volume expansion of 2 percent. Determine the maximum temperature rise allowed without jeopardizing safety.

2-36 Repeat Prob. 2-35 for a volume expansion of 1 percent for water.

2-37 The density of seawater at a free surface where the pressure is 98 kPa is approximately 1030 kg/m^3 . Taking the bulk modulus of elasticity of seawater to be $2.34 \times 10^9 \text{ N/m}^2$ and expressing variation of pressure with depth z as $dP = \rho g dz$ determine the density and pressure at a depth of 2500 m. Disregard the effect of temperature.

Viscosity

2-38C What is viscosity? What is the cause of it in liquids and in gases? Do liquids or gases have higher dynamic viscosities?

2-39C What is a Newtonian fluid? Is water a Newtonian fluid?

2-40C Consider two identical small glass balls dropped into two identical containers, one filled with water and the other with oil. Which ball will reach the bottom of the container first? Why?

2-41C How does the dynamic viscosity of (a) liquids and (b) gases vary with temperature?

2-42C How does the kinematic viscosity of (a) liquids and (b) gases vary with temperature?

2-43 A 50-cm \times 30-cm \times 20-cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. (a) Determine the force F that needs to be applied in the horizontal direction. (b) If a 0.4-mm-thick oil film with a dynamic viscosity of $0.012 \text{ Pa} \cdot \text{s}$ is applied between the block and inclined surface, determine the percent reduction in the required force.

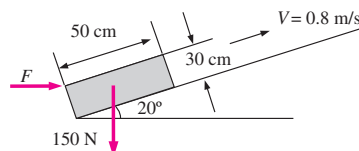


FIGURE P2-43

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FLUID MECHANICS

2-44 Consider the flow of a fluid with viscosity μ through a circular pipe. The velocity profile in the pipe is given as $u(r) = u_{\max}(1 - r^n/R^n)$, where u_{\max} is the maximum flow velocity, which occurs at the centerline; r is the radial distance from the centerline; and $u(r)$ is the flow velocity at any position r . Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe.

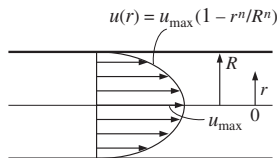


FIGURE P2-44

2-45 A thin 20-cm \times 20-cm flat plate is pulled at 1 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity of 0.3 m/s, as shown in Fig. P2-45. The dynamic viscosity of oil is 0.027 Pa \cdot s. Assuming the velocity in each oil layer to vary linearly, (a) plot the velocity profile and find the location where the oil velocity is zero and (b) determine the force that needs to be applied on the plate to maintain this motion.

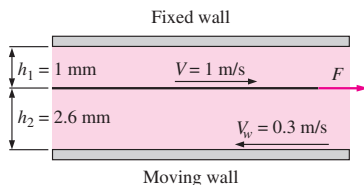


FIGURE P2-45

2-46 A frustum-shaped body is rotating at a constant angular speed of 200 rad/s in a container filled with SAE 10W oil

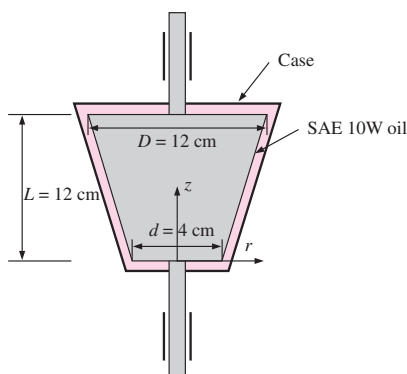


FIGURE P2-46

at 20°C ($\mu = 0.1$ Pa \cdot s), as shown in Fig. P2-46. If the thickness of the oil film on all sides is 1.2 mm, determine the power required to maintain this motion. Also determine the reduction in the required power input when the oil temperature rises to 80°C ($\mu = 0.0078$ Pa \cdot s).

2-47 The clutch system shown in Fig. P2-47 is used to transmit torque through a 3-mm-thick oil film with $\mu = 0.38$ N \cdot s/m² between two identical 30-cm-diameter disks. When the driving shaft rotates at a speed of 1450 rpm, the driven shaft is observed to rotate at 1398 rpm. Assuming a linear velocity profile for the oil film, determine the transmitted torque.

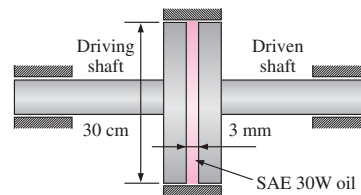



FIGURE P2-47

2-48  Reconsider Prob. 2-47. Using EES (or other) software, investigate the effect of oil film thickness on the torque transmitted. Let the film thickness vary from 0.1 mm to 10 mm. Plot your results, and state your conclusions.

2-49 The viscosity of some fluids changes when a strong electric field is applied on them. This phenomenon is known as the electrorheological (ER) effect, and fluids that exhibit such behavior are known as ER fluids. The Bingham plastic model for shear stress, which is expressed as $\tau = \tau_y + \mu(du/dy)$ is widely used to describe ER fluid behavior because of its simplicity. One of the most promising applications of ER fluids is the ER clutch. A typical multidisk ER clutch consists of several equally spaced steel disks of inner radius R_1 and outer radius R_2 , N of them attached to the input shaft. The gap h between the parallel disks is filled with a viscous fluid. (a) Find a relationship for the torque generated by the clutch when the output shaft is stationary

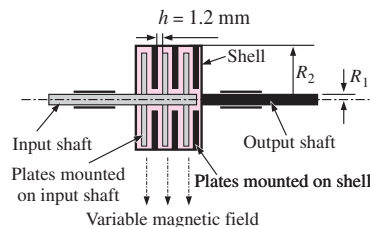


FIGURE P2-49

and (b) calculate the torque for an ER clutch with $N = 11$ for $R_1 = 50$ mm, $R_2 = 200$ mm, and $\dot{n} = 2400$ rpm if the fluid is SAE 10 with $\mu = 0.1$ Pa \cdot s, $\tau_y = 2.5$ kPa, and $h = 1.2$ mm.

Answer: (b) 2060 N \cdot m

2–50 The viscosity of some fluids, called magnetorheological (MR) fluids, changes when a magnetic field is applied. Such fluids involve micron-sized magnetizable particles suspended in an appropriate carrier liquid, and are suitable for use in controllable hydraulic clutches. See Fig. P2–49. The MR fluids can have much higher viscosities than the ER fluids, and they often exhibit shear-thinning behavior in which the viscosity of the fluid decreases as the applied shear force increases. This behavior is also known as pseudoplastic behavior, and can be successfully represented by Herschel–Bulkley constitutive model expressed as $\tau = \tau_y + K(du/dy)^m$. Here τ is the shear stress applied, τ_y is the yield stress, K is the consistency index, and m is the power index. For a Herschel–Bulkley fluid with $\tau_y = 900$ Pa, $K = 58$ Pa \cdot s^{*m*}, and $m = 0.82$, (a) find a relationship for the torque transmitted by an MR clutch for N plates attached to the input shaft when the input shaft is rotating at an angular speed of ω while the output shaft is stationary and (b) calculate the torque transmitted by such a clutch with $N = 11$ plates for $R_1 = 50$ mm, $R_2 = 200$ mm, $\dot{n} = 2400$ rpm, and $h = 1.2$ mm.

2–51 The viscosity of a fluid is to be measured by a viscometer constructed of two 75-cm-long concentric cylinders. The outer diameter of the inner cylinder is 15 cm, and the gap between the two cylinders is 0.12 cm. The inner cylinder is rotated at 200 rpm, and the torque is measured to be 0.8 N \cdot m. Determine the viscosity of the fluid.

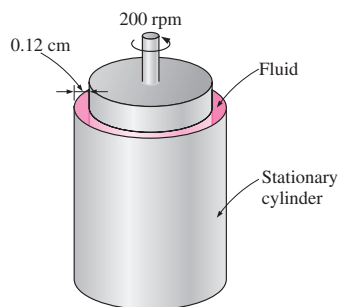


FIGURE P2–51

2–52E The viscosity of a fluid is to be measured by a viscometer constructed of two 3-ft-long concentric cylinders. The inner diameter of the outer cylinder is 6 in, and the gap between the two cylinders is 0.05 in. The outer cylinder is rotated at 250 rpm, and the torque is measured to be 1.2 lbf \cdot ft. Determine the viscosity of the fluid. **Answer:** 0.000648 lb \cdot s/ft²

2–53 In regions far from the entrance, fluid flow through a circular pipe is one-dimensional, and the velocity profile for

laminar flow is given by $u(r) = u_{\max}(1 - r^2/R^2)$, where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{\max} is the maximum flow velocity, which occurs at the center. Obtain (a) a relation for the drag force applied by the fluid on a section of the pipe of length L and (b) the value of the drag force for water flow at 20°C with $R = 0.08$ m, $L = 15$ m, $u_{\max} = 3$ m/s, and $\mu = 0.0010$ kg/m \cdot s.

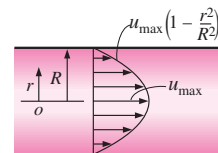


FIGURE P2–53

2–54 Repeat Prob. 2–53 for $u_{\max} = 5$ m/s. **Answer:** (b) 0.942 N

Surface Tension and Capillary Effect

2–55C What is surface tension? What is it caused by? Why is the surface tension also called surface energy?

2–56C Consider a soap bubble. Is the pressure inside the bubble higher or lower than the pressure outside?

2–57C What is the capillary effect? What is it caused by? How is it affected by the contact angle?

2–58C A small-diameter tube is inserted into a liquid whose contact angle is 110°. Will the level of liquid in the tube rise or drop? Explain.

2–59C Is the capillary rise greater in small- or large-diameter tubes?

2–60E A 0.03-in-diameter glass tube is inserted into kerosene at 68°F. The contact angle of kerosene with a glass surface is 26°. Determine the capillary rise of kerosene in the tube. **Answer:** 0.65 in

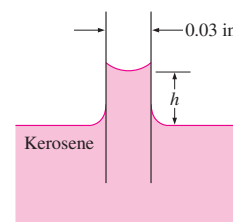



FIGURE P2–60E

2–61 A 1.9-mm-diameter tube is inserted into an unknown liquid whose density is 960 kg/m³, and it is observed that the liquid rises 5 mm in the tube, making a contact angle of 15°. Determine the surface tension of the liquid.

62
FLUID MECHANICS

2-62  Determine the gage pressure inside a soap bubble of diameter (a) 0.2 cm and (b) 5 cm at 20°C.

2-63 Nutrients dissolved in water are carried to upper parts of plants by tiny tubes partly because of the capillary effect. Determine how high the water solution will rise in a tree in a 0.005-mm-diameter tube as a result of the capillary effect. Treat the solution as water at 20°C with a contact angle of 15°. *Answer: 5.75 m*

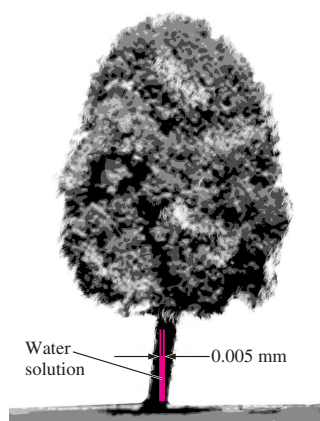


FIGURE P2-63

2-64 The surface tension of a liquid is to be measured using a liquid film suspended on a U-shaped wire frame with an 8-cm-long movable side. If the force needed to move the wire is 0.012 N, determine the surface tension of this liquid in air.

2-65 Contrary to what you might expect, a solid steel ball can float on water due to the surface tension effect. Determine the maximum diameter of a steel ball that would float on water at 20°C. What would your answer be for an aluminum ball? Take the densities of steel and aluminum balls to be 7800 kg/m³ and 2700 kg/m³, respectively.

Review Problems

2-66 The absolute pressure of an automobile tire is measured to be 290 kPa before a trip and 310 kPa after the trip. Assuming the volume of the tire remains constant at 0.022 m³, determine the percent increase in the absolute temperature of the air in the tire.

2-67 A 20-m³ tank contains nitrogen at 25°C and 800 kPa. Some nitrogen is allowed to escape until the pressure in the tank drops to 600 kPa. If the temperature at this point is 20°C, determine the amount of nitrogen that has escaped. *Answer: 42.9 kg*

2-68 The composition of a liquid with suspended solid particles is generally characterized by the fraction of solid parti-


cles either by weight or mass $C_{s, \text{mass}} = m_s/m_m$ or by volume, $C_{s, \text{vol}} = V_s/V_m$ where m is mass and V is volume. The subscripts s and m indicate solid and mixture, respectively. Develop an expression for the specific gravity of a water-based suspension in terms of $C_{s, \text{mass}}$ and $C_{s, \text{vol}}$.

2-69 The specific gravities of solids and carrier fluids of a slurry are usually known, but the specific gravity of the slurry depends on the concentration of the solid particles. Show that the specific gravity of a water-based slurry can be expressed in terms of the specific gravity of the solid SG_s and the mass concentration of the suspended solid particles $C_{s, \text{mass}}$ as

$$SG_m = \frac{1}{1 + C_{s, \text{mass}}(1/SG_s - 1)}$$

2-70E The pressure on the suction side of pumps is typically low, and the surfaces on that side of the pump are susceptible to cavitation, especially at high fluid temperatures. If the minimum pressure on the suction side of a water pump is 0.95 psia absolute, determine the maximum water temperature to avoid the danger of cavitation.

2-71 A closed tank is partially filled with water at 60°C. If the air above the water is completely evacuated, determine the absolute pressure in the evacuated space. Assume the temperature to remain constant.

2-72  The variation of the dynamic viscosity of water with absolute temperature is given as

T, K	$\mu, \text{Pa} \cdot \text{s}$
273.15	1.787×10^{-3}
278.15	1.519×10^{-3}
283.15	1.307×10^{-3}
293.15	1.002×10^{-3}
303.15	7.975×10^{-4}
313.15	6.529×10^{-4}
333.15	4.665×10^{-4}
353.15	3.547×10^{-4}
373.15	2.828×10^{-4}

Using tabulated data, develop a relation for viscosity in the form of $\mu = \mu(T) = A + BT + CT^2 + DT^3 + ET^4$. Using the relation developed, predict the dynamic viscosity of water at 50°C at which the reported value is $5.468 \times 10^{-4} \text{ Pa} \cdot \text{s}$. Compare your result with the results of Andrade's equation, which is given in the form of $\mu = D \cdot e^{B/T}$, where D and B are constants whose values are to be determined using the viscosity data given.

2-73 Consider laminar flow of a Newtonian fluid of viscosity μ between two parallel plates. The flow is one-dimensional, and the velocity profile is given as $u(y) = 4u_{\text{max}}[y/h - (y/h)^2]$, where y is the vertical coordinate from the bottom surface, h is the distance between the two plates, and u_{max} is the maximum flow velocity that occurs at midplane.

Develop a relation for the drag force exerted on both plates by the fluid in the flow direction per unit area of the plates.

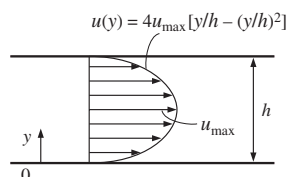


FIGURE P2-73

2-74 Some non-Newtonian fluids behave as a Bingham plastic for which shear stress can be expressed as $\tau = \tau_y + \mu(du/dr)$. For laminar flow of a Bingham plastic in a horizontal pipe of radius R , the velocity profile is given as $u(r) = (\Delta P/4\mu L)(r^2 - R^2) + (\tau_y/\mu)(r - R)$, where $\Delta P/L$ is the constant pressure drop along the pipe per unit length, μ is the dynamic viscosity, r is the radial distance from the centerline, and τ_y is the yield stress of Bingham plastic. Determine (a) the shear stress at the pipe wall and (b) the drag force acting on a pipe section of length L .

2-75 In some damping systems, a circular disk immersed in oil is used as a damper, as shown in Fig. P2-75. Show

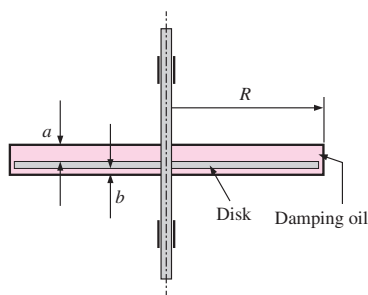


FIGURE P2-75

that the damping torque is proportional to angular speed in accordance with the relation $T_{\text{damping}} = C\omega$ where $C = 0.5\pi\mu(1/a + 1/b)R^4$. Assume linear velocity profiles on both sides of the disk and neglect the tip effects.

2-76E A 0.9-in-diameter glass tube is inserted into mercury, which makes a contact angle of 140° with glass. Determine the capillary drop of mercury in the tube at 68°F .
Answer: 0.0175 in

2-77 Derive a relation for the capillary rise of a liquid between two large parallel plates a distance t apart inserted into the liquid vertically. Take the contact angle to be ϕ .

2-78 Consider a 30-cm-long journal bearing that is lubricated with oil whose viscosity is $0.1 \text{ kg/m} \cdot \text{s}$ at 20°C at the beginning of operation and $0.008 \text{ kg/m} \cdot \text{s}$ at the anticipated steady operating temperature of 80°C . The diameter of the shaft is 8 cm, and the average gap between the shaft and the journal is 0.08 cm. Determine the torque needed to overcome the bearing friction initially and during steady operation when the shaft is rotated at 500 rpm.

Design and Essay Problems

2-79 Design an experiment to measure the viscosity of liquids using a vertical funnel with a cylindrical reservoir of height h and a narrow flow section of diameter D and length L . Making appropriate assumptions, obtain a relation for viscosity in terms of easily measurable quantities such as density and volume flow rate.

2-80 Write an essay on the rise of the fluid to the top of the trees by capillary and other effects.

2-81 Write an essay on the oils used in car engines in different seasons and their viscosities.

