Sheet 4: Chapter 4

4–4C  Show that 1 kPa \cdot m^3 = 1 kJ.

4–4C  \[ 1 \text{kPa} \cdot m^3 = 1 \text{k(N/m}^2\text{)} \cdot m^3 = 1 \text{kN} \cdot \text{m} = 1 \text{kJ} \]

4–5  A piston–cylinder device initially contains 0.07 m$^3$ of nitrogen gas at 130 kPa and 120°C. The nitrogen is now expanded polytropically to a state of 100 kPa and 100°C. Determine the boundary work done during this process.
4-5 A piston-cylinder device contains nitrogen gas at a specified state. The boundary work is to be determined for the polytropic expansion of nitrogen.

Properties The gas constant for nitrogen is 0.2968 kJ/kg.K (Table A-2).

Analysis The mass and volume of nitrogen at the initial state are

\[
m = \frac{PV}{RT} = \frac{(130 \text{ kPa})(0.07 \text{ m}^3)}{(0.2968 \text{ kJ/kg.K})(120 + 273 \text{ K})} = 0.07802 \text{ kg}
\]

\[
V_2 = \frac{mRT_2}{P_2} = \frac{(0.07802 \text{ kg})(0.2968 \text{ kPa.m}^3/\text{kg.K})(100 + 273 \text{ K})}{100 \text{ kPa}} = 0.08637 \text{ m}^3
\]

The polytropic index is determined from

\[
P_1 V_1^n = P_2 V_2^n \quad \rightarrow (130 \text{ kPa})(0.07 \text{ m}^3)^n = (100 \text{ kPa})(0.08637 \text{ m}^3)^n \quad \rightarrow n = 1.249
\]

The boundary work is determined from

\[
W_b = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{(100 \text{ kPa})(0.08637 \text{ m}^3) - (130 \text{ kPa})(0.07 \text{ m}^3)}{1 - 1.249} = 1.86 \text{ kJ}
\]

4-6 A piston–cylinder device with a set of stops initially contains 0.3 kg of steam at 1.0 MPa and 400°C. The location of the stops corresponds to 60 percent of the initial volume. Now the steam is cooled. Determine the compression work if the final state is (a) 1.0 MPa and 250°C and (b) 500 kPa. (c) Also determine the temperature at the final state in part (b).
A piston-cylinder device with a set of stops contains steam at a specified state. Now, the steam is cooled. The compression work for two cases and the final temperature are to be determined.

**Analysis**

(a) The specific volumes for the initial and final states are (Table A-6)

\[
\begin{align*}
R_1 &= 1 \text{ MPa} \\
T_1 &= 400^\circ\text{C} \\
\nu_1 &= 0.30661 \text{ m}^3/\text{kg} \\
R_2 &= 1 \text{ MPa} \\
T_2 &= 250^\circ\text{C} \\
\nu_2 &= 0.23275 \text{ m}^3/\text{kg}
\end{align*}
\]

Noting that pressure is constant during the process, the boundary work is determined from

\[
W_b = mP(\nu_1 - \nu_2) = (0.3 \text{ kg})(1000 \text{ kPa})(0.30661 - 0.23275)\text{m}^3/\text{kg} = 22.16 \text{ kJ}
\]

(b) The volume of the cylinder at the final state is 60% of initial volume. Then, the boundary work becomes

\[
W_b = mP(\nu_1 - 0.60\nu_1) = (0.3 \text{ kg})(1000 \text{ kPa})(0.30661 - 0.60 \times 0.30661)\text{m}^3/\text{kg} = 36.79 \text{ kJ}
\]

The temperature at the final state is

\[
\begin{align*}
\frac{P_2}{0.5 \text{ MPa}} &= \frac{\nu_2}{(0.60 \times 0.30661) \text{ m}^3/\text{kg}} \\
\nu_2 &= (0.60 \times 0.30661) \text{ m}^3/\text{kg}
\end{align*}
\]

\[T_2 = 151.8^\circ\text{C} \quad \text{(Table A-5)}\]

---

A piston–cylinder device initially contains 0.07 m³ of nitrogen gas at 130 kPa and 120°C. The nitrogen is now expanded to a pressure of 100 kPa polytropically with a polytropic exponent whose value is equal to the specific heat ratio (called isentropic expansion). Determine the final temperature and the boundary work done during this process.

4-7 A piston-cylinder device contains nitrogen gas at a specified state. The final temperature and the boundary work are to be determined for the isentropic expansion of nitrogen.

**Properties**

The properties of nitrogen are \( R = 0.2968 \text{ kJ/kg.K} \), \( k = 1.4 \) (Table A-2a).

**Analysis**

The mass and the final volume of nitrogen are

\[
m = \frac{P_1\nu_1}{RT_1} = \frac{(130 \text{ kPa})(0.07 \text{ m}^3)}{(0.2968 \text{ kJ/kg.K})(120 + 273 \text{ K})} = 0.07802 \text{ kg}
\]

\[
P_2\nu_2^k = P_1\nu_1^k 
\Rightarrow (130 \text{ kPa})(0.07 \text{ m}^3)^{1.4} = (100 \text{ kPa})\nu_2^{1.4} 
\Rightarrow \nu_2 = 0.08443 \text{ m}^3
\]

The final temperature and the boundary work are determined as

\[
T_2 = \frac{P_2\nu_2}{mR} = \frac{(100 \text{ kPa})(0.08443 \text{ m}^3)}{(0.07802 \text{ kg})(0.2968 \text{ kPa.m}^3/\text{kg.K})} = 364.6 \text{ K}
\]

\[
W_b = \frac{P_2\nu_2 - P_1\nu_1}{1-k} = \frac{(100 \text{ kPa})(0.08443 \text{ m}^3) - (130 \text{ kPa})(0.07 \text{ m}^3)}{1-1.4} = 1.64 \text{ kJ}
\]
A mass of 5 kg of saturated water vapor at 300 kPa is heated at constant pressure until the temperature reaches 200°C. Calculate the work done by the steam during this process. \textit{Answer: 165.9 kJ}

\textbf{4-8} Saturated water vapor in a cylinder is heated at constant pressure until its temperature rises to a specified value. The boundary work done during this process is to be determined.

\textit{Assumption} The process is quasi-equilibrium.

\textit{Properties} Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are (Tables A-4 through A-6)

\[
\begin{align*}
P_1 &= 300 \text{ kPa} \\
\text{Sat. vapor} & \quad \nu_1 = \nu_{g\text{ @ 300 kPa}} = 0.60582 \text{ m}^3/\text{kg} \\
P_2 &= 300 \text{ kPa} \\
T_2 &= 200^\circ\text{C} & \nu_2 = 0.71643 \text{ m}^3/\text{kg}
\end{align*}
\]

\textit{Analysis} The boundary work is determined from its definition to be

\[
W_{b,\text{out}} = \int_1^2 P\,dV = P(\nu_2 - \nu_1) = mP(\nu_2 - \nu_1)
\]

\[
= (5 \text{ kg})(300 \text{ kPa})(0.71643 - 0.60582) \text{ m}^3/\text{kg} \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right)
\]

\[
= 165.9 \text{ kJ}
\]

\textit{Discussion} The positive sign indicates that work is done by the system (work output).
A frictionless piston-cylinder device initially contains 200 L of saturated liquid refrigerant-134a. The piston is free to move, and its mass is such that it maintains a pressure of 900 kPa on the refrigerant. The refrigerant is now heated until its temperature rises to 70°C. Calculate the work done during this process. \textit{Answer:} 5571 \text{ kJ}

\textbf{FIGURE P4–9}

Refrigerant-134a in a cylinder is heated at constant pressure until its temperature rises to a specified value. The boundary work done during this process is to be determined.

\textit{Assumption} The process is quasi-equilibrium.

\textit{Properties} Noting that the pressure remains constant during this process, the specific volumes at the initial and the final states are (Tables A-11 through A-13)

\begin{align*}
P_1 &= 900 \text{ kPa} \\
\text{Sat. liquid} & \\
\nu_1 &= \nu_f @ 900 \text{ kPa} = 0.0008580 \text{ m}^3/\text{kg} \\
P_2 &= 900 \text{ kPa} \\
T_2 &= 70^\circ \text{C} \\
\nu_2 &= 0.027413 \text{ m}^3/\text{kg}
\end{align*}

\textit{Analysis} The boundary work is determined from its definition to be

\[
m = \frac{\nu_1}{\nu_1} = \frac{0.2 \text{ m}^3}{0.0008580 \text{ m}^3/\text{kg}} = 233.1 \text{ kg}
\]

and

\[
\begin{align*}
W_{b,\text{out}} &= \int_1^2 PdV = P(\nu_2 - \nu_1) = mP(\nu_2 - \nu_1) \\
&= (233.1 \text{ kg})(900 \text{ kPa})(0.027413 - 0.0008580)\text{ m}^3/\text{kg}\left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right) \\
&= 5571 \text{ kJ}
\end{align*}
\]

\textit{Discussion} The positive sign indicates that work is done by the system (work output).
4–12  A mass of 2.4 kg of air at 150 kPa and 12°C is contained in a gas-tight, frictionless piston–cylinder device. The air is now compressed to a final pressure of 600 kPa. During the process, heat is transferred from the air such that the temperature inside the cylinder remains constant. Calculate the work input during this process.  \textit{Answer: 272 \text{kJ}}

\textbf{Solution}  
Air in a cylinder is compressed at constant temperature until its pressure rises to a specified value. The boundary work done during this process is to be determined.

\textbf{Assumptions}  
1. The process is quasi-equilibrium.
2. Air is an ideal gas.

\textbf{Properties}  
The gas constant of air is \( R = 0.287 \text{kJ/kg.K} \) (Table A-1).

\textbf{Analysis}  
The boundary work is determined from its definition to be

\[
W_{b,\text{out}} = \int_{1}^{2} P \, dV = P_{1}V_{1}\ln\frac{V_{2}}{V_{1}} = mRT\ln\frac{P_{1}}{P_{2}}
\]

\[
= (2.4 \text{ kg})(0.287 \text{ kJ/kg.K})(285 \text{ K})\ln\frac{150 \text{ kPa}}{600 \text{ kPa}}
\]

\[
= -272 \text{ kJ}
\]

\textbf{Discussion}  
The negative sign indicates that work is done on the system (work input).

4–13  Nitrogen at an initial state of 300 K, 150 kPa, and 0.2 m\(^3\) is compressed slowly in an isothermal process to a final pressure of 800 kPa. Determine the work done during this process.
Solution  Nitrogen gas in a cylinder is compressed at constant temperature until its pressure rises to a specified value. The boundary work done during this process is to be determined.

Assumptions 1 The process is quasi-equilibrium. 2 Nitrogen is an ideal gas.

Analysis  The boundary work is determined from its definition to be

\[ W_{b, \text{out}} = \int_1^2 P \, dV = \frac{P_1 V_1}{V_2} \ln \frac{V_2}{V_1} = R \frac{V_1}{V_2} \ln \frac{P_1}{P_2} \]

\[ = (150 \, \text{kPa})(0.2 \, \text{m}^3) \left( \ln \frac{150 \, \text{kPa}}{800 \, \text{kPa}} \right) \left( \frac{1 \, \text{kJ}}{1 \, \text{kPa} \cdot \text{m}^3} \right) \]

\[ = -50.2 \, \text{kJ} \]

Discussion  The negative sign indicates that work is done on the system (work input).

4–14  A gas is compressed from an initial volume of 0.42 m³ to a final volume of 0.12 m³. During the quasi-equilibrium process, the pressure changes with volume according to the relation \( P = aV + b \), where \( a = -1200 \, \text{kPa/m}^3 \) and \( b = 600 \, \text{kPa} \). Calculate the work done during this process \((a)\) by plotting the process on a \( P-V \) diagram and finding the area under the process curve and \((b)\) by performing the necessary integrations.
Solution A gas in a cylinder is compressed to a specified volume in a process during which the pressure changes linearly with volume. The boundary work done during this process is to be determined by plotting the process on a $P-V$ diagram and also by integration.

**Assumption** The process is quasi-equilibrium.

**Analysis** (a) The pressure of the gas changes linearly with volume, and thus the process curve on a $P-V$ diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

\[
P_1 = aV_1 + b = (-1200 \text{ kPa/m}^3)(0.42 \text{ m}^3) + (600 \text{ kPa}) = 96 \text{ kPa}
\]

\[
P_2 = aV_2 + b = (-1200 \text{ kPa/m}^3)(0.12 \text{ m}^3) + (600 \text{ kPa}) = 456 \text{ kPa}
\]

and

\[
W_{\text{b, out}} = \text{Area} = \frac{P_1 + P_2}{2} (V_2 - V_1)
\]

\[
= \frac{(96 + 456) \text{ kPa}}{2} \left(0.12 - 0.42\right) \text{ m}^3 \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right)
\]

\[
= -82.8 \text{ kJ}
\]

(b) The boundary work can also be determined by integration to be

\[
W_{\text{b, out}} = \int_1^2 PdV = \int_1^2 (aV + b)dV = a \frac{V^2}{2} + b(V_2 - V_1)
\]

\[
= (-1200 \text{ kPa/m}^3) \left(\frac{0.12^2 - 0.42^2}{2}\right) \text{ m}^6 + (600 \text{ kPa})(0.12 - 0.42) \text{ m}^3
\]

\[
= -82.8 \text{ kJ}
\]

**Discussion** The negative sign indicates that work is done on the system (work input).
4–18 A frictionless piston–cylinder device contains 2 kg of nitrogen at 100 kPa and 300 K. Nitrogen is now compressed slowly according to the relation $PV^{1.4} = \text{const.}$ until it reaches a final temperature of 360 K. Calculate the work input during this process.  \textit{Answer: 89 kJ}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure_p4-18}
\caption{Figure P4–18}
\end{figure}

\textbf{Solution}  Nitrogen gas in a cylinder is compressed polytropically until the temperature rises to a specified value. The boundary work done during this process is to be determined.

\textit{Assumptions} 1 The process is quasi-equilibrium. 2 Nitrogen is an ideal gas.

\textit{Properties}  The gas constant for nitrogen is $R = 0.2968 \text{ kJ/kg.K}$ (Table A-2a).

\textit{Analysis}  The boundary work for this polytropic process can be determined from

\[
W_{b,\text{out}} = \int_{1}^{2} P \, dV = \frac{P_{2}U_{2} - P_{1}U_{1}}{1-n} = \frac{mR(T_{2} - T_{1})}{1-n} \\
= \frac{(2 \text{ kg})(0.2968 \text{ kJ/kg.K})(360 - 300)\text{K}}{1-1.4} \\
= -89.0 \text{ kJ}
\]

\textit{Discussion}  The negative sign indicates that work is done on the system (work input).

4–21 Carbon dioxide contained in a piston–cylinder device is compressed from 0.3 to 0.1 m³. During the process, the pressure and volume are related by $P = aV^{-2}$, where $a = 8 \text{ kPa} \cdot \text{m}^6$. Calculate the work done on the carbon dioxide during this process.  \textit{Answer: 53.3 kJ}
Solution CO₂ gas in a cylinder is compressed until the volume drops to a specified value. The pressure changes during the process with volume as $P = aV^{-2}$. The boundary work done during this process is to be determined.

**Assumption** The process is quasi-equilibrium.

**Analysis** The boundary work done during this process is determined from

$$W_{b,\text{out}} = \int_{V_1}^{V_2} P \, dV = \int_{V_1}^{V_2} \left( \frac{a}{V^2} \right) dV = -a \left( \frac{1}{V_2} - \frac{1}{V_1} \right)$$

$$= -(8 \text{ kPa} \cdot \text{m}^6) \left( \frac{1}{0.1 \text{ m}^3} - \frac{1}{0.3 \text{ m}^3} \right) \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right)$$

$$= -53.3 \text{ kJ}$$

**Discussion** The negative sign indicates that work is done on the system (work input).
A 0.5-m³ rigid tank contains refrigerant-134a initially at 160 kPa and 40 percent quality. Heat is now transferred to the refrigerant until the pressure reaches 700 kPa. Determine (a) the mass of the refrigerant in the tank and (b) the amount of heat transferred. Also, show the process on a $P$-$\nu$ diagram with respect to saturation lines.

**Solution** A rigid tank is initially filled with superheated R-134a. Heat is transferred to the tank until the pressure inside rises to a specified value. The mass of the refrigerant and the amount of heat transfer are to be determined, and the process is to be shown on a $P$-$\nu$ diagram.

**Assumptions** 1. The tank is stationary and thus the kinetic and potential energy changes are zero. 2. There are no work interactions.

**Analysis** (a) We take the tank as the system. This is a closed system since no mass enters or leaves. Noting that the volume of the system is constant and thus there is no boundary work, the energy balance for this stationary closed system can be expressed as

$$Q_{in} = \Delta U = m(u_2 - u_1) \quad \text{(since W = KE = PE = 0)}$$

Using data from the refrigerant tables (Tables A-11 through A-13), the properties of R-134a are determined to be

- $P_1 = 160 \text{ kPa}$
- $\nu_f = 0.0007437, \quad \nu_g = 0.12348 \text{ m}^3/\text{kg}$
- $x_1 = 0.4 \quad u_f = 31.09, \quad u_g = 190.27 \text{kJ/kg}$

$$\nu_1 = \nu_f + \nu_g x_1 = 0.0007437 + 0.4(0.12348 - 0.0007437) = 0.04984 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + \nu_g x_1 u_g = 31.09 + 0.4(190.27) = 107.19 \text{ kJ/kg}$$

- $P_2 = 700 \text{ kPa}$
- $\nu_2 = \nu_f$ (Superheated vapor)

Then the mass of the refrigerant is determined to be

$$m = \frac{\nu_f}{\nu_1} = \frac{0.5 \text{ m}^3}{0.04984 \text{ m}^3/\text{kg}} = 10.03 \text{ kg}$$

(b) Then the heat transfer to the tank becomes

$$Q_{in} = m(u_2 - u_1) = (10.03 \text{ kg})(376.99 - 107.19) \text{ kJ/kg} = 2707 \text{ kJ}$$
A 3-m³ rigid tank contains hydrogen at 250 kPa and 550 K. The gas is now cooled until its temperature drops to 350 K. Determine (a) the final pressure in the tank and (b) the amount of heat transfer.

Solution

The hydrogen gas in a rigid tank is cooled until its temperature drops to 300 K. The final pressure in the tank and the amount of heat transfer are to be determined.

Assumptions
1. Hydrogen is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -240°C and 1.30 MPa.
2. The tank is stationary, and thus the kinetic and potential energy changes are negligible. \( \Delta \text{ke} \approx \Delta \text{pe} \approx 0 \).

Properties
The gas constant of hydrogen is \( R = 4.124 \text{ kPa.m}^3/\text{kg.K} \) (Table A-1). The constant volume specific heat of hydrogen at the average temperature of 450 K is \( C_v = 10.377 \text{ kJ/kg.K} \) (Table A-2).

Analysis
(a) The final pressure of hydrogen can be determined from the ideal gas relation,

\[
P_1 V_1 / T_1 = P_2 V_2 / T_2 = \frac{P_2}{T_2} = \frac{T_2}{T_1} \cdot \frac{P_1}{T_1} = \frac{350 \text{ K}}{550 \text{ K}} \cdot (250 \text{ kPa}) = 159.1 \text{ kPa}
\]

(b) We take the hydrogen in the tank as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

\[
E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}} = \Delta U
\]

\[
\dot{Q}_{\text{out}} = \Delta U = m \left( u_2 - u_1 \right) = m C_v (T_1 - T_2)
\]

where

\[
m = \frac{P_1 V_1}{RT_1} = \frac{(250 \text{ kPa}) (3.0 \text{ m}^3)}{(4.124 \text{ kPa.m}^3/\text{kg.K})(550 \text{ K})} = 0.3307 \text{ kg}
\]

Substituting into the energy balance,

\[
\dot{Q}_{\text{out}} = (0.33307 \text{ kg})(10.377 \text{ kJ/kg.K})(550 - 350) \text{ K} = 686.2 \text{ kJ}
\]
An insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 4 kg of an ideal gas at 800 kPa and 50°C, and the other part is evacuated. The partition is now removed, and the gas expands into the entire tank. Determine the final temperature and pressure in the tank.

**Solution**

One part of an insulated rigid tank contains an ideal gas while the other side is evacuated. The final temperature and pressure in the tank are to be determined when the partition is removed.

**Assumptions**
1. The kinetic and potential energy changes are negligible, \( \Delta ke = \Delta pe = 0 \).
2. The tank is insulated and thus heat transfer is negligible.

**Analysis**

We take the entire tank as the system. This is a closed system since no mass crosses the boundaries of the system. The energy balance for this system can be expressed as

\[
\frac{E_{in} - E_{out}}{\text{Net energy transfer}} = \frac{\Delta E_{\text{system}}}{\text{Change in internal, kinetic, potential, etc. energies}}
\]

\[
0 = \Delta U = m(u_2 - u_1)
\]

Therefore,

\[
T_2 = T_1 = 50^\circ C
\]

Since \( u = u(T) \) for an ideal gas. Then,

\[
\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow P_2 = \frac{V_1}{V_2} P_1 = \frac{1}{2} (800 \text{ kPa}) = 400 \text{ kPa}
\]