Chapter #7
November XX, 2011
Announcements:

Dr. Walid’s e-mail and Office Hours
walid_aniss@yahoo.com

Office hours for Thermo 01 will be every Sunday and Tuesday from 9:00 – 12:00 am in Dr. Walid’s office (Room 5-213).

Text book:

Thermodynamics An Engineering Approach
Yunus A. Cengel & Michael A. Boles
Chapter 7

ENTROPY
Objectives of CH7: To

• Apply 2nd law of thermodynamics to processes.

• Define a new property called entropy to quantify 2nd law effects.

• Establish the increase of entropy principle.

• Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
* Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.

* Derive the *reversible steady-flow work relations*.

* Develop the *isentropic efficiencies* for various steady-flow devices.

* Introduce and apply the *entropy balance* to various systems.
consider a system connected to a thermal energy reservoir at a constant thermodynamic (i.e., absolute) temperature of $T_R$ through a reversible cyclic device (Fig. 7–1).
Fig. 7–1: The system considered in the development of the Clausius inequality.
The cyclic device receives heat $\delta Q_R$ from the reservoir and supplies heat $\delta Q$ to the system whose temperature at that part of the boundary is $T$ (a variable) while producing work $\delta W_{rev}$. 
A quantity whose cyclic integral is zero depends on the *state only* and not the process path, and thus it is a property. Therefore, the quantity \((\delta Q/T)_{\text{int, rev}}\) must represent a *property in the differential form*. 
Clausius realized in 1865 that he had discovered a new thermodynamic property, and he chose to name this property entropy. It is designated $S$ and is defined as

$$dS = \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \quad \text{(kJ/K)} \quad (7-4)$$
Entropy change of a system during a process can be determined by integrating Eq. 7–4 between initial and final states:

\[ \Delta S = S_2 - S_1 = \int_{1}^{2} \left( \frac{\delta Q}{T} \right)_{\text{int \ rev}} \] (kJ/K)  

(7-5)
A Special Case: Internally Reversible \textit{Isothermal} Heat Transfer Processes

\[ \Delta S = \int_1^2 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left( \frac{\delta Q}{T_0} \right)_{\text{int rev}} \]

\[ = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}} \]

which reduces to

\[ \Delta S = \frac{Q}{T_0} \quad (\text{kJ/K}) \quad (7-6) \]
EXAMPLE 7–1 Entropy Change during an Isothermal Process

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.
Solution:

\[ T = 300 \, \text{K} = \text{const.} \]

\[ \Delta S_{sys} = \frac{Q}{T} = 2.5 \, \frac{\text{kJ}}{\text{K}} \]

\[ Q = 750 \, \text{kJ} \]
The system undergoes an internally reversible, isothermal process, and thus its entropy change can be determined directly from Eq. 7–6 to be

\[ \Delta S_{\text{sys, isothermal}} = \frac{Q}{T_{\text{sys}}} \]

\[ = \frac{750 \text{ kJ}}{300 \text{ K}} = 2.5 \text{ kJ/K} \]
For an isolated system (or simply an adiabatic closed system), the heat transfer is zero, and the entropy change of an isolated system is:

$$\Delta S_{\text{isolated}} \geq 0$$  \hspace{1cm} (7-10)
The **inequality sign** in the preceding relation is a constant **reminder** that the **entropy change** of a closed system during an irreversible process is always greater than the entropy transfer. That is, **some entropy is generated** or **created during an irreversible process**, and this **generation** is **due entirely** to the presence of **irreversibilities**.
\[ S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0 \]
Entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.
A system and its surroundings form an isolated system.
This relation serves as a **criterion** in determining whether a process is **reversible, irreversible, or impossible**.

\[
S_{\text{gen}} \begin{cases} 
> 0 & \text{Irreversible process} \\
= 0 & \text{Reversible process} \\
< 0 & \text{Impossible process}
\end{cases}
\]
EXAMPLE 7–2 Entropy Generation during Heat Transfer Processes

A heat source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K and (b) 750 K. Determine which heat transfer process is more irreversible.
Solution:

(a) Source 800 K to Sink A 500 K

(b) Source 800 K to Sink B 750 K

2000 kJ
(a) For the heat transfer process to a sink at 500 K:

\[
\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000 \text{ kJ}}{800 \text{ K}} = -2.5 \text{ kJ/kg}
\]

\[
\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000 \text{ kJ}}{500 \text{ K}} = 4.0 \text{ kJ/kg}
\]

Hence,

\[
S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0) \text{ kJ/kg} = 1.5 \text{ kJ/kg}
\]
(b) For the heat transfer process to a sink at 750 K:

\[
\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000 \text{ kJ}}{800 \text{ K}} = -2.5 \text{ kJ/kg}
\]

\[
\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000 \text{ kJ}}{750 \text{ K}} = 2.67 \text{ kJ/kg}
\]

Hence,

\[
S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}}
\]

\[
= (-2.5 + 2.67) \text{ kJ/kg} = 0.12 \text{ kJ/kg}
\]
It is clear that,

$$S_{gen, A} > S_{gen, B}$$

Hence, process A is more irreversible
7–3 ■ ENTROPY CHANGE OF PURE SUBSTANCES

Since entropy is a property, the change in entropy of a system in going from one state to another is the same for all processes, both internally reversible and irreversible, between these two states. Thus, Eq. (7-5) allows determination of entropy change for all processes between 2 states.
The entropy values in the property tables are given relative to an arbitrary reference state. In steam tables, the entropy of saturated liquid $s_f$ at 0.01°C is assigned the value of zero. For refrigerant-134a, the zero value is assigned to saturated liquid at -40°C. The entropy values become negative at temperatures below the reference value.
- The value of entropy at a specified state is determined just like any other property.

- In the compressed liquid and superheated vapor regions, it can be obtained directly from the tables at the specified state.

- In the absence of compressed liquid data, the entropy of the compressed liquid can be approximated by the entropy of the saturated liquid at the given temperature:
\[
\begin{align*}
T & \quad s_1 = s_f \text{ at } T_1 \\
\frac{P_1}{T_1} & \quad s_2 = s_f + x_2 s_{fg} \\
T_3 & \quad s_3
\end{align*}
\]
In the saturated mixture region, it is determined from

\[ S_{T,P} \equiv S_{f,T} \]  \quad (\text{kJ/kg} \cdot \text{K})

\[ s = s_f + xs_{fg} \]  \quad (\text{kJ/kg} \cdot \text{K})

The entropy change of a specified mass \( m \) (a closed system) during a process is simply

\[ \Delta S = m\Delta s = m(s_2 - s_1) \]  \quad (\text{kJ/K}) (7-12)
$T$-$s$ diagram for water (see Figure A-9)
EXAMPLE 7–3 Entropy Change of a Substance in a Tank

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.
Solution:

\[ m = 5 \text{ kg} \]

Refrigerant-134a

\[ T_1 = 20^\circ\text{C} \]

\[ P_1 = 140 \text{ kPa} \]

\[ \Delta S = ? \]
Schematic and $T$-$s$ diagram for Example 7–3.
As refrigerant mass and tank volume are constant, hence \( v_2 = v_1 \).

From table A-12 for Saturated refrigerant 134a-Pressure table, for \( p_1 = 140 \text{ kPa} \), \( T_{sat} = -18.77 \, ^{\circ}C \).

As \( T_1 (= 20 \, ^{\circ}C) > T_{sat} (= -18.77^{\circ}C) \).
Hence, state 1 is superheated vapor.
From table A-13 for Superheated refrigerant-134a table, for $p_1 = 140 \text{ kPa}$
($= 0.14 \text{ Mpa}$) & $T_1 = 20 \text{ °C}$, Hence $v_1 = 0.16544$ $m^3/kg$ & $s_1 = 1.0624$ $kJ/kg.K$.

$v_2 = v_1 = 1.0624$ $kJ/kg.K$.

For $p_2 = 100 \text{ kPa}$ & $v_2 = 0.16544$ $m^3/kg$
From table A-12 for Saturated refrigerant 134a-Pressure table, for $p_2 = 100 \text{ kPa}$, $v_f = 0.0007259 \text{ m}^3/\text{kg} \& v_g = 0.19254 \text{ m}^3/\text{kg}$ and $s_f = 0.07188 \text{ kJ/kgK} \& s_g = 0.95183 \text{ kJ/kgK}$.

*It is clear that* $v_f \ (= 0.0007259 \text{ m}^3/\text{kg}) < v_2 \ (= 0.16544 \text{ m}^3/\text{kg}) < v_g \ (= 0.19254 \text{ m}^3/\text{kg})$

Hence, state 2 is Saturated liquid-vapor mixture
\[ x_2 = \frac{(v_2 - v_f)}{(v_g - v_f)} = \frac{(0.16544 - 0.0007259)}{(0.19254 - 0.0007259)} = 0.8587 \]

**But,**

\[ x_2 = \frac{(s_2 - s_f)}{(s_g - s_f)} \text{, Hence, } 0.8587 = \frac{(s_2 - 0.07188)}{(0.95183 - 0.07188)} \]

**Hence,** \( s_2 = 0.8275 \text{ kJ/kgK} \)
Hence, $\Delta s = s_2 - s_1 = 0.8275 \text{ kJ/kgK} - 1.0624 \text{ kJ/kg.K} = -0.2349 \text{ kJ/kg.K}$

Hence, $\Delta S = m \times \Delta s = 5 \text{ kg} \times -0.2349 \text{ kJ/kg.K} = -1.1745 \text{ kJ/K}$

-ve sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the entropy generation $S_{gen}$ that cannot be -ve.
Example 7-4 Entropy Change during a Constant-Pressure Process
A piston–cylinder device initially contains 1 kg of liquid water at 200 kPa and 10°C. The water is now heated at constant pressure by the addition of 3300 kJ of heat. Determine the entropy change of the water during this process.
Solution:

\[ p_1 = 200 \text{ kPa} \]

\[ T_1 = 10^\circ\text{C} \]
State 1: $p_1 = 200 \text{ kPa}, T_1 = 10^\circ \text{C}$.
From table A-5 (Saturated water-Pressure table): For $p_1 = 200 \text{ kPa}, T_{sat} = 120.21^\circ \text{C}$.

As $T_1 (= 10^\circ \text{C}) < T_{sat} (= 120.21^\circ \text{C})$.
Hence, State 1 is compressed (sub-cooled) liquid.

However, there is no data for compressed Liquid for $p_1 = 200 \text{ kPa}$.

Hence, $s_1 \approx s_f (T_1 = 10^\circ \text{C}) = 0.1511 \text{ kJ/kgK}$. 
& \ h_1 \approx \ h_f \ (T_1 = 10^{\circ}\text{C}) = 42.022 \ \text{kJ/kg}.

From Eq. (4-17)

\[ Q_{net, \ in} - W_{net, \ out} = \Delta E = \Delta U + \Delta PE + \Delta KE \]

i.e., \[ Q_{net, \ in} - W_{net, \ out} = \Delta U \ (\#-8) \]

\[ Q_{net, \ in} = Q_{in} - Q_{out} = Q_{in} \]

\[ W_{net, \ out} = W_b \ (\#-9) \]
Hence, Eq.(-8) can be re-written as

\[ Q_{in} = \Delta H = m (h_2 - h_1) \]

3300 kJ = 1 kg \((h_2 - 42.022)\) kJ/kg

Hence, \(h_2=3342.022\) kJ/kg

From Table (A-6) for superheated water for \(p = 200\) kPa (=0.2 Mpa)
$s_2$ is obtained by interpolation between $s = 8.2236 \text{ kJ/kgK}$ (corresponding to $h = 3277.0 \text{ kJ/kg}$) & $s = 8.5153 \text{ kJ/kgK}$ (corresponding to $h = 3487.7 \text{ kJ/kg}$)

\[
\frac{s_2 - 8.2236}{8.5153 - 8.2236} = \frac{3342.022 - 3277.0}{3487.7 - 3277.0}
\]

Hence, $s_2 = 8.313619 \text{ kJ/kgK}$
Hence,

$$\Delta S = m(s_2 - s_1) = \text{1 kg} \times (8.313619 - 0.1511)$$

kJ/kgK

Hence, $\Delta S = 8.162519 \text{ kJ/K}$
It is mentioned earlier that the entropy of a fixed mass can be changed by (1) heat transfer and (2) irreversibilities. Then it follows that the entropy of a fixed mass does not change during a process that is internally reversible and adiabatic (Fig. 7–14).
Isentropic process:

\[ \Delta s = 0 \]

or

\[ s_2 = s_1 \text{ (kJ/kg} \cdot \text{K)} \]  \hspace{1cm} (7–13)
EXAMPLE 7–5 Isentropic Expansion of Steam in a Turbine
Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.
Solution:

$T$ vs. $S$

- $P_1 = 5$ MPa
- $T_1 = 450°C$

Isentropic expansion

$s_2 = s_1$

$P_2 = 1.4$ MPa
For a single-stream (one-inlet and one-outlet) steady-flow system

\[
\dot{Q} - \dot{W} = \dot{m} \left[ h_2 - h_1 + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) \right]
\]

Where,

\[
\dot{Q} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}
\]

Hence,

\[
\dot{Q} = 0
\]
Hence, Eq. (5-38) can be re-written as:

$$-\dot{W}_{out} = \dot{m}(h_2 - h_1)$$

(*-1)
Dividing both sides of Eq. (\(\ast-1\)) by to get

\[-w_{\text{out}} = (h_2 - h_1)\]

Hence,

\[w_{\text{out}} = (h_1 - h_2)\]

\((\ast-2)\)
State 1:

\[ p_1 = 5 \text{ MPa and } T_1 = 450^\circ \text{C} & p_2 = 1.4 \text{ Mpa}. \]

From Table A-5 for \( p_1 = 5 \text{ Mpa} \), \( T_{sat} = 263.94^\circ \text{C} \)

As \( T_1 = (450^\circ \text{C}) > T_{sat} = (263.94^\circ \text{C}) \). Hence, point 1 is superheated steam

From Table A-6, for superheated steam for \( p_1 = 5 \text{ MPa} \) and \( T_1 = 450^\circ \text{C} \)
\[ h_1 = 3317.2 \text{ kJ/kg and } s_1 = 6.821 \text{ kJ/kg.K} \]

As process 1-2 is reversible adiabatic (isentropic), hence \( s_2 = s_1 = 6.821 \text{ kJ/kg.K} \)

From Table A-5 for \( p_2 = 1.4 \text{ Mpa} \), \( T_{sat} = 195.04^\circ \text{C} \), \( s_f = 2.2835 \text{ kJ/kg.K} \), \( s_g = 6.4675 \text{ kJ/kg.K} \)

As \( s_2 (=6.821 \text{ kJ/kg.K}) > s_g (=6.4675 \text{ kJ/kg.K}) \), hence point 2 is superheated steam
From Table A-6 for superheated steam, $p_2 = 1.4$ Mpa, $s_2 = 6.821$ kJ/kg.K, $h_2$ is evaluated by interpolation between $h = 2927.9$ kJ/kg & 3040.9 kJ/kg corresponding to $s = 6.7488$ kJ/kg.K & 6.9553 kJ/kg.K

$$\frac{h_2 - 2927.9}{3040.9 - 2927.9} = \frac{6.821 - 6.7488}{6.9553 - 6.7488}$$

Hence, $h_2 = 2967.4$ kJ/kg
From Eq.(*-2), \( w_{out} = h_1 - h_2 = 3317.2 \text{ kJ/kg} - 2967.4 \text{ kJ/kg} = 349.8 \text{ kJ/kg} \)
From Eq. (7-4)

\[ dS = \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \text{ (kJ/K)} \]  

(7-4)

Hence,

\[ \delta Q_{\text{int rev}} = T \, dS \text{ (kJ)} \]
On a $T$-$S$ diagram, the area under the process curve represents the heat transfer for internally reversible processes.
\[ Q_{\text{int rev}} = \int_{1}^{2} T \, dS \quad \text{(kJ)} \quad (7-15) \]

\[ \delta q_{\text{int rev}} = T \, ds \quad \text{(kJ/kg)} \quad (7-16) \]

\[ q_{\text{int rev}} = \int_{1}^{2} T \, ds \quad \text{(kJ/kg)} \quad (7-17) \]
Special cases,

a) Isentropic (Reversible adiabatic) process,

\[ s_2 = s_1 \]

\[ q_{\text{int,rev}} = 0 \]
b) *internally reversible isothermal process*,

\[ T_2 = T_1 \]
\[ Q_{\text{int rev}} = T_0 \Delta S \quad (\text{kJ}) \quad (7-18) \]

\[ q_{\text{int rev}} = T_0 \Delta s \quad (\text{kJ/kg}) \quad (7-19) \]
*T-*s diagram for water:

In superheated vapor region of *T-*s diagram, const. *h* lines become nearly horizontal as *p* ↓.

*T*-s diagram of water is given in Fig. A–9.
The *h-s diagram for water (Mollier diagram)* is given in the appendix in Fig. A–10.

\[ T = 240°C \]

\[ p = 0.01 \text{ MPa} \]

\[ p = 0.1 \text{ MPa} \]

\[ s_2 = s_1 \]
h-s diagram for water (Mollier diagram)
The T-S Diagram of the Carnot Cycle

\[ Q_H = T_H (S_2 - S_1) \]
\[ Q_L = T_L (S_2 - S_1) \]
\[ W_{net} = Q_H - Q_L = (T_H - T_L)(S_2 - S_1) \]
\[ \eta_{Carnot} = \frac{W_{net}}{Q_H} = \frac{(T_H - T_L)}{T_H} \]

The T-S diagram of a Carnot cycle
Differential form of the conservation of energy equation for a closed stationary system (a fixed mass) is

\[ \delta Q_{\text{int \ rev}} - \delta W_{\text{int \ rev}} = dE = dKE + dPE + dU \]  

\[ (\ast - 3) \]

In the absence of overall system motion & gravity effects

Hence,

\[ \delta Q_{\text{int \ rev}} - \delta W_{\text{int \ rev}} = dU \]  

\[ (7.21) \]
But, \( \delta W \)_{\text{int, rev}} = p \, dV \quad \text{(KJ)} \quad (**-4)

Eq. (6.2b) can be put in the following form:

But,

\[ \delta q_{\text{int, rev}} = T \, ds \quad \text{(kJ/kg)} \quad (7-16) \]

Hence, Eq. (7.16) can be re-written as:

\[ \delta Q_{\text{int, rev}} = T \, dS \quad \text{(**-5)} \]
By substituting by $\delta Q_{\text{int, rev}}$ and $\delta W_{\text{int, rev}}$ from Eqs. (\textit{-5})& (\textit{-4}) in Eq. (7-21) to get

$$\delta Q_{\text{int, rev}} - \delta W_{\text{int, rev}} = dU$$

Hence,

$$TdS - pdV = dU$$

For compressible substances
Thus, \[ TdS = dU + pdV \] (kJ) \hspace{1cm} (7-22)

On mass basis,

Hence, \[ Tds = du + pdv \] (kJ/kg) \hspace{1cm} (7-23)

Gibbs Eqn.

But, \[ h = u + pv \] \hspace{1cm} (*-6)

Hence, \[ dh = du + pdv + vdp \]

Thus,

Hence, \[ du + pdv = dh - vdp \] \hspace{1cm} (*-7)
From Eqs. (7-23) & (7-7) to get

\[ T ds = du + pdv = dh - \nu dp \]  

(7-24)

\[ T ds \] relations are valid for both reversible & irreversible processes and for both closed & open systems
Dividing both sides of Eqs. (7-23) & (7-24) to get

\[ ds = \frac{du}{T} + \frac{P \, dv}{T} \]  \hspace{1cm} (7-25)

and

\[ ds = \frac{dh}{T} - \frac{\nu \, dP}{T} \]  \hspace{1cm} (7-26)
For ideal gases, $du = c_v \, dT$ or $dh = c_p \, dT$ & $pv = RT$, change of entropy can be obtained by integration of Eq. (7-23) or (7-24).
**7–8  ■ ENTRPY CHANGE OF LIQUIDS AND SOLIDS**

**Incompressible substances** \((dv \cong 0)\)

Hence, Eq.(7-25) can be re-written as

\[
\frac{ds}{T} = \frac{du}{T} + \frac{P}{T} \, dv
\]

(7-25)
Then,

$$ds = \frac{du}{T}.$$  \hspace{1cm} (7-27)

since $c_p = c_v = c$ and $du = c \, dT$ for incompressible substances. Then the entropy change during a process is determined by integration to be
Liquids, solids:

\[ s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \approx c_{avg} \ln \frac{T_2}{T_1} \] (kJ/kg . K)  

(7-29)

where \( c_{avg} \) is the average specific heat of the substance over the given temperature interval.
EXAMPLE 7–7 Effect of Density of a Liquid on Entropy

Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or 82°C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7–1.
Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (a) using tabulated properties and (b) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?
<table>
<thead>
<tr>
<th>Temp., $T$, K</th>
<th>Pressure, $P$, MPa</th>
<th>Density, $\rho$, kg/m$^3$</th>
<th>Enthalpy, $h$, kJ/kg</th>
<th>Entropy, $s$, kJ/kg · K</th>
<th>Specific heat, $c_p$, kJ/kg · K</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>0.5</td>
<td>425.3</td>
<td>208.3</td>
<td>4.878</td>
<td>3.476</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>425.8</td>
<td>209.0</td>
<td>4.875</td>
<td>3.471</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>426.6</td>
<td>210.5</td>
<td>4.867</td>
<td>3.460</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>429.1</td>
<td>215.0</td>
<td>4.844</td>
<td>3.432</td>
</tr>
<tr>
<td>120</td>
<td>0.5</td>
<td>410.4</td>
<td>243.4</td>
<td>5.185</td>
<td>3.551</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>411.0</td>
<td>244.1</td>
<td>5.180</td>
<td>3.543</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>412.0</td>
<td>245.4</td>
<td>5.171</td>
<td>3.528</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>415.2</td>
<td>249.6</td>
<td>5.145</td>
<td>3.486</td>
</tr>
</tbody>
</table>
**Solution:**

(a) Solution using tabulated properties.

*From Table 7-1, for* $T_1 = 110$ K, $p_1 = 1$ MPa; $s_1 = 4.875$ kJ/kg.K

& for $T_2 = 120$ K, $p_2 = 5$ MPa; $s_2 = 5.145$ kJ/kg.K

*Hence,* $\Delta s = 5.145$ kJ/kg.K - $4.875$ kJ/kg.K = $0.270$ kJ/kg.K
(b) Solution by approximating liquid methane as an incompressible substance.

As $T_1 (=110 \text{ K})$ & $T_2 (=120 \text{ K}) < T_{cr}$ ($=191\text{K}$), hence, methane is Liquid at states 1 & 2

From Table 7-1, for $T_1 = 110 \text{ K}$, $p_1 = 1 \text{ MPa} ; \ c_{p1} = 3.471 \text{ kJ/kg.K}$

From Table 7-1, for $T_2 = 120 \text{ K}$, $p_2 = 5 \text{ MPa} ; \ c_{p2} = 3.486 \text{ kJ/kg.K}$
\[ \text{Hence; } c_{avg} = \frac{(c_{p1} + c_{p2})}{2} = \frac{(3.471 + 3.486)}{2} = 3.4785 \text{ kJ/kg.K} \]

Then,

\[ \Delta s = c_{avg} \ln \frac{T_2}{T_1} = (3.4785 \text{ kJ/kg \cdot K}) \ln \frac{120 \text{ K}}{110 \text{ K}} \]

\[ = 0.303 \text{ kJ/kg \cdot K} \]
\[
% \ \text{Error} = \left| \frac{\Delta s_{\text{approximate}} - \Delta s_{\text{exact}}}{\Delta s_{\text{exact}}} \right| \times 100 = \left( \frac{0.303 - 0.270}{0.270} \right) \times 100 = 12.2\%
\]
EXAMPLE 7–8 Economics of Replacing a Valve by a Turbine

A cryogenic manufacturing facility handles liquid methane at 115 K and 5 MPa at a rate of 0.280 m³/s. A process requires dropping the pressure of liquid methane to 1 MPa, which is done by throttling the liquid methane by passing it through a flow resistance such as a valve. A recently hired engineer proposes to
replace the throttling valve by a turbine in order to produce power while dropping the pressure to 1 MPa. Using data from Table 7–1, determine the maximum amount of power that can be produced by such a turbine. Also, determine how much this turbine will save the facility from electricity usage costs per year if the turbine operates continuously (8760 h/yr) and the facility pays $0.075/kWh for electricity.
**Solution:**

From Eq. (5-38)

For a single-stream (one-inlet and one-outlet) steady-flow system

\[
\dot{Q} - \dot{W} = m \left[ h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]
\]

(5-38)
Hence, Eq. (5-38) can be re-written as;

\[ -\bar{W}_{\text{out}} = \dot{m}(h_2 - h_1) \]

\[ W_{\text{out}} = \dot{m}(h_1 - h_2) \quad (\ast-8) \]

The maximum amount of power implies that the process in the turbine is isentropic (reversible adiabatic); i.e.

\[ S_2 = S_1 \quad (\ast-9) \]
From Fig. (7-30);

State 1:
\[
\begin{align*}
P_1 &= 5 \text{ MPa} \\
T_1 &= 115 \text{ K}
\end{align*}
\]
\[
\begin{align*}
&h_1 = 232.3 \text{ kJ/kg} \\
&s_1 = 4.9945 \text{ kJ/kg} \cdot \text{K} \\
&\rho_1 = 422.15 \text{ kg/s}
\end{align*}
\]

State 2:
\[
\begin{align*}
P_2 &= 1 \text{ MPa} \\
&s_2 = s_1
\end{align*}
\]
\[
\begin{align*}
h_2 &= 222.8 \text{ kJ/kg}
\end{align*}
\]

Also, the mass flow rate of liquid methane is
\[
\dot{m} = \rho_1 \dot{V}_1 = (422.15 \text{ kg/m}^3)(0.280 \text{ m}^3/\text{s}) = 118.2 \text{ kg/s}
\]

Hence;
\[
\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)
\]
\[
= (118.2 \text{ kg/s})(232.3 - 222.8) \text{ kJ/kg}
\]
\[
= 1123 \text{ kW}
\]
For continuous operation \((365 \times 24 = 8760 \text{ h})\), the amount of power produced per year is

\[
\text{Annual power production} = \dot{W}_{\text{out}} \times \Delta t = (1123 \text{ kW})(8760 \text{ h/yr})
\]
\[
= 0.9837 \times 10^7 \text{ kWh/yr}
\]

At $0.075/\text{kWh}$, the amount of money this turbine can save the facility is

\[
\text{Annual power savings} = (\text{Annual power production})(\text{Unit cost of power})
\]
\[
= (0.9837 \times 10^7 \text{ kWh/yr})(0.075/\text{kWh})
\]
\[
= $737,800/\text{yr}$
\]
THE ENTROPY CHANGE OF IDEAL GASES

Substituting by $du = c_v \, dT$ and $p = RT/v$ in Eq. (7-25) to get

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$  \hspace{1cm} (7-30)

The entropy change for a process is obtained by integrating this relation between the end states:

$$s_2 - s_1 = \int_{1}^{2} c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$  \hspace{1cm} (7-31)
Similarly, substituting by $dh = c_p \, dT$ and $p = RT/v$ in Eq. (7-26) to get

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$  \hspace{1cm} (7-32)

The entropy change for a process is obtained by integrating this relation between the end states:

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{p_2}{p_1}$$  \hspace{1cm} (7-32')

The integrals in Eqs. 7–31 and 7–32 cannot be performed unless the dependence of $c_v$ and $c_p$ on temperature is known.
Constant Specific Heats (Approximate Analysis)

Assuming constant specific heats for ideal gases is a common approximation,

The entropy-change relations for ideal gases under the constant-specific heat assumption are easily obtained by replacing \( c_v(T) \) and \( c_p(T) \) in Eqs. 7–31 and 7–32 by \( c_{v,avg} \) and \( c_{p,avg} \), respectively, and performing the integrations. We obtain
\[ s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{\nu_2}{\nu_1} \quad \text{(kJ/kg \cdot K)} \]  
(7-33)

\[ s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{(kJ/kg \cdot K)} \]  
(7-34)
EXAMPLE 7–9 Entropy Change of an Ideal Gas

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process assuming average specific heats.

Solution:

The entropy change of air during this process can be determined approximately from Eq. 7–34 by using a $c_p$ value at the average temperature of 37°C (Table A–2b) and treating it as a constant.
\[ s_2 - s_1 = c_{p,avg} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \]

\[ = (1.006 \text{ kJ/kg} \cdot \text{K}) \ln \frac{330 \text{ K}}{290 \text{ K}} - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \]

\[ = -0.3842 \text{ kJ/kg} \cdot \text{K} \]
Isentropic Processes of Ideal Gases

(Reversible adiabatic) \( p \, v^n = c \)

\( n = k = \frac{c_p}{c_v} \) \& \( R = c_p - c_v \)

Hence, \( p \, v^k = c \)

Also, for Ideal gas,

\( p \, v = RT \)
Hence; for isentropic process for ideal gas

\[ p \, V^k = c \] \hspace{1cm} (7-47)

\[ T \, V^{k-1} = c \] \hspace{1cm} (7-45)

\[ T \, p^{(1-k)/k} = c \] \hspace{1cm} (7-46)
EXAMPLE 7–10 Isentropic Compression of Air in a Car Engine

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio $V_1/V_2$ of this engine is 8, determine the final temperature of the air.
Solution:

\[
\frac{V_1}{V_2} = \frac{v_1}{v_2} = \frac{8}{8}
\]

\[
T_1 = 22°C = 22 + 273 = 295 \text{ K}
\]

\[
p_1 = 95 \text{ kPa}
\]

\[
T_2 = ?
\]

From Eq.(7-45)

\[
T v^{k-1} = c \quad (7-45)
\]
Hence,

\[ T_2 \nu_2^{k-1} = T_1 \nu_1^{k-1} \]

i.e., \[ T_2 = T_1 \left( \frac{\nu_1}{\nu_2} \right)^{k-1} \]

From Table (A-2), \( k \) (for Air) for \( T = T_1 = 295 \) K

Hence,

\[ T_2 = 295 \text{ K} \left( 8 \right)^{1.4-1} = 295 \text{ K} \left( 8 \right)^{1.4-1} \]

\[ T_2 = 677.732 \text{ K} \]
Evaluation of more accurate value of $T_2$

$$T_{\text{avg}} = \frac{(T_1 + T_2)}{2} = \frac{(295 + 677.732)}{2} = 486.37 \, K$$

From Table (A-2), $k$ (for Air) for $T = (T_{\text{avg}} = 486.37 \, K)$

$$\frac{(k - 1.391)}{(1.387 - 1.391)} = \frac{(486.37 - 450)}{(500 - 450)}$$

Hence, $k = 1.38809$
Hence,

\[ T_2 = 295 \, \text{K} \times (8)^{1.38809 \times -1} = 295 \, \text{K} \times (8)^{0.38809} \]

\[ = 661.15 \, \text{K} \]
EXAMPLE 7–11 Isentropic Compression of an Ideal Gas

Helium gas is compressed by an adiabatic compressor from an initial state of 100 kPa and 10°C to a final temperature of 150°C in a reversible manner. Determine the exit pressure of helium.
Solution:

\[ T_1 = 10^\circ C = 10 + 273 \text{ K} = 283 \text{ K} \]

\[ p_1 = 100 \text{ kPa} \]

\[ T_2 = 150^\circ C = 150 + 273 \text{ K} = 423 \text{ K} \]

\[ p_2 = ? \]

\[ Tp^{(1-k)/k} = c \quad (7-46) \]

Hence, \[ T_1 p_1^{(1-k)/k} = T_2 p_2^{(1-k)/k} \]

Then, \[ p_2 = p_1 \left[ \frac{T_1}{T_2} \right]^{k/(1-k)} \]
Hence,

\[ p_2 = 100 \text{ kPa } \left[ \frac{283 \text{ K}}{423 \text{ K}} \right]^{k/(1-k)} \]

From Table A-2, \( k \) for Helium = 1.667

Hence,

\[ p_2 = 100 \text{ kPa } \left[ \frac{283}{423} \right]^{1.667/(1-1.667)} \]

\[ p_2 = 273.06 \text{ kPa} \]
The work done during a process depends on the path followed as well as on the properties at the end states. Recall that reversible (quasi-equilibrium) moving boundary work associated with closed systems is

\[ W_b = \int_{1}^{2} P \, dV \]
By butting Eq. (5-39) in differential form to get

\[ q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \]  \hspace{1cm} (5-39)

\[ \delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe \]  \hspace{1cm} (*-10)

But from Eqs. (7-16) and (7-24)

\[ \delta q_{\text{rev}} = T \, ds \]  \hspace{1cm} (Eq. 7–16)

\[ T \, ds = dh - \nu \, dP \]  \hspace{1cm} (Eq. 7–24) \hspace{1cm} \begin{cases} \delta q_{\text{rev}} = dh - \nu \, dP \end{cases} \hspace{1cm} (*-11)
By substituting by $\delta q_{rev}$ from Eq. (\textsuperscript{-11}) in Eq. (\textsuperscript{-10}) to get

$$dh - vdp - \delta w_{rev} = dh + dKe + dPE$$

Hence,

$$-\delta w_{rev} = v \; dP + dke + dpe$$

Integrating, we find

$$w_{rev} = -\int_{1}^{2} v \; dP - \Delta ke - \Delta pe \quad (kJ/kg) \quad (7-51)$$
When the changes in K.E. and P.E. are negligible, this equation reduces to

\[ w_{\text{rev}} = -\int_{1}^{2} \nu \, dP \quad (\text{kJ/kg}) \]  

\[ (7-52) \]

Eqs. 7–51 and 7–52 are relations for the *reversible work output associated with an internally reversible process in a steady-flow device*.
When the working fluid is *incompressible*, the specific volume $v$ remains constant during the process, Eq. 7–51 can be rewritten as,

$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta ke - \Delta pe \quad \text{(kJ/kg)} \quad (7-54)$$

*Fig. 7-42 Reversible Steady-Flow Work, $\Delta ke \approx 0, \Delta pe \approx 0$*
EXAMPLE 7–12 Compressing a Substance in the Liquid versus Gas Phases

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (a) saturated liquid and (b) saturated vapor at the inlet state.
Solution:

\[ p_1 = 100 \text{ kPa} \quad \& \quad p_2 = 1 \text{ MPa} \]

\[ W_{\text{input, reversible}} (\text{required to compress the steam isentropically}) = ? \]

Assuming that

a) Steam exists as saturated liquid at the inlet state; (Compression by pump).

b) Steam exists as saturated vapor at the inlet state; (Compression by compressor).
Schematic and $T$-$s$ diagram for Example 7–12.
(a) In this case, steam is a saturated liquid initially, and its specific volume is

\[ v_1 = v_{f@100 \text{kPa}} = 0.001043 \text{ m}^3/\text{kg} \quad \text{(Table A-5)} \]

which remains essentially constant during the process. Thus,

\[
\begin{align*}
\omega_{\text{rev, in}} &= \int_{1}^{2} v \, dP \approx v_1 (P_2 - P_1) \\
&= (0.001043 \text{ m}^3/\text{kg})[ (1000 - 100) \text{ kPa} ] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\
&= 0.94 \text{ kJ/kg}
\end{align*}
\]
(b) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how $v$ varies with $p$ to perform the integration in Eq. 7–53. This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the second $T \, ds$ relation by setting $ds = 0$: 
Thus,

\[
T \, ds = dh - \nu \, dP \quad \text{(Eq. 7-24)} \]
\[
ds = 0 \quad \text{(isentropic process)} \quad \nu \, dP = dh
\]

\[
\mathcal{W}_{\text{rev,in}} = \int_{1}^{2} \nu \, dP = \int_{1}^{2} dh = h_2 - h_1
\]

This result could also be obtained from the energy balance relation for an isentropic steady-flow process. Next we determine the enthalpies:
**Discussion:** Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.
Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

A reversible turbine delivers more work than an irreversible one if both operate between the same end states.
Therefore, work-producing devices such as turbines \((w \text{ is } +ve)\) deliver more work, and work-consuming devices such as pumps and compressors \((w \text{ is } -ve)\) require less work when they operate reversibly.

\[ w_{\text{rev}} \geq w_{\text{act}} \]
MINIMIZING THE COMPRESSOR WORK

Work I/P to a compressor is minimized when the compression process is executed in an internally reversible manner. When the changes in K.E. and P.E. are negligible, the compressor work is given by (Eq. 7–53)

\[ w_{\text{rev, in}} = \int_{1}^{2} v \, dP \]  

(7–56)
a) For Isentropic process; \((P^k = \text{constant})\):

\[
\omega_{\text{comp, in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right] \quad (7–57a)
\]

b) For Polytropic process; \((P^n = \text{constant})\):

\[
\omega_{\text{comp, in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] \quad (7–57b)
\]

c) For Isothermal process; \((P = \text{constant})\):

\[
\omega_{\text{comp, in}} = RT \ln \frac{P_2}{P_1} \quad (7–57c)
\]
\[ w_{\text{rev, in}} = \int_{1}^{2} \nu \, dP \]

\textit{p-v diagrams of isentropic, polytropic, and isothermal} compression processes between the same pressure limits.
Multistage Compression with Intercooling

$p-v$ and $T-s$ diagrams for a two-stage steady-flow compression process.
The total work input for a two-stage compressor is the sum of the work inputs for each stage of compression, as determined from Eq. 7–57b:

\[
w_{\text{comp, in}} = w_{\text{comp I, in}} + w_{\text{comp II, in}} \quad (7–58)
\]

\[
w_{\text{comp, in}} = \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]
\]

The only variable in this equation is \( p_x \). The \( p_x \) value that minimizes the total work is determined by differentiating this expression w. r. t. \( p_x \) and setting the resulting expression equal to zero.
That is, to minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same. When this condition is satisfied, the compression work at each stage becomes identical, that is,

\[ \frac{\partial (w_{\text{comp,in}})}{\partial (P_x)} = 0 \]

\[ P_x = (P_1P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x} \quad (7-59) \]
EXAMPLE 7–13 Work Input for Various Compression Processes

Air is compressed steadily by a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa. Determine the compressor work per unit mass for (a) isentropic compression with \( k = 1.4 \), (b) polytropic compression with \( n = 1.3 \), (c) isothermal compression, and (d) ideal two stage compression with intercooling with a polytropic exponent of 1.3.
Solution:

\[ p_1 = 100 \text{ kPa} \quad \& \quad T_1 = 300 \text{ K} \]

\[ p_2 = 900 \text{ KPa} \]

(a) isentropic compression with \( k = 1.4 \)

\((Pv^k = \text{constant}):\)

\[
w_{\text{comp, in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]
\]

\[
= \frac{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.4 - 1} \left[ \left( \frac{900 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.4-1)/1.4} - 1 \right]
\]

Thus, \( W_{\text{comp, in}} = 263.2 \text{ kJ/kg} \)
(b) *polytropic* compression with $n = 1.3$

$$w_{\text{comp, in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$ (7–57b)

Thus, $W_{\text{comp, in}} = 246.4$ kJ/kg

(c) *isothermal* compression, $(P\nu = \text{constant})$:

$$w_{\text{comp, in}} = RT \ln \frac{P_2}{P_1}$$ (7–57c)
Thus, \( W_{\text{comp, in}} = (0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K}) \ln \frac{900 \text{ kPa}}{100 \text{ kPa}} \)

\[ = 189.2 \text{ kJ/kg} \]

\textit{(d) ideal two stage compression with intercooling with a polytropic exponent of 1.3. (}n = 1.3\text{): In this case, the pressure ratio across each stage is the same, and its value is}

\[ P_x = (P_1P_2)^{1/2} = [ (100 \text{ kPa})(900 \text{ kPa}) ]^{1/2} = 300 \text{ kPa} \]
The compressor work across each stage is also the same. Thus the total compressor work is twice the compression work for a single stage:

\[
\begin{align*}
\omega_{\text{comp,in}} &= 2 \omega_{\text{comp 1,in}} = 2 \frac{nRT_1}{n - 1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] \\
&= \frac{2(1.3)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.3 - 1} \left[ \left( \frac{300 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.3-1)/1.3} - 1 \right] \\
&= 215.3 \text{ kJ/kg}
\end{align*}
\]
It is clear that,
\[ W_{\text{comp, in})\text{isentropic}} = 263.2 \text{ kJ/kg} \] > \[ W_{\text{comp, in})\text{adiabatic}} = 246.4 \text{ kJ/kg} \] > \[ W_{\text{comp, in})\text{ideal two stage compression with intercooling}} = 215.3 \text{ kJ/kg} \] > \[ W_{\text{comp, in})\text{isothermal}} = 189.2 \text{ kJ/kg} \]
Isentropic Efficiency of Turbines

*Isentropic efficiency of a turbine* is defined as the ratio of the actual work O/P of the turbine to the work O/P that would be achieved *if* the process between the inlet state and the exit pressure *were isentropic*:
The $h$-$s$ diagram for the actual and isentropic processes of an adiabatic turbine.
\[ \eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \]  \hspace{1cm} (7–60)

\[ \eta_T \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \]  \hspace{1cm} (7–61)
EXAMPLE 7–14 Isentropic Efficiency of a Steam Turbine

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power O/P of the turbine is 2 MW, determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the steam flowing through the turbine.
Solution:

\[ p_1 = 3 \text{ MPa}, \quad T_1 = 400^\circ \text{C}, \]
\[ p_2 = 50 \text{ kPa}, \quad T_2 = 100^\circ \text{C}. \]

\[ \dot{W}_{\text{out}} = 2 \text{ MW} \]

\[ \eta_T = ? \]

\[ \dot{m} = ? \]
Schematic and $T$-$s$ diagram for Example 7–14.
Analysis A sketch of the system and the T-s diagram of the process r given in Fig. 7–50.

(a) The enthalpies at various states are

<table>
<thead>
<tr>
<th>State 1:</th>
<th>$P_1 = 3$ MPa</th>
<th>$T_1 = 400°C$</th>
<th>$h_1 = 3231.7$ kJ/kg</th>
<th>$s_1 = 6.9235$ kJ/kg·K</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 2a:</td>
<td>$P_{2a} = 50$ kPa</td>
<td>$T_{2a} = 100°C$</td>
<td>$h_{2a} = 2682.4$ kJ/kg</td>
<td></td>
</tr>
</tbody>
</table>

The exit enthalpy of the steam for the isentropic process $h_{2S}$ is determined from the requirement that the entropy of the steam remain constant ($s_{2S} = s_1$):
Obviously, at the end of the isentropic process steam exists as a saturated mixture since \( s_f < s_{2s} < s_g \). Thus we need to find the quality at state 2s first:

\[
x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897
\]

and

\[
h_{2s} = h_f + x_{2s}h_{fg} = 340.54 + 0.897(2304.7) = 2407.9 \text{ kJ/kg}
\]
By substituting these enthalpy values into Eq. 7–61, the isentropic efficiency of this turbine is determined to be

\[ w_a = h_1 - h_2 = 3231.7 - 2682.4 \text{ kJ/kg} = 547.3 \text{ kJ/kg} \]

\[ w_s = h_1 - h_{2s} = 3231.7 - 2407.9 \text{ kJ/kg} = 823.8 \text{ kJ/kg} \]

\[ \eta_T = \frac{w_a}{w_s} = \frac{547.3}{823.8} = 0.6644 \]

\[ \eta_T \% = 0.6644 \times 100\% = 66.44\% \]
(b) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

\[ W_{\text{out}} = \dot{m} W_a \]

\[ = 2 \text{ MW} = 2000 \text{ kW} \]

\[ 2000 \text{ kW} = \dot{m} \times 547.3 \text{ kJ/kg} \]

\[ \dot{m} = \frac{2000}{547.3} = 3.654 \text{ kg/s} \]
Isentropic Efficiencies of Compressors and Pumps

a) For Compressors.

h-s diagram of actual and isentropic processes of an adiabatic compressor.

\[ \eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a} \]  

\( \Delta ke \cong 0, \Delta pe \cong 0 \)  

\[ \eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad (7-63) \]
b) For Pumps.

\[
\eta_P = \frac{w_s}{w_a} = \frac{\nu(P_2 - P_1)}{h_{2a} - h_1}
\]

(7–64)
EXAMPLE 7–15 Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (a) the exit temperature of air and (b) the required power input to the compressor.
Solution:

\[ p_1 = 100 \text{ kPa} , \ T_1 = 12^\circ \text{C} , \ p_2 = 800 \text{ kPa} . \]

\[ T_1 = 12 + 273 = 285 \text{ K} \]

\[ \dot{m} = 0.2 \text{ kg/s} \quad \eta_c = 0.8, \ T_2 = ? \quad W_{a,in} = ? \]

As process 1-2s is isentropic.

Hence,

\[ \dot{m} p_1 \frac{(1-k)}{k} = c \]

\[ T_1 p_1 \frac{(1-k)}{k} = T_{2s} p_{2s} \frac{(1-k)}{k} \]
Hence,

\[ T_{2s} = T_1 \left[ \frac{p_1}{p_{2s}} \right]^{(1-k)/k} \]

\[ T_{2s} = 285 \left[ \frac{100}{800} \right]^{(1-1.4)/1.4} K \]

\[ T_{2s} = 516.26 K \]

Hence, \[ T_{avg} = \frac{(T_1 + T_{2s})}{2} \]

\[ = \frac{(285 + 516.26)}{2} = 400.63 K \]

From Table A-2, \( k \) (for \( T_{avg} = 400.63 K \))

\[ = 1.395, \ c_{p,avg} = 1.013 \text{kJ/kg.K} \]
Hence,

\[ T_{2s} = 285 \left[ \frac{100}{800} \right] \frac{(1 - 1.395)}{1.395} K \]

Hence,

\[ T_{2s} = 513.5 K \]

\[ w_s = c_{p,avg}( T_{2s} - T_1 ) \]

= 1.013 kJ/kg.K(513.5 K - 285 K)

= 231.5 kJ/kg
\[ \eta_c = \left( \frac{w_s}{w_a} \right) \]

Hence,

\[ 0.8 = \left( \frac{231.5 \text{ kJ/kg}}{w_a} \right) \]

\[ w_a = \left( \frac{231.5 \text{ kJ/kg}}{0.8} \right) = 289.4 \text{ kJ/kg} \]

\[ w_a = c_p(T_2 - T_1) \]

From, Table (A-2), \( c_p = 1.005 \text{ kJ/kg.K} \)

Hence,

\[ 289.4 \text{ kJ/kg} = 1.005 \text{ kJ/kg.K}(T_2 - 285 \text{ K}) \]
289.4 kJ/kg = 1.005 kJ/kg.K( T_2 – 285 K)

Hence,

\[ T_2 = 572.92 \text{ K} \approx 573 \text{ K} \]

Hence,

\[ T_{\text{avg}} = \frac{(T_1 + T_2)}{2} = \frac{(285 + 573 \text{ K})}{2} = 429 \text{ K} \]

\[ [c_{p, \text{avg}} - c_p)T = 400 \text{ K}] / [c_p)T = 450 \text{ K} - c_p)T = 400 \text{ K}] \]

\[ = \frac{[429 - 400]}{[450 - 400]} \]
Hence,

\[ \frac{c_{p, \text{avg}} - 1.013 \text{ kJ/kg.K}}{1.020 \text{ kJ/kg.K} - 1.013 \text{ kJ/kg.K}} = \frac{29}{50} \]

Hence,

\[ c_{p, \text{avg}} = 1.01706 \text{ kJ/kg.K} \]

Hence,

\[ 289.4 \text{ kJ/kg} = 1.01706 \text{ kJ/kg.K}(T_2 - 285 \text{ K}) \]

Hence,

\[ T_2 = 1.01706 \text{ kJ/kg.K}(T_2 - 285 \text{ K}) \]

\[ T_2 = 569.55 \text{ K} \]
\[ W_{a,\text{in}} = \dot{m} \dot{w}_a \]

\[ = 0.2 \text{ kg/s} \times 289.4 \text{ kJ/kg} = 57.88 \text{ kW} \]

\[ \approx 58 \text{ kW} \]
Isentropic Efficiency of Nozzles

The $h$-$s$ diagram of the actual and isentropic processes of an adiabatic nozzle.
\[ \eta_N \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \]  

\[ \eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \]
EXAMPLE 7–16 Effect of Efficiency on Nozzle Exit Velocity

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 80 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine (a) the maximum possible exit velocity, (b) the exit temperature, and (c) the actual exit velocity of the air. Assume constant specific heats for air.
Solution:

\[ p_1 = 200 \text{ kPa} , \quad T_1 = 950 \text{ K} , \quad p_2 = 80 \text{ kPa} . \quad \eta_N = 0.92 , \quad V_{2s} = ? \ (b) \quad T_2 = ? \ (c) \quad V_{2a} = ? . \]

From Eq. (7-46)

\[ T p \left( \frac{1-k}{k} \right) = c \]

Hence,

\[ T_1 p_1 \left( \frac{1-k}{k} \right) = T_{2s} p_2 \left( \frac{1-k}{k} \right) \]

Hence,

\[ T_{2s} = T_1 \left[ \frac{p_1}{p_2} \right]^{(1-k)/k} \]
From Table A-2, For $T_f= 950$ K ,

\[
\left[ k, T= 950 \text{ K} - k, T= 900 \text{ K} \right] / \left[ k, T= 1000 \text{ K} - k, T= 900 \text{ K} \right] \\
= \left[ 950 - 900 \right] / \left[ 1000 - 900 \right]
\]

\[
\left[ k, T= 950 \text{ K} - 1.344 \right] / \left[ 1.336 - 1.344 \right] = 0.5
\]

\[ k, T= 950 \text{ K} = 1.34 \]

\[
\left[ c_p, T= 950 \text{ K} - c_p, T= 900 \text{ K} \right] / \left[ c_p, T= 1000 \text{ K} - c_p, T= 900 \text{ K} \right] \\
= \left[ 950 - 900 \right] / \left[ 1000 - 900 \right]
\]

\[
\left[ c_p, T= 950 \text{ K} - 1.121 \right] / \left[ 1.142 - 1.121 \right] = 0.5
\]

\[ c_p, T= 950 \text{ K} = 1.1315 \text{ kJ/kg.K} \]
Then,

\[ T_{2s} = T_1 \left[ \frac{p_1}{p_2} \right]^{(1-k)/k} \]

\[ = 950 \text{ K} \left[ \frac{200 \text{ kPa}}{80 \text{ kPa}} \right]^{(1-1.34)/1.34} \]

\[ = 753 \text{ K} \]

\[ h_f - h_{2s} = c_p [T_1 - T_{2s}] = 1.1315 [950 - 753] = 222.9 \text{ kJ/kg} = (V_{2s}^2 / 2). \]

\[ V_{2s} = [2*(h_f - h_{2s})]^{0.5} = [2*222.9*1000]^{0.5} \]

\[ = 667.7 \text{ m/s} \]
\[ \eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \]

\[ V_{2a} = V_{2s} \left[ \eta_N \right]^{0.5} = 667.73 \, \text{m/s} \left[ 0.92 \right]^{0.5} \]
\[ = 640.42 \, \text{m/s} \]
Homework