**Introduction**

The two major problems that the modern power systems are facing are voltage and angle stabilities. There are various approaches to overcome the problem of stability arising due to small signal oscillations in an interconnected power system. As mentioned in the previous chapter, installing power system stabilizers with generator excitation control system provides damping to these oscillations. However, with the advancement in the power electronic technology, various reactive power control equipment are increasingly used in power transmission systems.

A power network is mostly reactive. A synchronous generator usually generates active power that is specified by the mechanical power input. The reactive power supplied by the generator is dictated by the network and load requirements. A generator usually does not have any control over it. However the lack of reactive power can cause voltage collapse in a system. It is therefore important to supply/absorb excess reactive power to/from the network. Shunt compensation is one possible approach of providing reactive power support.

A device that is connected in parallel with a transmission line is called a **shunt compensator**, while a device that is connected in series with the transmission line is called a **series compensator**. These are referred to as compensators since they compensate for the reactive power in the ac system. We shall assume that the shunt compensator is always connected at the midpoint of transmission system, while the series compensator can be connected at any point in the line. We shall demonstrate that such connections in an SMIB power system improves

- voltage profile
- power-angle characteristics
- stability margin
- damping to power oscillations

A **static var compensator (SVC)** is the first generation shunt compensator. It has been around since 1960s. In the beginning it was used for load compensation such as to provide var support for large industrial loads, for flicker mitigation etc. However with the advancement of semiconductor technology, the SVC started appearing in the transmission systems in 1970s. Today a large number of SVCs are connected to many transmission systems all over the world. An SVC is constructed using the thyristor technology and therefore does not have gate turn off capability.

With the advancement in the power electronic technology, the application of a gate turn off thyristor (GTO) to high power application became commercially feasible. With this the second generation shunt compensator device was conceptualized and constructed. These devices use synchronous voltage sources for generating or absorbing reactive power. A synchronous voltage source (SVS) is constructed using a voltage source converter (VSC). Such a shunt compensating device is called **static compensator or STATCOM**. A STATCOM usually contains an SVS that is driven from a dc storage capacitor and the SVS is connected to the ac system bus through an interface transformer. The transformer steps the ac system voltage down such that the voltage rating of the SVS switches are within specified limit. Furthermore, the leakage reactance of the transformer plays a very significant role in the operation of the STATCOM.

Like the SVC, a **thyristor controlled series compensator (TCSC)** is a thyristor based series compensator that connects a **thyristor controlled reactor (TCR)** in parallel with a fixed capacitor. By varying the firing angle of the anti-parallel thyristors that are connected in series with a reactor in the TCR, the fundamental frequency inductive reactance of the TCR can be changed. This effects a change in the reactance of the TCSC and it can be controlled to produce either inductive or capacitive reactance.

Alternatively a **static synchronous series compensator or SSSC** can be used for series compensation. An SSSC is an SVS based all GTO based device which contains a VSC. The VSC is driven by a dc capacitor. The output of the VSC is connected to a three-phase transformer. The other end of the transformer is connected in series with the transmission line. Unlike the TCSC, which changes the impedance of the line, an SSSC injects a voltage in the line in quadrature with the line current.
making the SSSC voltage to lead or lag the line current by 90°, the SSSC can emulate the behavior of an inductance or capacitance.

In this chapter, we shall discuss the ideal behavior of these compensating devices. For simplicity we shall consider the ideal models and broadly discuss the advantages of series and shunt compensation.

### Section I: Ideal Shunt Compensator

- **Improving Voltage Profile**
- **Improving Power-Angle Characteristics**
- **Improving Stability Margin**
- **Improving Damping to Power Oscillations**

The ideal shunt compensator is an ideal current source. We call this an ideal shunt compensator because we assume that it only supplies reactive power and no real power to the system. It is needless to say that this assumption is not valid for practical systems. However, for an introduction, the assumption is more than adequate. We shall investigate the behavior of the compensator when connected in the middle of a transmission line. This is shown in Fig. 10.1, where the shunt compensator, represented by an ideal current source, is placed in the middle of a lossless transmission line. We shall demonstrate that such a configuration improves the four points that are mentioned above.

---

**Fig. 10.1 Schematic diagram of an ideal, midpoint shunt compensation**

**Improving Voltage Profile**

Let the sending and receiving voltages be given by \( V_s \) and \( V_r \) respectively. The ideal shunt compensator is expected to regulate the midpoint voltage to

\[
V_M = \frac{V_s + V_r}{2}
\]

against any variation in the compensator current. The voltage current characteristic of the compensator is shown in Fig. 10.2. This ideal behavior however is not feasible in practical systems where we get a slight droop in the voltage characteristic. This will be discussed later.

---

**Fig. 10.2 Voltage-current characteristic of an ideal shunt compensator.**
Under the assumption that the shunt compensator regulates the midpoint voltage tightly as given by

\[ I_s = \frac{V \angle \delta - V \angle (\delta/2)}{jX/2} \]  
(10.2)
\[ I_x = \frac{V \angle (\delta/2) - V}{jX/2} \]  
(10.3)

(10.1), we can write the following expressions for the sending and receiving end currents

\[ I_s + I_G = I_R \]  
(10.4)

Again from Fig. 10.1 we write

\[ I_G = -j \frac{A \pi r}{X} \{1 - \cos(\delta/2)\} \angle (\delta/2) \]  
(10.5)

Combing (10.2)-(10.4) and solving we get

We thus have to generate a current that is in phase with the midpoint voltage and has a magnitude of

\[ F_s + jQ_s = V_s^* \left[ \frac{V \angle \delta - V \angle (\delta/2)}{-jX/2} \right] - \frac{V^2 - V \angle (\delta/2)}{-jX/2} \]  
(10.6)

\[ (4V/X) \{1 - \cos(\delta/2)\}. \] The apparent power injected by the shunt compensator to the ac bus is then

Since the real part of the injected power is zero, we conclude that the ideal shunt compensator injects only reactive power to the ac system and no real power.

### Improving Power-Angle Characteristics

\[ P_s + jQ_s = V_s^* \left[ \frac{V \angle \delta - V \angle (\delta/2)}{-jX/2} \right] - \frac{V^2 - V \angle (\delta/2)}{-jX/2} \]  
(10.7)

\[ = \frac{2V^2 \sin(\delta/2)}{X} + j \frac{2V^2 \{1 - \cos(\delta/2)\}}{X} \]

The apparent power supplied by the source is given by

Similarly the apparent power delivered at the receiving end is
\[ P_k + jQ_k = V_k^* = V \left[ \frac{-j \frac{\delta}{2} - j \frac{\gamma}{2}}{-j \frac{X}{2}} \right] \]
\[ = \frac{2V^2 \sin \left( \frac{\delta}{2} \right)}{X} + j \frac{2V^2 \cos \left( \frac{\delta}{2} \right) - 1}{X} \]  \hspace{1cm} (10.8)

\[ P_x = P_e = P_R = \frac{2V^2}{X} \sin \left( \frac{\delta}{2} \right) \]  \hspace{1cm} (10.9)

\[ Q_e = Q_2 - Q_0 = \frac{2V^2}{X} \left( 1 - \cos \left( \frac{\delta}{2} \right) \right) \]  \hspace{1cm} (10.10)

Hence the real power transmitted over the line is given by

Combining (10.6)-(10.8), we find the reactive power consumed by the line as

The power-angle characteristics of the shunt compensated line are shown in Fig. 10.3. In this figure \( P_{\text{max}} = V^2/X \) is chosen as the power base.

Fig. 10.3 Power-angle characteristics of ideal shunt compensated line.

Fig. 10.3 depicts \( P_e - \delta \) and \( Q_e - \delta \) characteristics. It can be seen from fig 10.4 that for a real power transfer of 1 per unit, a reactive power injection of roughly 0.5359 per unit will be required from the shunt compensator if the midpoint voltage is regulated as per (10.1). Similarly for increasing the real power transmitted to 2 per unit, the shunt compensator has to inject 4 per unit of reactive power. This will obviously increase the device rating and may not be practical. Therefore power transfer enhancement using midpoint shunt compensation may not be feasible from the device rating point of view.
Let us now relax the condition that the midpoint voltage is regulated to 1.0 per unit. We then obtain some very interesting plots as shown in Fig. 10.5. In this figure, the x-axis shows the reactive power available from the shunt device, while the y-axis shows the maximum power that can be transferred over the line without violating the voltage constraint. There are three different P-Q relationships given for three midpoint voltage constraints. For a reactive power injection of 0.5 per unit, the power transfer can be increased from about 0.97 per unit to 1.17 per unit by lowering the midpoint voltage to 0.9 per unit. For a reactive power injection greater than 2.0 per unit, the best power transfer capability is obtained for $V_M = 1.0$ per unit. Thus there will be no benefit in reducing the voltage constraint when the shunt device is capable of injecting a large amount of reactive power. In practice, the level to which the midpoint voltage can be regulated depends on the rating of the installed shunt device as well as the power being transferred.

**Improving Stability Margin**

This is a consequence of the improvement in the power angle characteristics and is one of the major benefits of using midpoint shunt compensation. As mentioned before, the stability margin of the system pertains to the regions of acceleration and deceleration in the power-angle curve. We shall use this concept to delineate the advantage of mid point shunt compensation.
Consider the power angle curves shown in Fig. 10.6.

The curve of Fig. 10.6 (a) is for an uncompensated system, while that of Fig. 10.6 (b) for the compensated system. Both these curves are drawn assuming that the base power is \( V^2/X \). Let us assume that the uncompensated system is operating on steady state delivering an electrical power equal to \( P_m \) with a load angle of \( \delta_0 \) when a three-phase fault occurs that forces the real power to zero. To obtain the critical clearing angle for the uncompensated system is \( \delta_{cr} \), we equate the accelerating area \( A_1 \) with

\[
A_1 = \int_{\delta_0}^{\delta_{max}} P_m \, d\delta = P_m (\delta_{cr} - \delta_0)
\]

the decelerating area \( A_2 \), where

\[
A_2 = \int_{\delta_{cr}}^{\delta_{max}} (\sin \delta - F_m) \, d\delta = (\cos \delta_{cr} - \cos \delta_{max}) - F_m (\delta_{max} - \delta_{cr})
\]

Let us now consider that the midpoint shunt compensated system is working with the same mechanical power input \( P_m \). The operating angle in this case is \( \delta_1 \) and the maximum power that can be transferred in this case is 2 per unit. Let the fault be cleared at the same clearing angle \( \delta_{cr} \) as before. Then equating

\[
A_3 = \int_{\delta_{cr}}^{\delta_{max}} P_m \, d\delta = P_m (\delta_{cr} - \delta_1)
\]

areas \( A_3 \) and \( A_4 \) in Fig. 10.6 (b) we get \( \delta_2 \), where

\[
\delta_{cr} = \cos^{-1} \left[ F_m (\delta_{max} - \delta_0) + \cos \delta_{max} \right] \tag{10.11}
\]

Example 10.1
Let an uncompensated SMIB power system is operating in steady state with a mechanical power input \( P_m \) equal to 0.5 per unit. Then \( \delta_0 = 30^\circ = 0.5236 \text{ rad} \) and \( \delta_{\text{max}} = 150^\circ = 2.6180 \text{ rad} \). Consequently, the critical clearing angle is calculated as (see Chapter 9) \( \delta_{\text{cr}} = 79.56^\circ = 1.3886 \text{ rad} \).

Let us now put an ideal shunt compensator at the midpoint. The pre-fault steady state operating angle of the compensated system can be obtained by solving \( 2 \sin(\delta/2) = 0.5 \), which gives \( \delta_1 = 28.96^\circ = 0.5054 \text{ rad} \). Let us assume that we use the same critical clearing angle as obtained above for clearing a fault in the compensated system as well.

The accelerating area is then given by \( A_3 = 0.4416 \). Equating with area \( A_4 \) we get a nonlinear equation of the form

\[
0.4416 = 3.0740 - 4 \cos(\delta/2) - 0.5 \delta_2 + 0.6943
\]

Solving the above equation we get \( \delta_2 = 104.34^\circ = 1.856 \text{ rad} \). It is needless to say that the stability margin has increased significantly in the compensated system.

**Improving Damping to Power Oscillations**

The swing equation of a synchronous machine is given by (9.14). For any variation in the electrical quantities, the mechanical power input remains constant. Assuming that the magnitude of the midpoint voltage of the system is controllable by the shunt compensating device, the accelerating power in (9.14) becomes a function of two independent variables, \( |V_M| \) and \( \delta \). Again since the mechanical power is constant, its perturbation with the independent variables is zero. We then get the following small perturbation expression of the swing equation

\[
\frac{2 \dot{I}_q}{\omega} \frac{d^2 \Delta S}{dt^2} + \frac{\partial F_e}{\partial |V_M|} \Delta |V_M| + \frac{\partial F_e}{\partial \delta} \Delta \delta = 0
\]  

(10.12)

where \( \Delta \) indicates a perturbation around the nominal values.

If the mid point voltage is regulated at a constant magnitude, \( \Delta |V_M| \) will be equal to zero. Hence the above equation will reduce to
Chapter 10: Compensation of Power Transmission Systems

The second order differential equation given in (10.13) can be written in the Laplace domain by neglecting the initial conditions as

$$\frac{2H}{\omega} \frac{d^2 \Delta \delta}{dt^2} + \frac{\partial P_e}{\partial \delta} \Delta \delta = 0$$  \hspace{1cm} (10.13)

The roots of the above equation are located on the imaginary axis of the s-plane at locations $\pm j \omega_m$ where

$$\omega_m = \sqrt{\left(\omega/2H\right)\left(\partial P_e/\partial \delta\right)}$$

(10.14)

This implies that the load angle will oscillate with a constant frequency of $\omega_m$. Obviously, this solution is not acceptable. Thus in order to provide damping, the mid point voltage must be varied according to in sympathy with the rate of change in $\Delta \delta$. We can then write

$$\Delta V_M = K_M \frac{d \Delta \delta}{dt}$$  \hspace{1cm} (10.15)

sympathy with the rate of change in $\Delta \delta$. We can then write

$$\frac{2H}{\omega} \frac{d^2 \Delta \delta}{dt^2} + \frac{\partial P_e}{\partial \delta} \left[ K_M \frac{d \Delta \delta}{dt} + \frac{\partial P_e}{\partial \delta} \Delta \delta \right] = 0$$  \hspace{1cm} (10.16)

where $K_M$ is a proportional gain. Substituting (10.15) in (10.12) we get

Provided that $K_M$ is positive definite, the introduction of the control action (10.15) ensures that the roots of the second order equation will have negative real parts. Therefore through the feedback, damping to power swings can be provided by placing the poles of the above equation to provide the necessary damping ratio and undamped natural frequency of oscillations.

Example 10.2

Consider the SMIB power system shown in Fig. 10.7. The generator is connected to the infinite bus through a double circuit transmission line. At the midpoint bus of the lines, a shunt compensator is connected. The shunt compensator is realized by the voltage source $V_F$ that is connected to the midpoint bus through a pure inductor $X_F$, also known as an interface inductor. The voltage source $V_F$ is driven such that it is always in phase with the midpoint voltage $V_M$. The current $I_0$ is then purely inductive, its direction being dependent on the relative magnitudes of the two voltages. If the magnitude of the midpoint voltage is higher than the voltage source $V_F$, inductive current will flow from the ac system to the voltage source. This implies that the source is absorbing var in this configuration. On the other hand, the source will generate var if its magnitude is higher than that of the midpoint voltage.

The system is simulated in MATLAB. The three-phase transmission line equations are simulated using their differential equations, while the generator is represented by a pure voltage source. The second order swing equation is simulated in which the mechanical power input is chosen such that the initial operating angle of the generator voltage is $(0.6981 \text{ rad})$. The instantaneous electrical power is computed from the dot product of the three-phase source current vector and source voltage vector. The system parameters chosen for simulation are:
Two different tests are performed. In the first one, the midpoint voltage is regulated to 1 per unit using a proportional-plus-integral (PI) controller. The magnitude of the midpoint voltage is first calculated using the d-q transformation of the three phase quantities. The magnitude is then compared with the set reference (1.0) and the error is passed through the PI controller to determine the magnitude of the source voltage, ie,

\[
|V_F| = K_P \left( 1 - \frac{|V_M|}{|V_F|} \right) + K_I \int \left( 1 - \frac{|V_M|}{|V_F|} \right) dt \tag{10.17}
\]

The source voltage is then generated by phase locking it with the midpoint voltage using

\[
V_F = \frac{|V_M|}{|V_F|} V_M \tag{10.18}
\]

The system quantities when the system is perturbed for its nominal operating condition. The proportional gain (K_P) is chosen as 2.0, while the integral gain (K_I) is chosen as 10.

Fig. 10.8 depicts the system quantities when the system is perturbed for its nominal operating condition.

The load angle undergoes sustained oscillation and this oscillation is in phase with the injected reactive power. It can be seen that the load angle undergoes sustained oscillation and this oscillation is in phase with the injected reactive power. This implies that, by tightly regulating the midpoint voltage though a high gain integral controller, the injected reactive power oscillates in sympathy with the load angle. Therefore to damp out the rotor oscillation, a controller must be designed such that the injected reactive power is in phase opposition with the load angle. It is to be noted that the source voltage also modulates in sympathy with the injected reactive power. This however is not evident from Fig. 10.8 (a) as the time axis has been shortened here.
To improve damping, we now introduce a term that is proportional to the deviation of machine speed in the feedback loop such that the control law is given by

$$\|V_p\| = K_p(1 - |V_M|) + K_I \int (1 - V_M) dt + C_P \frac{d \Delta \delta}{dt}$$  \hspace{1cm} (10.19)

the feedback loop such that the control law is given by

The values of proportional gain $K_p$ and integral gain $K_I$ chosen are same as before, while the value of $C_P$ chosen is 50. With the system operating on steady state, delivering power at a load angle of 40° for 50 ms, breaker $B$ (see Fig. 10.7) opens inadvertently. The magnitude of the midpoint voltage is shown in Fig. 10.9 (a). It can be seen that the magnitude settles to the desired value of 1.0 per unit once the initial transients die down. Fig. 10.9 (b) depicts perturbations in load angle and reactive power injected from their prefault steady state values. It can be seen that these two quantities have a phase difference of about 90° and this is essential for damping of power oscillations.

**Fig 10.9 System response with the damping controller**

**Section II: Ideal Series Compensator**

- Impact of Series Compensator on Voltage Profile
- Improving Power-Angle Characteristics
- An Alternate Method of Voltage Injection
- Improving Stability Margin
- Comparisons of the Two Modes of Operation
• **Power Flow Control and Power Swing Damping**

### Ideal Series Compensator

Let us assume that the series compensator is represented by an ideal voltage source. This is shown in Fig. 10.10. Let us further assume that the series compensator is ideal, i.e., it only supplies reactive power and no real power to the system. It is needless to say that this assumption is not valid for practical systems. However, for an introduction, the assumption is more than adequate. It is to be noted that, unlike the shunt compensator, the location of the series compensator is not crucial, and it can be placed anywhere along the transmission line.

![Fig. 10.10 Schematic diagram of an ideal series compensated system.](image)

### Impact of Series Compensator on Voltage Profile

In the equivalent schematic diagram of a series compensated power system is shown in Fig. 10.10, the receiving end current is equal to the sending end current, i.e., $I_S = I_R$. The series voltage $V_Q$ is injected in such a way that the magnitude of the injected voltage is made proportional to that of the line current.

$$V_Q = \lambda I_S e^{j90^\circ}$$  \hspace{1cm} (10.20)

Furthermore, the phase of the voltage is forced to be in quadrature with the line current. We then have

The ratio $\lambda/X$ is called the **compensation level** and is often expressed in percentage. This compensation level is usually measured with respect to the transmission line reactance. For example, we shall refer the compensation level as 50% when $\lambda = X/2$. In the analysis presented below, we assume that the injected voltage lags the line current. The implication of the voltage leading the current will be discussed later.

Applying KVL we get

$$V_S - V_Q - V_R = jX I_S \Rightarrow V_S - V_R = \mp j\lambda I_S + jX I_S$$

$$I_S = \frac{V_S - V_R}{j(X + \lambda)}$$  \hspace{1cm} (10.21)

Assuming $V_S = V < \delta$ and $V_R = V < 0^\circ$, we get the following expression for the line current

When we choose $V_Q = \lambda I_S e^{j90^\circ}$, the line current equation becomes

$$I_S = \frac{V_S - V_R}{j(X - \lambda)}$$
Thus we see that $\lambda$ is subtracted from $X$. This choice of the sign corresponds to the voltage source acting as a pure capacitor. Hence we call this as the **capacitive mode of operation**. In contrast, if we choose $V_Q = \lambda I_S e^{j90^\circ}$, $\lambda$ is added to $X$, and this mode is referred to as the **inductive mode of operation**. Since this voltage injection using (10.20) add $\lambda$ to or subtract $\lambda$ from the line reactance, we shall refer it as voltage injection in **constant reactance mode**. We shall consider the implication of series voltage injection on the transmission line voltage through the following example.

**Example 10.3**

Consider a lossless transmission line that has a 0.5 per unit line reactance ($X$). The sending end and receiving end voltages are given by $1<\delta$ and $1<0^\circ$ per unit respectively where $\delta$ is chosen as $30^\circ$. Let us choose $\lambda = 0.5$ and operation in the capacitive mode. For this line, this implies a 30% level of line impedance compensation. The line current is then given from (10.21) as $I_S = 1.4797 < 15^\circ$ per unit and the injected voltage calculated from (10.20) is $V_Q = 0.2218 < -75^\circ$ per unit. The phasor diagrams of the two end voltages, line current and injected voltage are shown in Fig. 10.11 (a). We shall now consider a few different cases.

Let us assume that the series compensator is placed in the middle of the transmission line. We then define two voltages, one at either side of the series compensator. These are:

Voltage on the left: $V_{QL} = V_S - jX I_S / 2 = 0.9723 < 8.45^\circ$ per unit
Voltage on the right: $V_{QR} = V_R + jX I_S / 2 = 0.9723 < 21.55^\circ$ per unit

The difference of these two voltages is the **injected voltage**. This is shown in Fig. 10.11 (b), where the angle $\theta = 8.45^\circ$. The worst case voltage along the line will then be at the two points on either side of the series compensator where the voltage phasors are aligned with the line current phasor. These two points are equidistant from the series compensator. However, their particular locations will be dependent on the system parameters.

As a second case, let us consider that the series compensator is placed at the end of the transmission line, just before the infinite bus. We then have the following voltage

Voltage on the left of the compensator: $V_{QL} = V_R + V_Q = 1.0789 < -11.46^\circ$ per unit

This is shown in Fig. 10.11 (c). The maximum voltage rise in the line is then to the immediate left of the compensator, i.e., at $V_{QL}$. The maximum voltage drop however still occurs at the point where the voltage phasor is aligned with the line current phasor.
As a third case, let us increase the level of compensation from 30% to 70% (i.e., change $\lambda$ from 0.15 to 0.35). We however, do not want to change the level of steady state power transfer. The relation between power transfer and compensation level will be discussed in the next subsection. It will however suffice to say that this is accomplished by lowering the value of the angle $\delta$ of the sending end voltage to 12.37°. Let us further assume that the series compensator is placed in the middle of the transmission line. We then have $V_{Ql} = 1.0255 < -8.01^\circ$ per unit and $V_{QR} = 1.0255 < 20.38^\circ$ per unit. This is shown in Fig. 10.11 (d). It is obvious that the voltage along the line rises to a maximum level at either side of the series compensator.

**Improving Power-Angle Characteristics**

\[
P_s + jQ_s = V_s I_s^* = V \angle \delta = V \left[ \frac{\sqrt{V^2 - \delta - V}}{-j(X + j\lambda)} \right] = \frac{V^2 - \delta}{-j(X + j\lambda)}
\]

\[= \frac{V^2 \sin \delta}{X + j\lambda} + j \frac{V^2 (1 - \cos \delta)}{X + j\lambda}
\]

Noting that the sending end apparent power is $V_s I_s^*$, we can write

\[
P_r + jQ_r = V_r I_r^* = V \left[ \frac{\sqrt{V^2 - \delta - V}}{-j(X + j\lambda)} \right]
\]

\[= \frac{V^2 \sin \delta}{X + j\lambda} + j \frac{V^2 (\cos \delta - 1)}{X + j\lambda}
\]

Similarly the receiving end apparent power is given by

Hence the real power transmitted over the line is given by
The power-angle characteristics of a series compensated power system are given in Fig. 10.12. In this figure the base power is chosen as \( V^2 / X \). Three curves are shown, of which the curve \( P_0 \) is the **power-angle curve** when the line is not compensated. Curves which have maximum powers greater than the base power pertain to capacitive mode of operation. On the other hand, all curves the inductive mode of operation will have maximum values less than 1. For example, in Fig. 10.12, the curve \( P_1 \) is for capacitive mode and the curve \( P_2 \) is for inductive mode of operation.

\[
P_s = P_R = P_e = \frac{V^2}{X + \lambda} \sin \delta \quad (10.24)
\]

Let us now have a look at the reactive power. For simplicity let us restrict our attention to capacitive mode of operation only as this represents the normal mode of operation in which the power transfer over the line is enhanced. From (10.20) and (10.21) we get the reactive power supplied by the compensator as

\[
Q_D = V_e (\delta - \delta_p) - j \frac{V_e^2}{(X - \lambda)} \left( \frac{V_e - \delta - V}{j(X - \lambda)} \right)
\]

In Fig. 10.13, the reactive power injected by the series compensator is plotted against the maximum power transfer as the compensation level changes from 10% to 60%. As the compensation level increases, the maximum power transfer also increases. However, at the same time, the reactive injection requirement from the series compensator also increases. It is interesting to note that at 50% compensation level, the reactive power injection requirement from a series compensator is same that from shunt compensator that is regulating the midpoint voltage to 1.0 per unit.

\[
Q_D = -j \frac{2V^2}{(X - \lambda)^2} (1 - \cos \delta) \quad (10.25)
\]

Solving the above equation we get
An Alternate Method of Voltage Injection

So far we have assumed that the series compensator injects a voltage that is in quadrature with the line current and its magnitude is proportional to the magnitude of the line current. A set of very interesting equations can be obtained if the last assumption about the magnitude is relaxed. The injected voltage is

\[ V_2 = \frac{I_S}{|I_S|} e^{\pi/2} \]  

then given by

\[ \frac{V_2}{I_S} = \frac{\lambda}{|I_S|} e^{\pi/2} = \frac{\lambda}{|I_S|} e^{\pi/2} = \frac{\lambda}{X} \]

We can then write the above equation as

i.e., the voltage source in quadrature with the current is represented as a pure reactance that is either inductive or capacitive. Since in this form we injected a constant voltage in quadrature with the line current, we shall refer this as constant voltage injection mode. The total equivalent inductance of the line is then

\[ X_{eq} = X + X_\theta \]

line is then

\[ F = \frac{\lambda}{X_{eq}} \sin \delta = \frac{\lambda}{X + X_\theta} \sin \delta \]

Defining \( V_S = V < \delta \) and \( V_R < 0^\circ \), we can then write the power transfer equation as

Since \( |V_0| / |I_S| = X_\alpha \), we can modify the above equation as...
Consider the phasor diagram of Fig. 10.14 (a), which is for capacitive operation of the series compensator. From this diagram we get

\[
|I_s| X = |V_2| + 2V \sin (\delta/2)
\]

Similarly from the inductive operation phasor diagram shown in Fig. 10.14 (b), we get

\[
|I_s| X = -|V_2| + 2V \sin (\delta/2)
\]

Substituting the above two equations in (10.28) and rearranging we get

\[
F_s = \frac{V^2}{X(1 + \frac{|I_s| X}{|V_2|})} \sin \delta
\]

where the positive sign is for capacitive operation.

\[
(10.28)
\]

\[
F_s = \frac{V^2}{X} \sin \delta \frac{|I_s| X}{|V_2| + 2V \sin (\delta/2)} = \frac{V^2}{X} \sin \delta \frac{|V_2| + 2V \sin (\delta/2)}{2V \sin (\delta/2)}
\]

\[
= \frac{V^2}{X} \sin \delta \frac{|V_2|}{2V \cos (\delta/2)} = \frac{V^2}{X} \sin \delta \frac{|V_2| \cos (\delta/2)}{2V \sin (\delta/2)}
\]

\[
(10.29)
\]

where the positive sign is for capacitive operation.

Fig. 10.14 Phasor diagram of series compensated system: (a) capacitive operation and (b) inductive operation.

...Contd...An Alternate Method of Voltage Injection

The power-angle characteristics of this particular series connection are given in Fig. 10.15. In this figure the base power is chosen as \( V^2/X \). Three curves are shown, of which the curve \( P_0 \) is the power-angle curve when the line is not compensated. Curves which have maximum powers greater than the base power pertain to capacitive mode of operation. On the other hand, all curves the inductive mode of operation will have maximum values less than 1. For example, in Fig. 10.15, the curve \( P_1 \) is for capacitive mode and the curve \( P_2 \) is for inductive mode of operation.
Fig. 10.15 Power-angle characteristics for constant voltage mode.

\[
Q_{\theta} = \frac{V_{\theta}}{X} \left| J_{S} \right|
\]  
(10.30)

The reactive power supplied by the compensator in this case will be

**Improving Stability Margin**

From the power-angle curves of Figs. 10.13 and 10.15 it can be seen that the same amount of power can be transmitted over a capacitive compensated line at a lower load angle than an uncompensated system. Furthermore, an increase in the height in the power-angle curve means that a larger amount of decelerating area is available for a compensated system. Thus improvement in stability margin for a capacitive series compensated system over an uncompensated system is obvious.

**Comparisons of the Two Modes of Operation**

As a comparison between the two different modes of voltage injection, let us first consider the constant reactance mode of voltage injection with a compensation level of 50%. Choosing \( \frac{V^2}{X} \) as the base power, the power-angle characteristic reaches a maximum of 2.0 per unit at a load angle \( \frac{\pi}{2} \). Now \( |V_{Q}| \) in constant voltage mode is chosen such that the real power is 2.0 per unit at a load angle of \( \frac{\pi}{2} \). This is accomplished using (10.29) where we get

\[
\left| V_{\theta} \right| = \frac{2 \sin 90^\circ}{\cos 45^\circ} = 1.4142 \text{ per unit}
\]

The power-angle characteristics of the two different modes are now drawn in Fig. 10.16 (a). It can be seen that the two curves match at \( \frac{\pi}{2} \). However, the maximum power for constant voltage case is about 2.1 per unit and occurs at an angle of 67°.

Fig. 10.16 (b) depicts the line current for the two cases. It can be seen that the increase in line current in either case is monotonic. This is not surprising for the case of constant reactance mode since as the load angle increases, both real power and line currents increase. Now consider the case of constant voltage control. When the load angle moves backwards from \( \frac{\pi}{2} \) to 67°, the power moves from 2.0 per unit to its peak value of 2.1 per unit. The line current during this stage decreases from about 2.83 to 2.50 per unit. Thus, even though the power through the line increases, the line current decreases.
Power Flow Control and Power Swing Damping

One of the major advantages of series compensation is that through its use real power flow over transmission corridors can be effectively controlled. Consider, for example, the SMIB system shown in Fig. 10.17 in which the generator and infinite bus are connected through a double circuit transmission line, labeled line-1 and line-2. Of the two transmission lines, line-2 is compensated by a series compensator. The compensator then can be utilized to regulate power flow over the entire system.

For example, let us consider that the system is operating in the steady state delivering a power of \( P_{m0} \) at a load angle of \( \delta_0 \). Lines 1 and 2 are then sending power \( P_{e1} \) and \( P_{e2} \) respectively, such that \( P_{m0} = P_{e1} + P_{e2} \). The mechanical power input suddenly goes up to \( P_{m1} \). There are two ways of controlling the power in this situation:

- **Regulating Control**: Channeling the increase in power through line-1. In this case the series compensator maintains the power flow over line-2 at \( P_{e2} \). The load angle in this case goes up in sympathy with the increase in \( P_{e1} \).
- **Tracking Control**: Channeling the increase in power through line-2. In this case the series compensator helps in maintaining the power flow over line-1 at \( P_{e1} \) while holding the load angle to \( \delta_0 \).

Let us illustrate these two aspects with the help of a numerical example.

**Example 10.4**

Let us consider the system of Fig. 7.8 where the system parameters are given by

\[
\text{System Frequency} = 50 \text{ Hz, } |V_S| = |V_R| = 1.0 \text{ per unit, } X = 0.5 \text{ per unit and } d_0 = 30^\circ /\
\]

It is assumed that the series compensator operates in constant reactance mode with a compensation level of 30%. We then have

\[
P_{e1} = 1.0 \text{ per unit, } P_{e2} = 1.43 \text{ per unit, } P_m = 2.43 \text{ per unit}\
\]
Chapter 10: Compensation of Power Transmission Systems

The objective of the control scheme here is to maintain the power through line-2 to a pre-specified value, $P_{ref}$. To accomplish this a proportional-plus-integral (PI) controller is placed in the feedback loop of $P_{e2}$. In addition, to improve damping a term that is proportional to the deviation of machine speed is introduced in the feedback loop. The control law is then given by

$$C_L = K_P \left( P_{e2} - P_{ref} \right) + K_I \int \left( P_{e2} - P_{ref} \right) dt + C_P \frac{d\delta}{dt}$$  \hspace{1cm} (10.31)

where $C_L = \lambda / X$ is the compensation level. For the simulation studies performed, the following controller parameters are chosen

$K_P = 0.1$, $K_I = 1.0$ and $C_P = 75$

**Regulating Control:** With the system operating in the nominal steady state, the mechanical power input is suddenly raised by 10%. It is expected that the series compensator will hold the power through line-2 constant at line-2 at $P_{e2}$ such that entire power increase is channeled through line-1. We then expect that the power $P_{e1}$ will increase to 1.243 per unit and the load angle to go up to 0.67 rad. The compensation level will then change to 13%.

The time responses for various quantities for this test are given in Fig. 10.18. In Fig. 10.18 (a), the power through the two line is plotted. It can be seen that while the power through line-2 comes back to its nominal value following the transient, the power through the other line is raised to expected level. Similarly, the load angle and the compensation level reach their expected values, as shown in Figs. 10.18 (b) and (c), respectively. Finally, Fig. 10.18 (d) depicts the last two cycles of phase-a of the line current and injected voltage. It can be clearly seen that these two quantities are in quadrature, with the line current leading the injected voltage.

**Tracking Control:** With the system operating in the nominal steady state, the mechanical power input is suddenly raised by 25%. It is expected that the series compensator will make the entire power increase to flow through line-2 such that both $P_{e1}$ and load angle are maintained constant at their nominal values. The power $P_{e2}$ through line-2 will then increase to about 2.04 per unit and the compensation level will change to 51%.

The time responses for various quantities for this test are given in Fig. 10.19. It can be seen that while the power through line-1 comes back to its nominal value following the transient, the power through the other line is raised to level expected. Similarly, the load angle comes back to its nominal value and the compensation level is raised 51%, as shown in Figs. 10.19 (b) and (c), respectively. Finally, Fig. 7.19 (d) depicts the last two cycles of phase-a of the line current and injected voltage.

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**Fig. 10.18 System response with regulating power flow controller.**

**Fig. 10.19 System response with tracking power flow controller.**
Fig. 10.19 System response with regulating power flow controller.