

# **Thermodynamics I**

Spring 1432/1433H (2011/2012H)

Saturday, Wednesday 8:00am -  
10:00am & Monday 8:00am - 9:00am  
MEP 261 Class ZA

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Chapter #10  
December XX, 2011

Announcements:

***Dr. Walid's e-mail and Office Hours***

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Office hours for Thermo 01 will be every Sunday and Tuesday from **9:00 – 12:00 am** in ***Dr. Walid's*** office (Room 5-213).

Text book:

***Thermodynamics An Engineering Approach***

Yunus A. Cengel & Michael A. Boles

7<sup>th</sup> Edition, McGraw-Hill Companies,

ISBN-978-0-07-352932-5, 2008

# **Chapter 10**

# **VAPOR AND COMBINED POWER CYCLES**

## *Objectives of CH10: To*

- *Analyze vapor power cycles in which the working fluid is alternately vaporized and condensed.*
- Analyze power generation coupled with process heating called *cogeneration*.
- *Investigate ways to modify the basic Rankine vapor power cycle to increase the cycle thermal efficiency.*
- *Analyze the reheat and regenerative vapor power cycles*

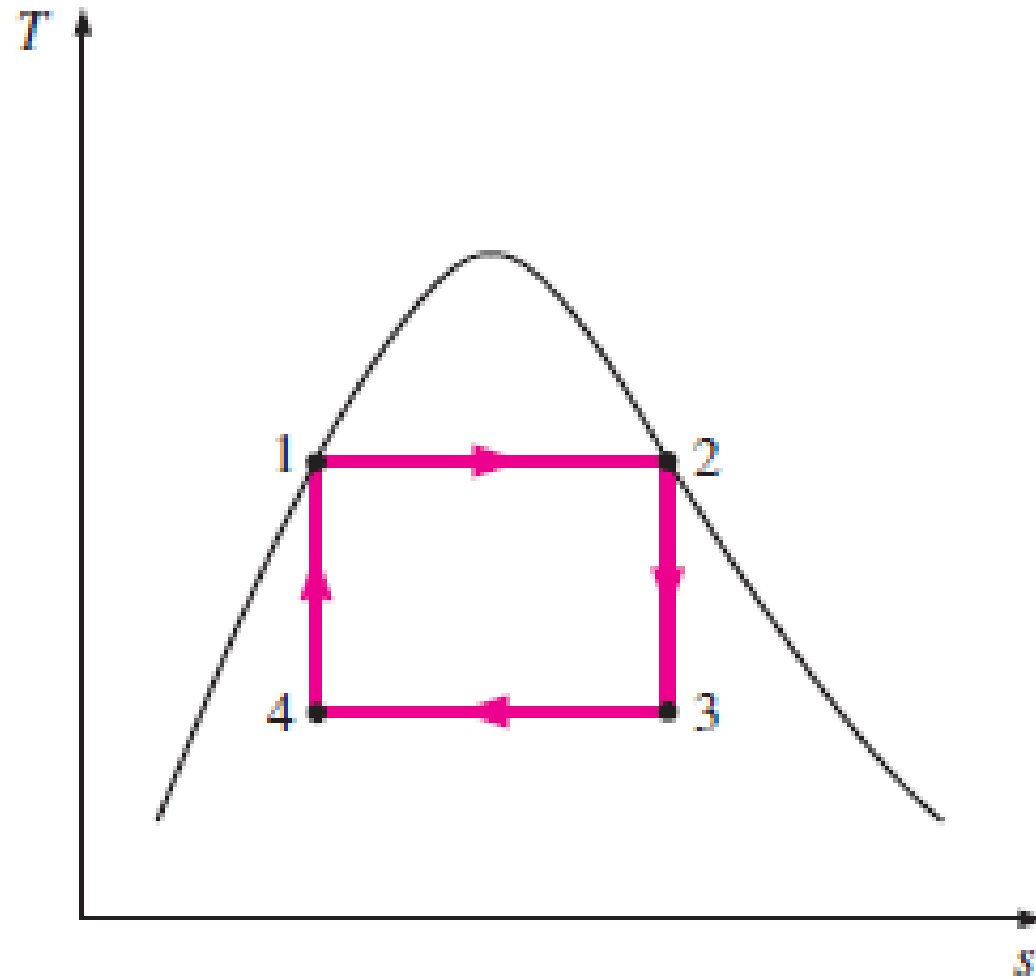
- *Analyze power cycles that consist of two separate cycles known as combined cycles and binary cycles*

# Chapter 10

## VAPOR AND COMBINED POWER CYCLES

### *10-1 ■ THE CARNOT VAPOR CYCLE*

We have mentioned repeatedly that the Carnot cycle is the most efficient cycle operating between two specified temperature limits. Thus it is natural to look at the Carnot cycle first as a prospective ideal cycle for vapor power plants.



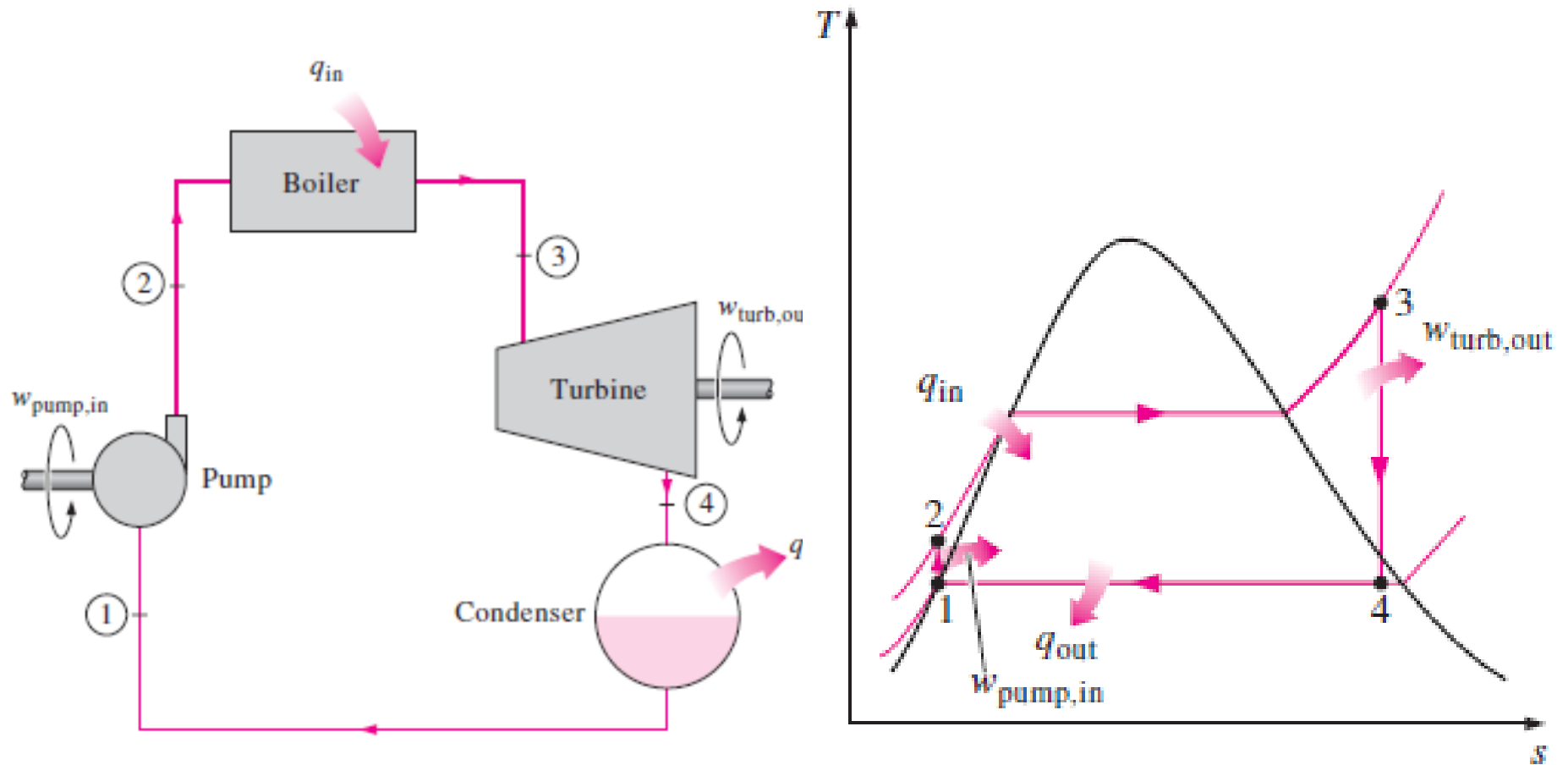
**FIGURE 10–1**

*$T$ - $s$  diagram of a Carnot vapor cycle.*

Consider a steady-flow *Carnot cycle executed within the saturation dome* of a pure substance, as shown in Fig. 10-1. *The fluid is heated reversibly and isothermally* in a boiler (*process 1-2*), *expanded isentropically* in a turbine (*process 2-3*), condensed reversibly and isothermally *in a condenser* (process 3-4), and *compressed isentropically* by a compressor *to the initial state (process 4-1)*.



## 10-2 ■ RANKINE CYCLE: THE IDEAL CYCLE FOR VAPOR POWER CYCLES



**FIGURE 10-2**

The simple ideal Rankine cycle.

**Rankine cycle, is the ideal cycle for vapor power plants. The ideal Rankine cycle does not involve any internal irreversibilities and consists of the following four processes:**

*1-2 Isentropic compression* in a pump.

*2-3 Constant pressure heat addition* in a boiler.


3-4 Isentropic expansion in a turbine.

*4-1 Constant pressure heat rejection* in a condenser.

Water enters the *pump at state 1 as saturated liquid* and is compressed isentropically to the operating pressure of the boiler. The water temperature increases somewhat during this isentropic compression process due to a slight decrease in the specific volume of water. The vertical distance between states 1 and 2 on the *T-s diagram is greatly exaggerated for clarity.* (If water were truly incompressible, would there be a temperature change at all during this process?)




**Water enters the *boiler as a compressed liquid at state 2 and leaves as a superheated vapor at state 3*.** The boiler is basically a large heat exchanger where the heat originating from combustion gases, nuclear reactors, or other sources is transferred to the water essentially at constant pressure. **The boiler, together with the section where the steam is superheated (the superheater), is often called the *steam generator*.**

The **superheated vapor at state 3** enters the turbine, *where it expands isentropically and produces work* by rotating the shaft connected to an electric generator. The *pressure and the temperature* of steam *drop during this process* to the values at state 4, where steam enters the condenser. At this state, **steam is usually a saturated liquid–vapor mixture with a high quality.**



**Steam** is **condensed at constant pressure in** the **condenser**, which is basically a large heat exchanger, *by rejecting heat to a cooling medium such as a lake, a river, or the atmosphere.* **Steam** leaves the **condenser as saturated liquid** and enters the pump, *completing the cycle.*

**In areas where water is precious**, the power plants are cooled by air instead of water. This method of cooling, which is also used in car engines, is *called dry cooling*. Several power plants in the world, including some in the United States, use dry cooling to conserve water.

Remembering that the **area under** the process **curve** on a  ***$T$ - $s$  diagram*** represents the heat transfer for internally reversible processes, we see that the **area under process curve 2-3**  represents the heat transferred to the water in the boiler and the **area under** the process **curve 4-1**  represents the heat rejected in the condenser. The **difference between these two**  (the *area enclosed by the cycle curve*) is the net work produced during the cycle.

## Energy Analysis of the Ideal Rankine Cycle

All four components associated with the Rankine cycle (the pump, boiler, turbine, and condenser) are steady-flow devices, and *thus* **all four processes** that make up the Rankine cycle can be *analyzed as steady-flow processes*.

The **kinetic and potential energy changes** of the **steam** are usually small *relative to* the *work and heat transfer* terms and are **therefore** *usually neglected*.



Then the *steady-flow energy equation per unit mass of steam* reduces to

$$(q_{in} - q_{out}) + (W_{in} - W_{out}) = h_e - h_i \text{ (kJ/kg)} \quad (10-1)$$

The **boiler and** the **condenser** do not involve any work, and the **pump and** the **turbine** are assumed to be isentropic. Then the conservation of energy relation for each device can be expressed as follows:

$$\text{Pump } (q = 0): \quad w_{\text{pump,in}} = h_2 - h_1 \quad (10-2)$$

or

$$w_{\text{pump,in}} = v(P_2 - P_1) \quad (10-3)$$

where,

$$h_1 = h_f @ P_1 \quad \text{and} \quad v \cong v_1 = v_f @ P_1 \quad (10-4)$$

Boiler ( $w = 0$ ):

$$q_{\text{in}} = h_3 - h_2 \quad (10-5)$$

Turbine ( $q = 0$ ):

$$w_{\text{turb,out}} = h_3 - h_4 \quad (10-6)$$

*Condenser ( $w = 0$ ):*

$$q_{\text{out}} = h_4 - h_1 \quad (10-7)$$

The *thermal efficiency* of the Rankine cycle is determined from

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} \quad (10-8)$$

where,

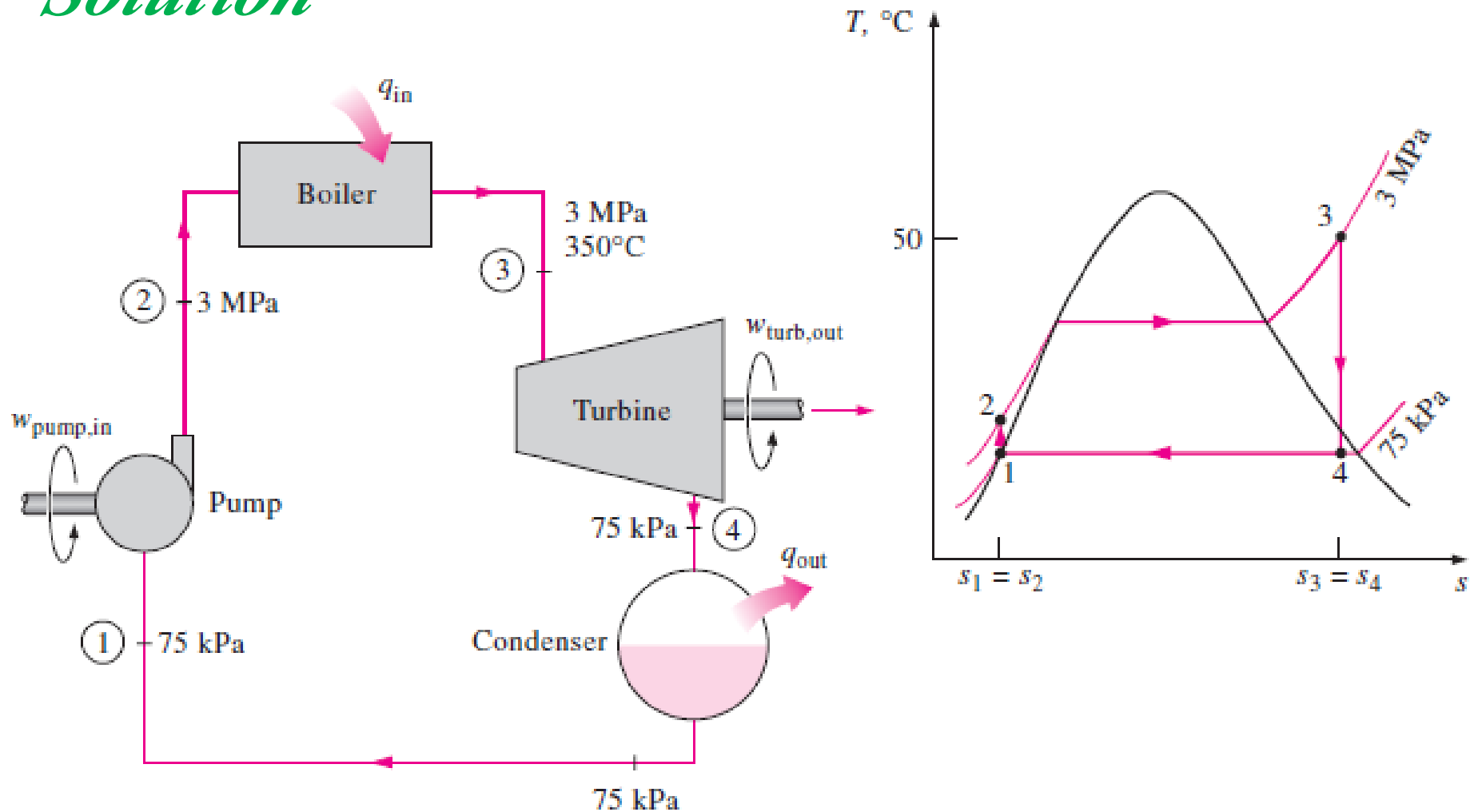
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = w_{\text{turb,out}} - w_{\text{pump,in}}$$

## *EXAMPLE 10–1 The Simple Ideal Rankine Cycle*

Consider a steam power plant operating on the simple ideal Rankine cycle.

Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle.

## *Solution*



**FIGURE 10–3**

Schematic and  $T-s$  diagram for Example 10–1.

### *State 1:*

$$\left. \begin{array}{l} p_1 = 75 \text{ kPa} \\ \text{Sat. liquid} \end{array} \right\} \begin{array}{l} h_1 = h_f @ 75 \text{ kPa} = 384.44 \text{ kJ/kg} \\ v_1 = v_f @ 75 \text{ kPa} = 0.001037 \text{ m}^3/\text{kg} \\ T_1 = T_{\text{saturation}} @ 75 \text{ kPa} = 91.76 \text{ }^\circ\text{C} \end{array}$$

### *State 2:*

$$\left. \begin{array}{l} p_2 = 3 \text{ MPa} \\ s_2 = s_1 \end{array} \right\}$$

$$\begin{aligned} w_{\text{pump},in} &= v_1 (p_2 - p_1) = 0.001037 \text{ m}^3/\text{kg} [(3000-75) \text{ kPa}] \\ &= 3.03 \text{ kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pump},in} = (384.44 + 3.03) \text{ kJ/kg} = 387.47 \text{ kJ/kg}$$

### *State 3:*

$$\left. \begin{array}{l} p_3 = 3 \text{ MPa} \\ T_3 = 350 \text{ }^\circ\text{C} \end{array} \right\} \begin{array}{l} h_3 = 3116.1 \text{ kJ/kg} \\ s_3 = 6.745 \text{ kJ/kg.K} \end{array}$$

### *State 4:*

$$p_4 = 75 \text{ kPa (sat. Mixture)}$$

$$s_f = 1.2132 \text{ kJ/kg.K}, s_g = 7.4558 \text{ kJ/kg.K}$$

$$h_f = 388.44 \text{ kJ/kg}, h_g = 2662.4 \text{ kJ/kg}$$

$$s_4 = s_3$$

}

$$x_4 = (s_4 - s_f)/(s_g - s_f) = (6.745 - 1.2132)/(7.4558 - 1.2132) = 0.8861$$

$$h_4 = h_f + x_4(h_g - h_f) = 384.44 + 0.8861(2662.4 - 384.44) = 2403.0 \text{ kJ/kg}$$

$$\text{Thus, } q_{in} = h_3 - h_2 = 3116.1 - 387.47 \text{ kJ/kg} = 2728.6 \text{ kJ/kg}$$

$$\text{and, } q_{out} = h_4 - h_1 = 2403.0 - 384.44 \text{ kJ/kg} = 2018.6 \text{ kJ/kg}$$

$$\text{and, } \eta_{th} = 1 - (q_{out}/q_{in}) = 1 - (2018.6 / 2728.6) = 0.260 \text{ or } 26 \%$$



The thermal efficiency;  $\eta_{th}$  can also be determined from

$$W_{turb,out} = h_3 - h_4 = (3116.1 - 2403.0) \text{ kJ/kg} = 713.1 \text{ kJ/kg}$$

$$W_{net} = W_{turb,out} - W_{pump,in} = (713.1 - 3.03) \text{ kJ/kg} = 710.07 \text{ kJ/kg}$$

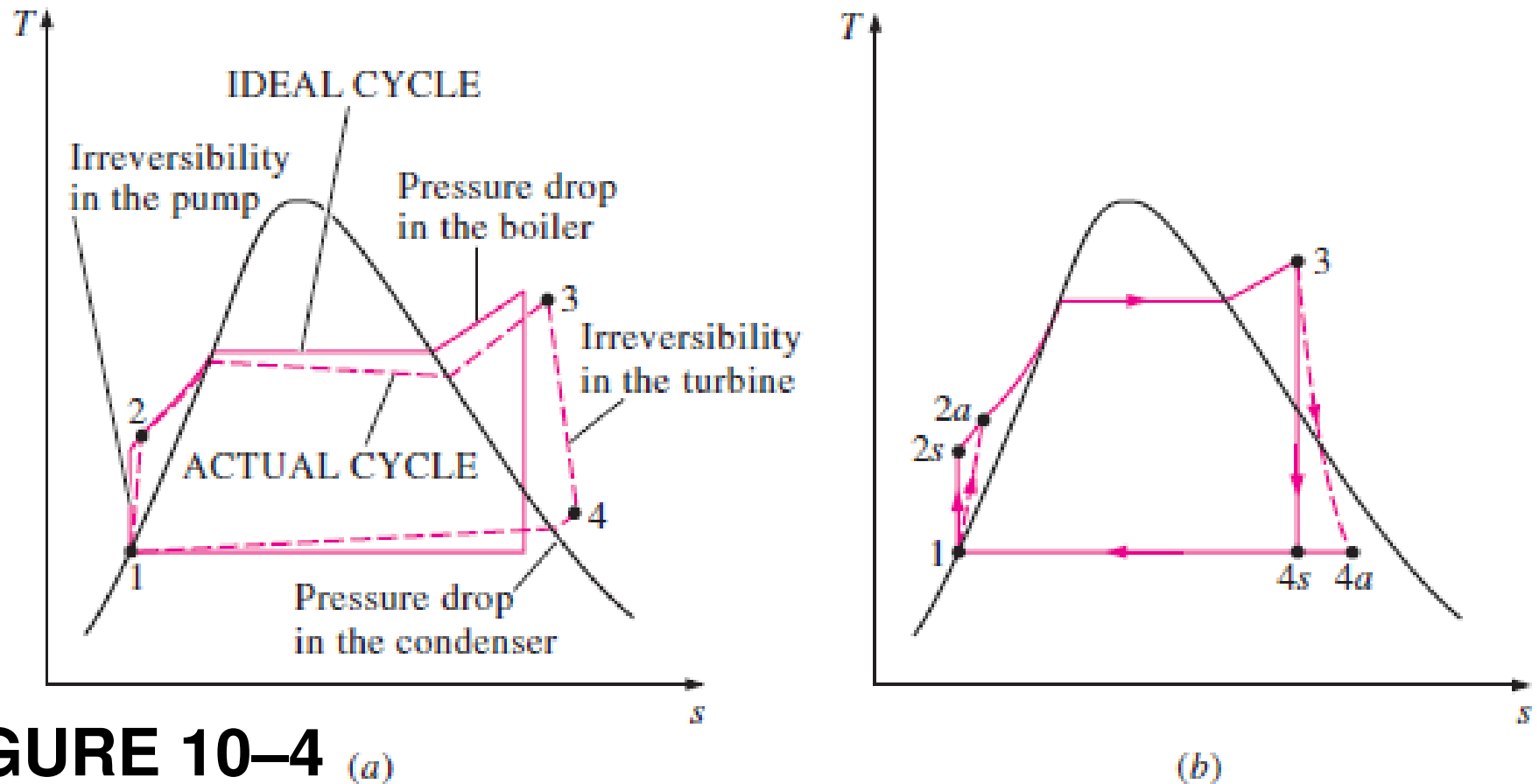
$$\eta_{th} = W_{net} / q_{in} = 710.07 / 2728.6 = 0.260 \text{ or } 26 \%$$

The Carnot thermal efficiency;  $\eta_{th, Carnot}$  operating between the two temperature limits,  $T_{max} = T_3 = 350^\circ\text{C} = 350 + 273 = 623 \text{ K}$  &  $T_{min} = T_1 = 91.76^\circ\text{C} = 91.76 + 273 = 364.76 \text{ K}$  &

*But,  $\eta_{th, Carnot} = 1 - (T_{min} / T_{max}) = 1 - (364.76 / 623) = 0.415$  or 41.5 %*

It is clear that the thermal efficiency;  $\eta_{th}$  of simple ideal Rankine cycle (= 26 %) is less than the Carnot thermal efficiency;  $\eta_{th, Carnot}$  operating between the two temperature limits, (= 41.5 %)

## 10-3 ■ DEVIATION OF ACTUAL VAPOR POWER CYCLES FROM IDEALIZED ONES



**FIGURE 10-4** (a)

(a) Deviation of actual vapor power cycle from the ideal Rankine cycle. (b) The effect of pump and turbine irreversibilities on the ideal Rankine cycle.

$$\eta_P = \frac{w_s}{w_a} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad (10-10)$$

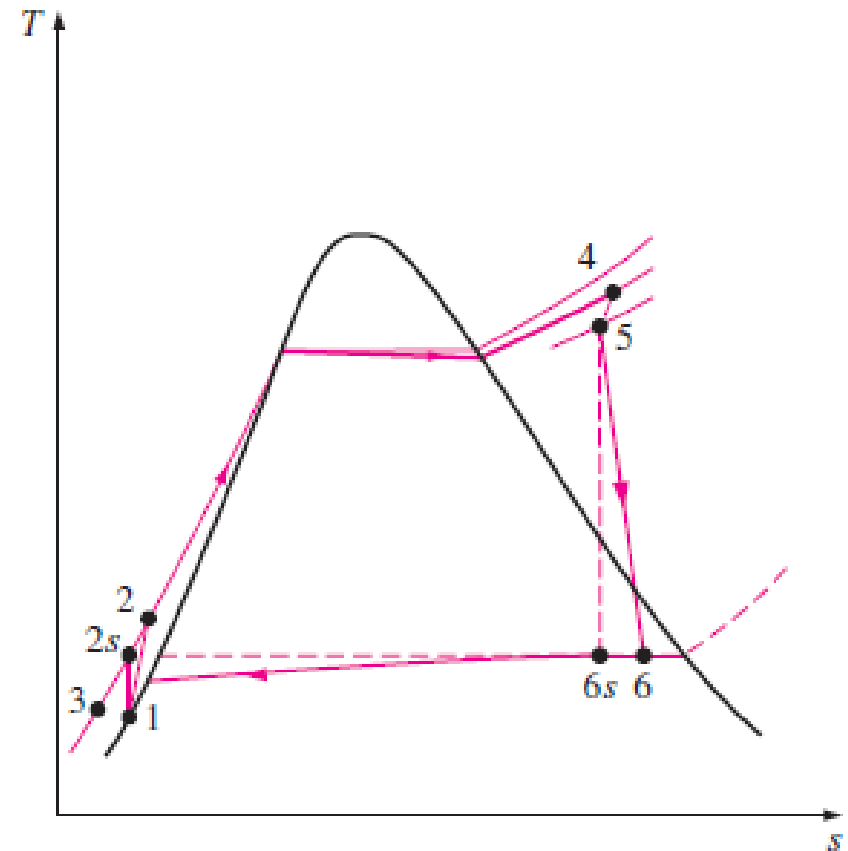
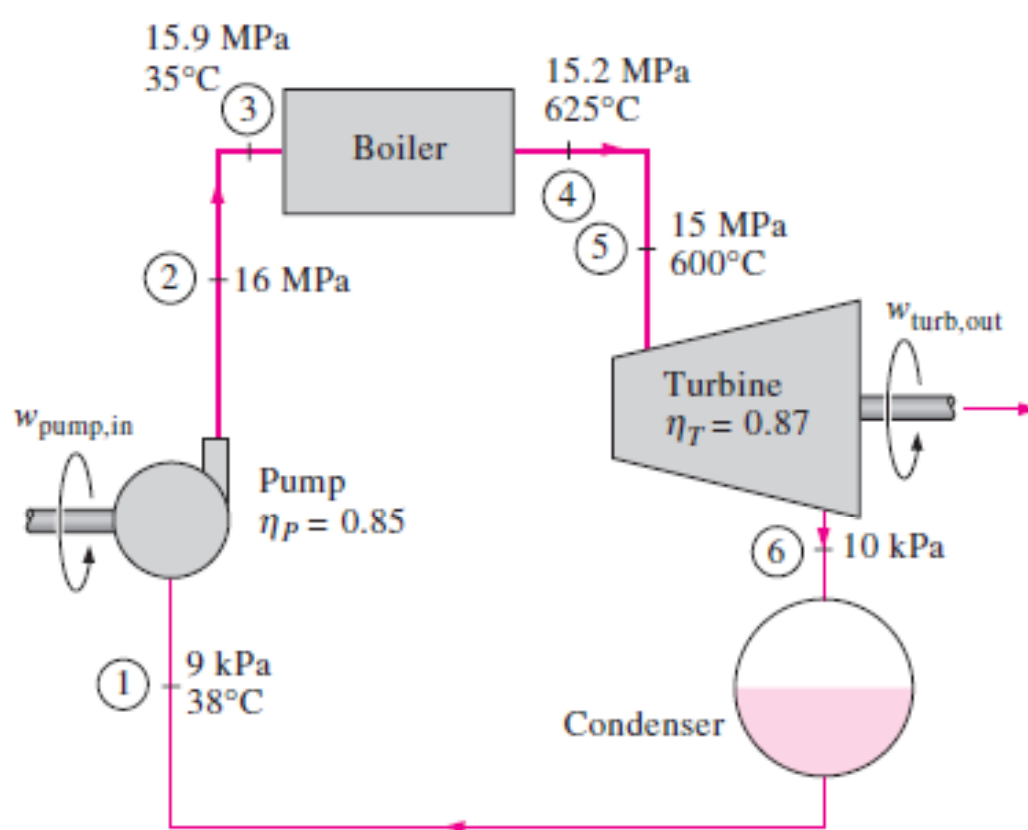
and

$$\eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \quad (10-11)$$

## *EXAMPLE 10–2 An Actual Steam Power Cycle*

A steam power plant operates on the cycle shown in Fig. 10–5. If the isentropic efficiency of the turbine is 87 percent and the isentropic efficiency of the pump is 85 percent, determine (a) the thermal efficiency of the cycle and (b) the net power output of the plant for a mass flow rate of 15 kg/s.

## *Solution*



**FIGURE 10–5**

Schematic and  $T-s$  diagram for Example 10–2.

### ***Solution:***

**A steam power cycle with specified turbine and pump efficiencies is considered. The thermal efficiency and the net power output are to be determined.**

***Assumptions :*** **1) Steady operating conditions exist.**  
**2 )Kinetic and potential** energy changes are negligible.

*Analysis:* The schematic of the power plant and the  $T$ - $s$  diagram of the cycle are shown in Fig. 10–5. The temperatures and pressures of steam at various points are also indicated on the figure. We note that the power plant involves steady-flow components and operates on the Rankine cycle, but the imperfections at various components are accounted for.

(a) *The thermal efficiency of a cycle is the ratio of the net work output to the heat input, and it is determined as follows:*



## *Pump work*

$$\begin{aligned}w_{\text{pump,in}} &= \frac{w_{s,\text{pump,in}}}{\eta_p} = \frac{v_1(P_2 - P_1)}{\eta_p} \\&= \frac{(0.001009 \text{ m}^3/\text{kg})[(16,000 - 9) \text{ kPa}]}{0.85} \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\&= 19.0 \text{ kJ/kg}\end{aligned}$$

*Turbine work output:*

$$\begin{aligned}w_{\text{turb,out}} &= \eta_T w_{s,\text{turb,out}} \\&= \eta_T (h_5 - h_{6s}) = 0.87 (3583.1 - 2115.3) \text{ kJ/kg} \\&= 1277.0 \text{ kJ/kg}\end{aligned}$$

*Boiler heat input:*  $q_{\text{in}} = h_4 - h_3 = (3647.6 - 160.1) \text{ kJ/kg} = 3487.5 \text{ kJ/kg}$

Thus,

$$w_{\text{net}} = w_{\text{turb,out}} - w_{\text{pump,in}} = (1277.0 - 19.0) \text{ kJ/kg} = 1258.0 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1258.0 \text{ kJ/kg}}{3487.5 \text{ kJ/kg}} = \mathbf{0.361 \text{ or } 36.1\%}$$

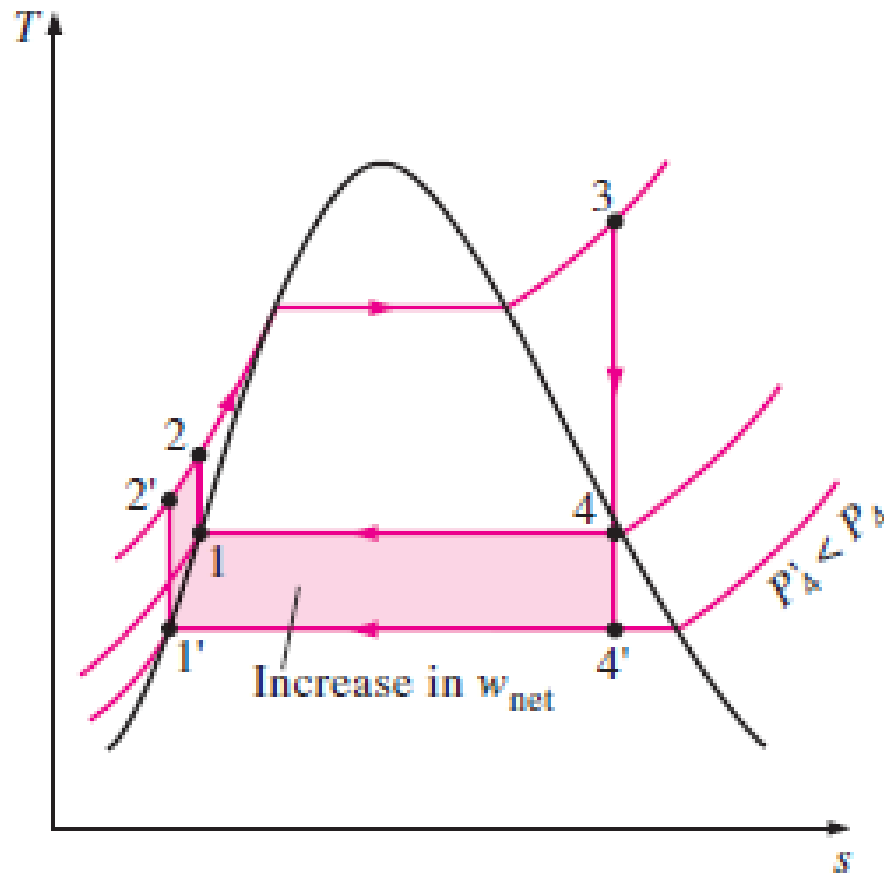
(b) The power produced by this power plant is

$$\dot{W}_{\text{net}} = \dot{m}(w_{\text{net}}) = (15 \text{ kg/s})(1258.0 \text{ kJ/kg}) = \mathbf{18.9 \text{ MW}}$$

**Discussion** Without the irreversibilities, the thermal efficiency of this cycle would be 43.0 percent (see Example 10–3c).

## 10-4 ■ HOW CAN WE INCREASE THE EFFICIENCY OF THE RANKINE CYCLE?

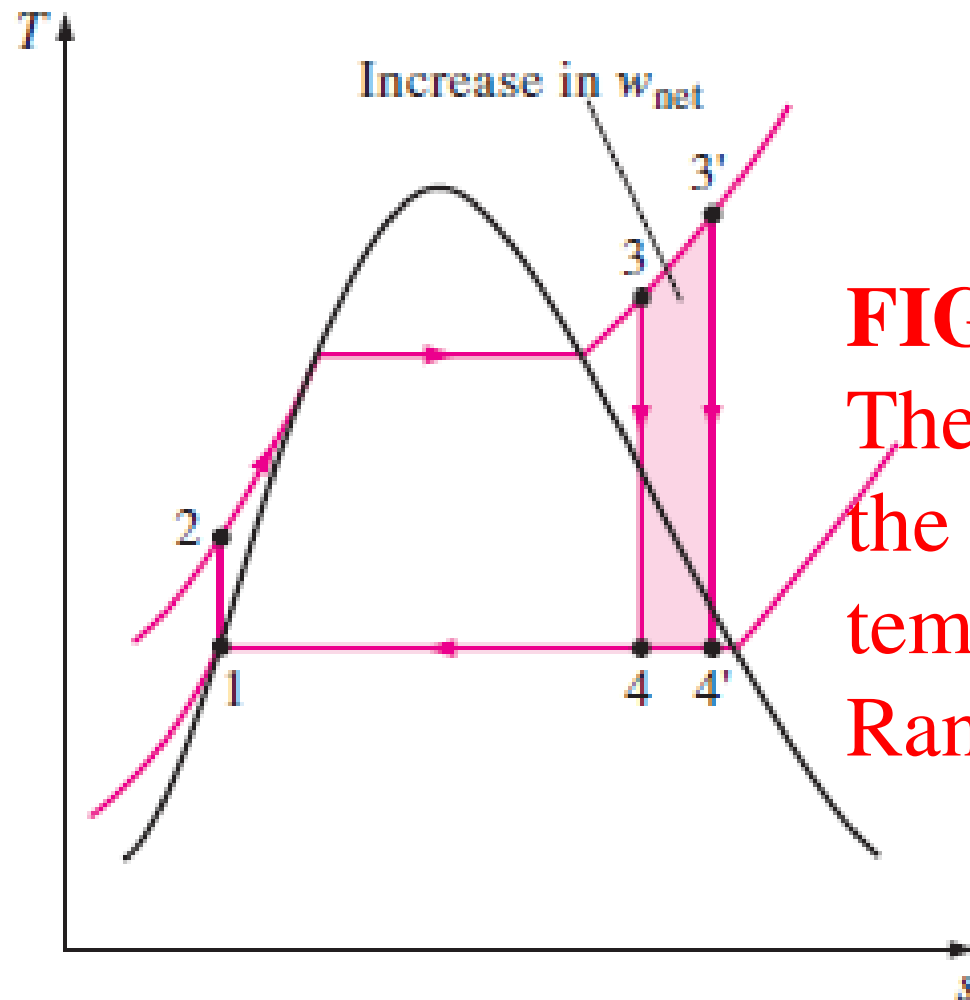
Lowering the Condenser Pressure (Lowers  $T_{low,avg}$ )



**FIGURE 10-6**

The effect of lowering the condenser pressure on the ideal Rankine cycle.

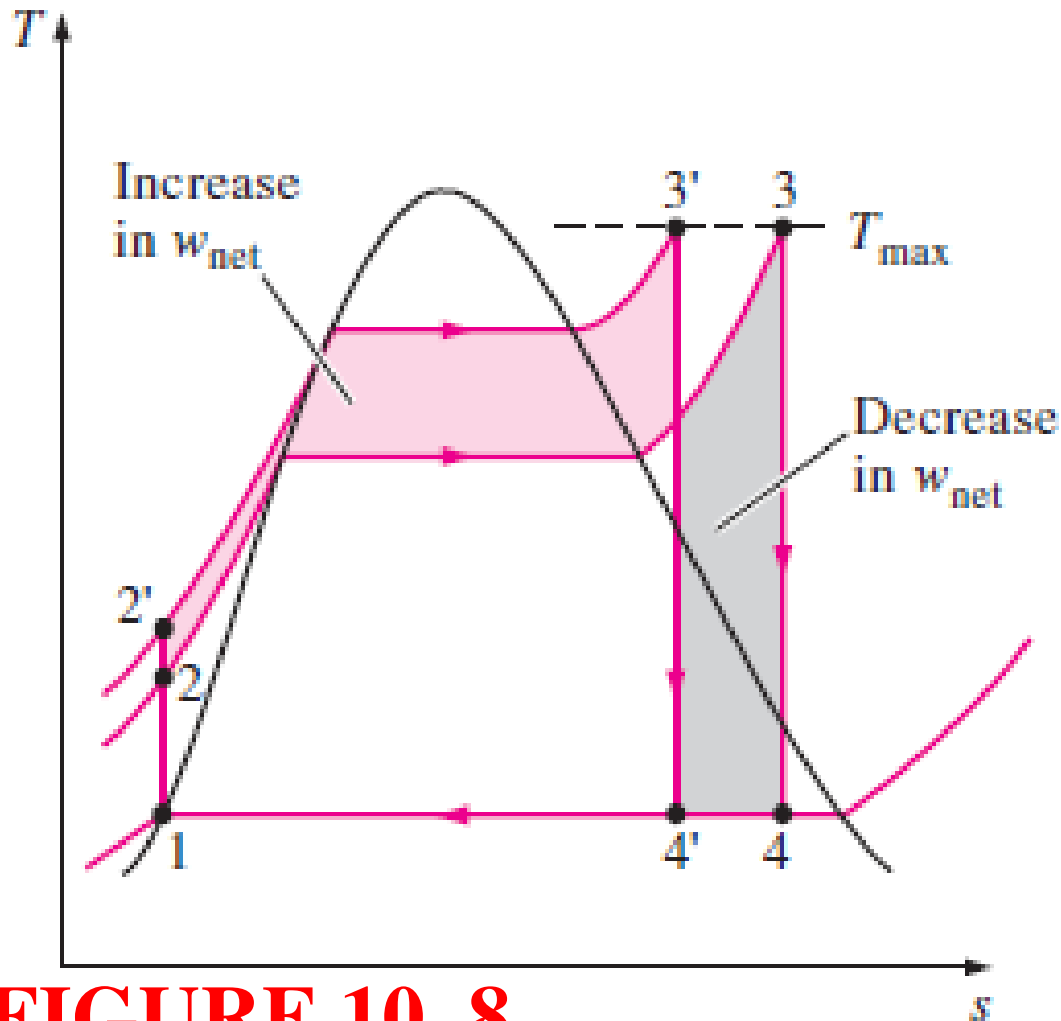
# Superheating the Steam to High Temperatures (Increases $T_{high,avg}$ )



**FIGURE 10–7**

The effect of superheating the steam to higher temperatures on the ideal Rankine cycle.

## Increasing the Boiler Pressure (Increases $T_{high,avg}$ )



**FIGURE 10–8**

The effect of increasing the boiler pressure on the ideal Rankine cycle.

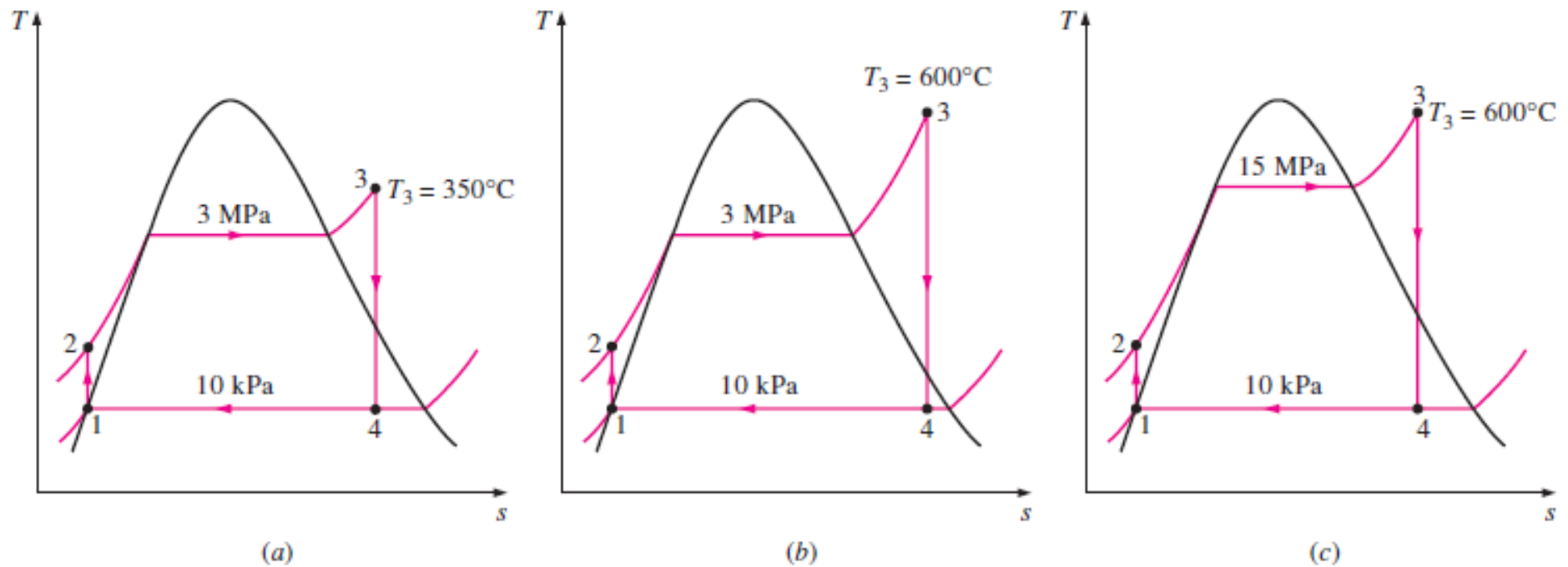
### *EXAMPLE 10–3 Effect of Boiler Pressure and Temperature on Efficiency*

A steam power plant operates on the cycle shown in Fig. 10–5. If the isentropic efficiency of the turbine is 87 percent and the isentropic efficiency of the pump is 85 percent, determine (a) the thermal efficiency of the cycle and (b) the net power output of the plant for a mass flow rate of 15 kg/s.

***Solution:*** A steam power plant operating on the ideal Rankine cycle is considered.

The effects of superheating the steam to a higher temperature and raising the boiler pressure on thermal efficiency are to be investigated.

***Analysis:*** The *T-s* diagrams of the cycle for all three cases are given in Fig. 10–10.



**FIGURE 10–10**

*$T$ - $s$  diagrams of the three cycles discussed in Example 10–3.*



(a) This is the steam power plant discussed in Example 10–1, except that the condenser pressure is lowered to 10 kPa. The thermal efficiency is determined in a similar manner:

$$\text{State 1: } \left. \begin{array}{l} P_1 = 10 \text{ kPa} \\ \text{Sat. liquid} \end{array} \right\} \begin{array}{l} h_1 = h_f @ 10 \text{ kPa} = 191.81 \text{ kJ/kg} \\ v_1 = v_f @ 10 \text{ kPa} = 0.00101 \text{ m}^3/\text{kg} \end{array}$$

$$\text{State 2: } \begin{array}{l} P_2 = 3 \text{ MPa} \\ s_2 = s_1 \end{array}$$

$$\begin{aligned} w_{\text{pump,in}} &= v_1(P_2 - P_1) = (0.00101 \text{ m}^3/\text{kg})[(3000 - 10) \text{ kPa}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= 3.02 \text{ kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pump,in}} = (191.81 + 3.02) \text{ kJ/kg} = 194.83 \text{ kJ/kg}$$

$$\text{State 3: } \left. \begin{array}{l} P_3 = 3 \text{ MPa} \\ T_3 = 350^\circ\text{C} \end{array} \right\} \begin{array}{l} h_3 = 3116.1 \text{ kJ/kg} \\ s_3 = 6.7450 \text{ kJ/kg} \cdot \text{K} \end{array}$$

*State 4:*

$$p_4 = 10 \text{ kPa (sat. Mixture)}$$

$$s_f = 0.6492 \text{ kJ/kg.K}, s_g = 8.1488 \text{ kJ/kg.K}$$

$$h_f = 191.81 \text{ kJ/kg}, h_g = 2583.9 \text{ kJ/kg}$$

$$s_4 = s_3$$

$$x_4 = (s_4 - s_f)/(s_g - s_f) = (6.745 - 0.6492)/(8.1488 - 0.6492) = 0.8128$$

$$h_4 = h_f + x_4(h_g - h_f) = 191.81 + 0.8128(2583.9 - 191.81) = 2136.1 \text{ kJ/kg}$$

$$\text{Thus, } q_{in} = h_3 - h_2 = 3116.1 - 194.83 \text{ kJ/kg} = 2921.3 \text{ kJ/kg}$$

*and,  $q_{out} = h_4 - h_1 = 2136.1 - 191.81 \text{ kJ/kg} = 1994.3 \text{ kJ/kg}$*

*and,  $\eta_{th} = 1 - (q_{out}/q_{in}) = 1 - (1994.3 / 2921.3) = 0.334$*   
or 33.4 %

*(b) States 1 and 2 remain the same in this case, and the enthalpies at state 3 (3 MPa and 600°C) and state 4 (10 kPa and  $s_4 = s_3$ ) are determined to be*

$$h_3 = 3682.8 \text{ kJ/kg}$$

$$h_4 = 2380.3 \text{ kJ/kg} \quad (x_4 = 0.915)$$

Thus,

$$q_{\text{in}} = h_3 - h_2 = 3682.8 - 194.83 = 3488.0 \text{ kJ/kg}$$

$$q_{\text{out}} = h_4 - h_1 = 2380.3 - 191.81 = 2188.5 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2188.5 \text{ kJ/kg}}{3488.0 \text{ kJ/kg}} = \mathbf{0.373 \text{ or } 37.3\%}$$

Therefore, the thermal efficiency increases from 33.4 to 37.3 percent as a result of superheating the steam from 350 to 600°C. At the same time, the quality of the steam increases from 81.3 to 91.5 percent (in other words, the moisture content decreases from 18.7 to 8.5 percent).

(c) State 1 remains the same in this case, but the other states change. The enthalpies at state 2 (15 MPa and  $s_2 = s_1$ ), state 3 (15 MPa and 600°C), and state 4 (10 kPa and  $s_4 = s_3$ ) are determined in a similar manner to be

$$h_2 = 206.95 \text{ kJ/kg}$$

$$h_3 = 3583.1 \text{ kJ/kg}$$

$$h_4 = 2115.3 \text{ kJ/kg} \quad (x_4 = 0.804)$$

Thus,

$$q_{\text{in}} = h_3 - h_2 = 3583.1 - 206.95 = 3376.2 \text{ kJ/kg}$$

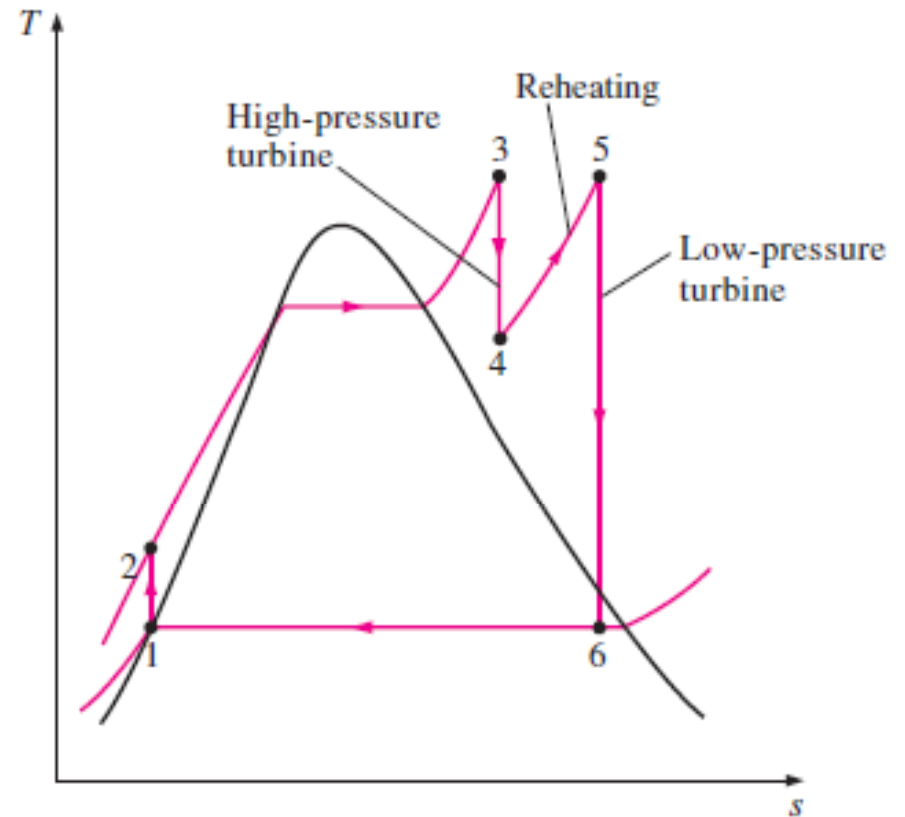
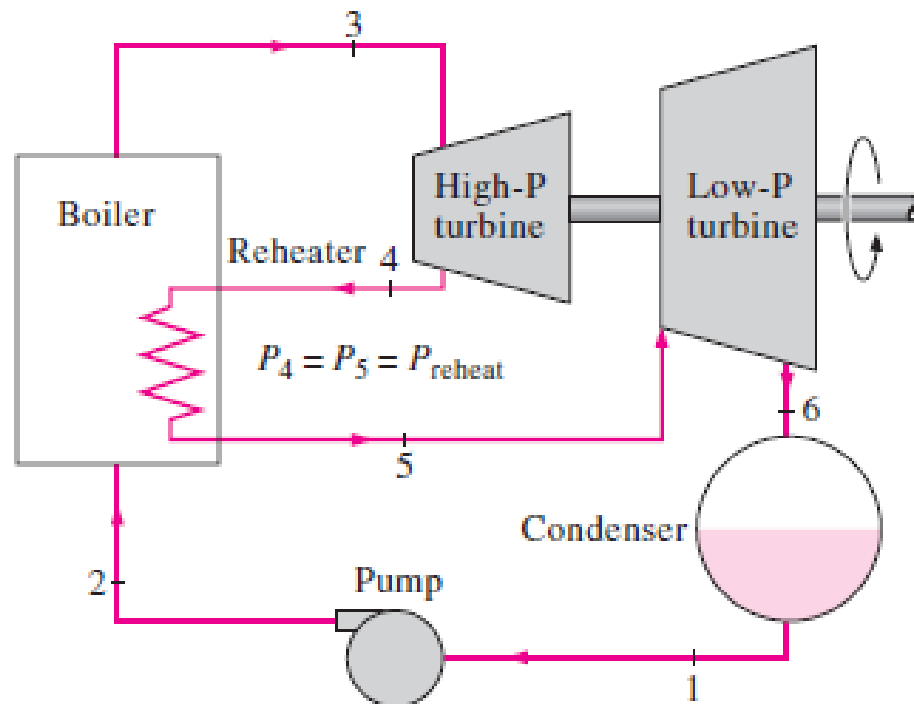
$$q_{\text{out}} = h_4 - h_1 = 2115.3 - 191.81 = 1923.5 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1923.5 \text{ kJ/kg}}{3376.2 \text{ kJ/kg}} = \mathbf{0.430 \text{ or } 43.0\%}$$

***Discussion:*** *The thermal efficiency increases from 37.3 to 43.0 percent as a result of raising the boiler pressure from 3 to 15 MPa while maintaining the turbine inlet temperature at 600°C. At the same time, however, the quality of the steam decreases from 91.5 to 80.4 percent (in other words, the moisture content increases from 8.5 to 19.6 percent).*

# 10-5 ■ THE IDEAL REHEAT RANKINE CYCLE



**FIGURE 10-11**

The ideal reheat Rankine cycle.



The total heat input and the total turbine work output for a reheat cycle become

$$q_{\text{in}} = q_{\text{primary}} + q_{\text{reheat}} = (h_3 - h_2) + (h_5 - h_4)$$

and

(10-12)

$$w_{\text{turb,out}} = w_{\text{turb,I}} + w_{\text{turb,II}} = (h_3 - h_4) + (h_5 - h_6)$$

(10-13)

### ***EXAMPLE 10–4 The Ideal Reheat Rankine Cycle***

Consider a steam power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10.4 percent, determine *(a) the pressure at which the steam should be reheated and (b) the thermal efficiency of the cycle.* Assume the steam is reheated to the inlet temperature of the high-pressure turbine.

### *Solution:*

**A steam power plant operating on the ideal reheat Rankine cycle is considered. For a specified moisture content at the turbine exit, the reheat pressure and the thermal efficiency are to be determined.**

***Assumptions*** *1) Steady operating conditions exist.*  
*2) Kinetic and potential energy changes are negligible.*

***Analysis*** *The schematic of the power plant and the T-s diagram of the cycle are shown in Fig. 10–13. We note that the power plant operates on the ideal reheat Rankine cycle. Therefore, the pump and*

the turbines are isentropic, there are no pressure drops in the boiler and condenser, and steam leaves the condenser and enters the pump as saturated liquid at the condenser pressure.

*(a) The reheat pressure is determined from the requirement that the entropies at states 5 and 6 be the same:*

*State 6:*  $P_6 = 10 \text{ kPa}$

$$x_6 = 0.896 \quad (\text{sat. mixture})$$

$$s_6 = s_f + x_6 s_{fg} = 0.6492 + 0.896(7.4996) = 7.3688 \text{ kJ/kg} \cdot \text{K}$$

Also,

$$h_6 = h_f + x_6 h_{fg} = 191.81 + 0.896(2392.1) = 2335.1 \text{ kJ/kg}$$

Thus,

$$\begin{array}{lcl} \text{State 5:} & \left. \begin{array}{l} T_5 = 600^\circ\text{C} \\ s_5 = s_6 \end{array} \right\} & \begin{array}{l} P_5 = \mathbf{4.0 \text{ MPa}} \\ h_5 = 3674.9 \text{ kJ/kg} \end{array} \end{array}$$

Therefore, steam should be reheated at a pressure of 4 MPa or lower to prevent a moisture content above 10.4 percent.

*(b) To determine the thermal efficiency, we need to know the enthalpies at all other states:*

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 10 \text{ kPa} \\ \text{Sat. liquid} \end{array} \right\} \quad \begin{array}{l} h_1 = h_{f@ 10 \text{ kPa}} = 191.81 \text{ kJ/kg} \\ v_1 = v_{f@ 10 \text{ kPa}} = 0.00101 \text{ m}^3/\text{kg} \end{array}$$

$$\text{State 2:} \quad P_2 = 15 \text{ MPa}$$

$$s_2 = s_1$$

$$\begin{aligned} w_{\text{pump,in}} &= v_1 (P_2 - P_1) = (0.00101 \text{ m}^3/\text{kg}) \\ &\quad \times [(15,000 - 10) \text{ kPa}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= 15.14 \text{ kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pump,in}} = (191.81 + 15.14) \text{ kJ/kg} = 206.95 \text{ kJ/kg}$$

$$\text{State 3:} \quad \left. \begin{array}{l} P_3 = 15 \text{ MPa} \\ T_3 = 600^\circ\text{C} \end{array} \right\} \quad \begin{array}{l} h_3 = 3583.1 \text{ kJ/kg} \\ s_3 = 6.6796 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{State 4:} \quad \left. \begin{array}{l} P_4 = 4 \text{ MPa} \\ s_4 = s_3 \end{array} \right\} \quad \begin{array}{l} h_4 = 3155.0 \text{ kJ/kg} \\ (T_4 = 375.5^\circ\text{C}) \end{array}$$

Thus

$$\begin{aligned}q_{\text{in}} &= (h_3 - h_2) + (h_5 - h_4) \\&= (3583.1 - 206.95) \text{ kJ/kg} + (3674.9 - 3155.0) \text{ kJ/kg} \\&= 3896.1 \text{ kJ/kg} \\q_{\text{out}} &= h_6 - h_1 = (2335.1 - 191.81) \text{ kJ/kg} \\&= 2143.3 \text{ kJ/kg}\end{aligned}$$

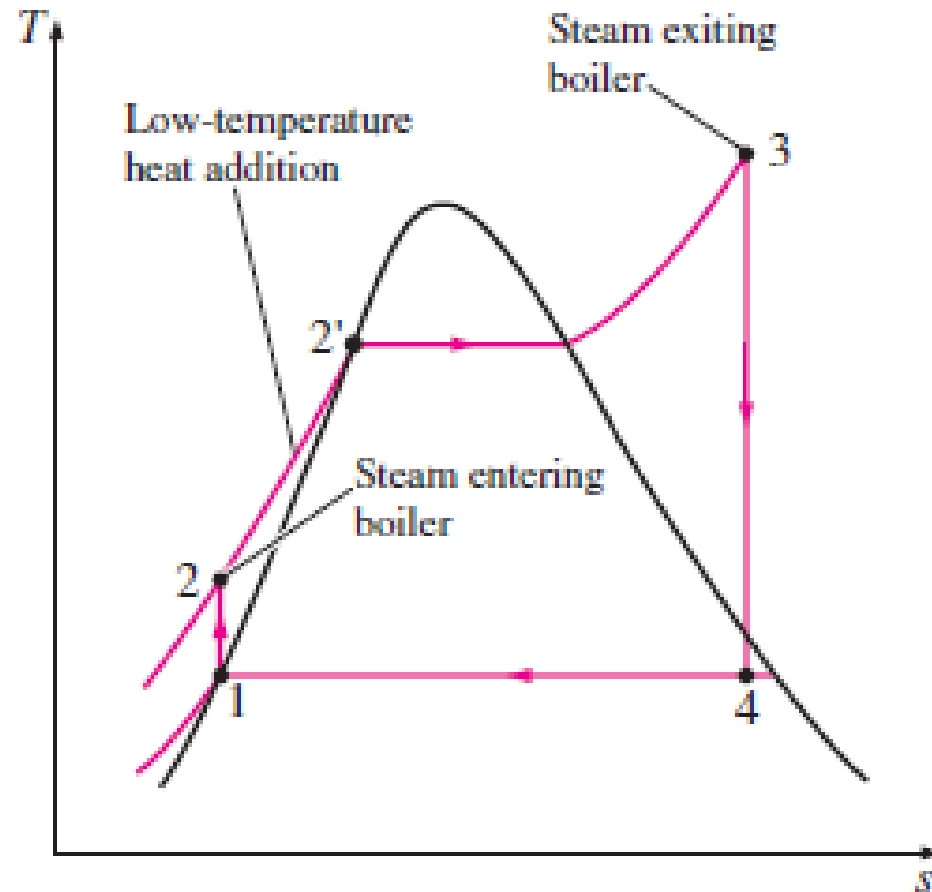
and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{2143.3 \text{ kJ/kg}}{3896.1 \text{ kJ/kg}} = \mathbf{0.450 \text{ or } 45.0\%}$$

*Discussion* This problem was solved in Example 10–3c for the same pressure and temperature limits but without the reheat process. A comparison of the two results reveals that reheating reduces the moisture content from 19.6 to 10.4 percent while increasing the thermal efficiency from 43.0 to 45.0 percent.



## 10–6 ■ THE IDEAL REGENERATIVE RANKINE CYCLE



**FIGURE 10–14**

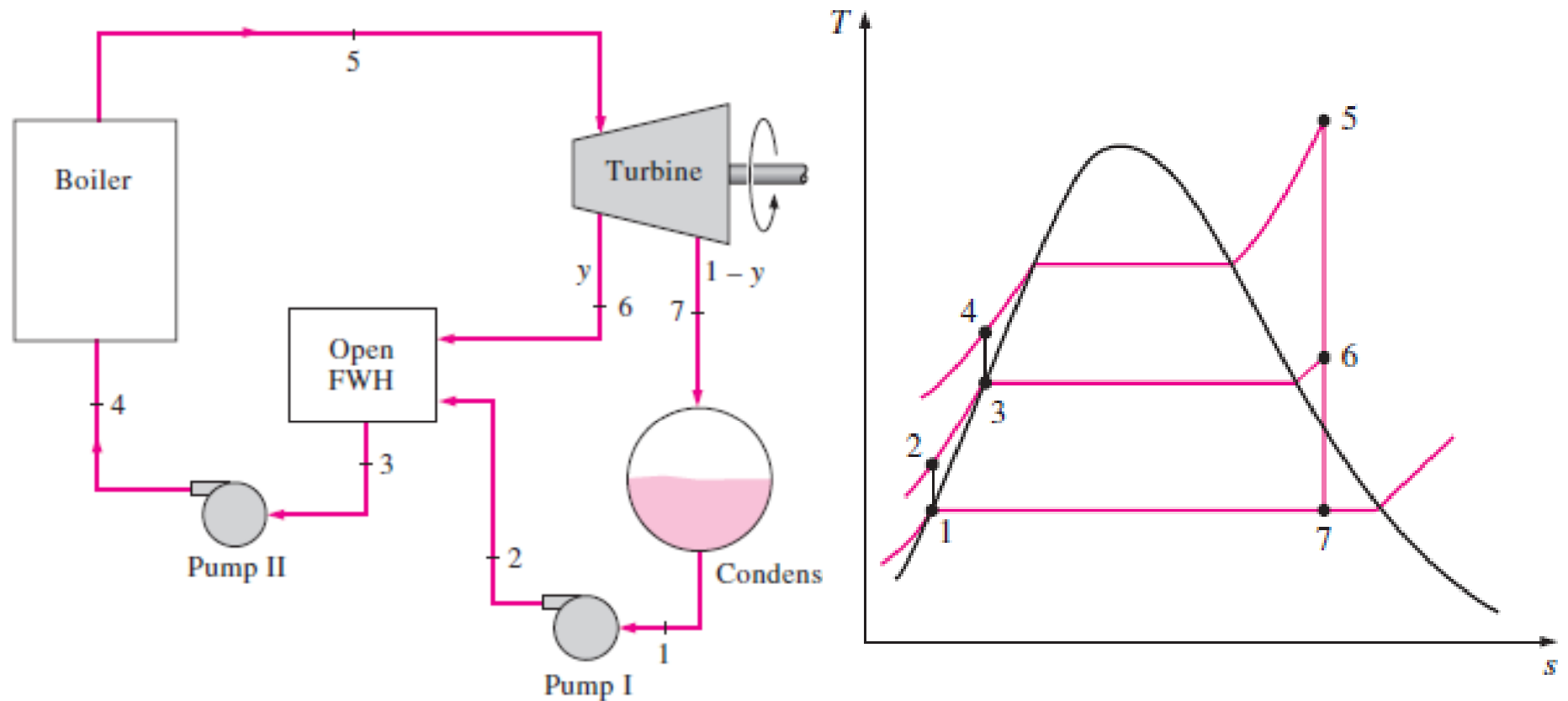
The first part of the heat-addition process in the boiler takes place at relatively low temperatures.

To remedy **this shortcoming**, we look for ways to raise the temperature of the liquid leaving the pump (called the feedwater) *before it enters the boiler*.

A feedwater heater is basically a heat exchanger where heat is transferred from the steam to the feedwater either by mixing the two fluid streams (*open feedwater heaters*) or without mixing them (*closed feedwater heaters*).

Regeneration with both types of feedwater heaters is discussed below.

## *Open feedwater heaters*



**FIGURE 10–15**

The ideal regenerative Rankine cycle with an open feedwater heater.

In the analysis of steam power plants, it is more convenient to work with quantities expressed per unit mass of the steam flowing through the boiler.

For each 1 kg of steam leaving the boiler,  $y$  kg *expands partially in the turbine* and is extracted at state 6. The remaining  $(1 - y)$  kg *expands completely to the condenser pressure*.

Therefore, the mass flow rates are different in different components. If the mass flow rate through the boiler is  $\dot{m}$  , e.g., it is  $(1 - y)\dot{m}$  through the condenser. *This aspect of the regenerative Rankine cycle should be considered in the analysis of the cycle as well as in the interpretation of the areas on the  $T$ - $s$  diagram.*

*In light of Fig. 10–15, the heat and work interactions of a regenerative Rankine cycle with one feedwater heater can be expressed per unit mass of steam flowing through the boiler as follows:*

$$q_{in} = h_5 - h_4 \quad (10-14)$$

$$q_{out} = (1 - y) (h_7 - h_1) \quad (10-15)$$

$$W_{turb,out} = (h_5 - h_6) + (1 - y_2) (h_6 - h_7) \quad (10-16)$$

$$W_{pump,in} = (1 - y) W_{pump\ I,in} + W_{pump\ II,in} \quad (10-17)$$

where,  $y = \dot{m}_6/\dot{m}_5$  (fraction of steam extracted)

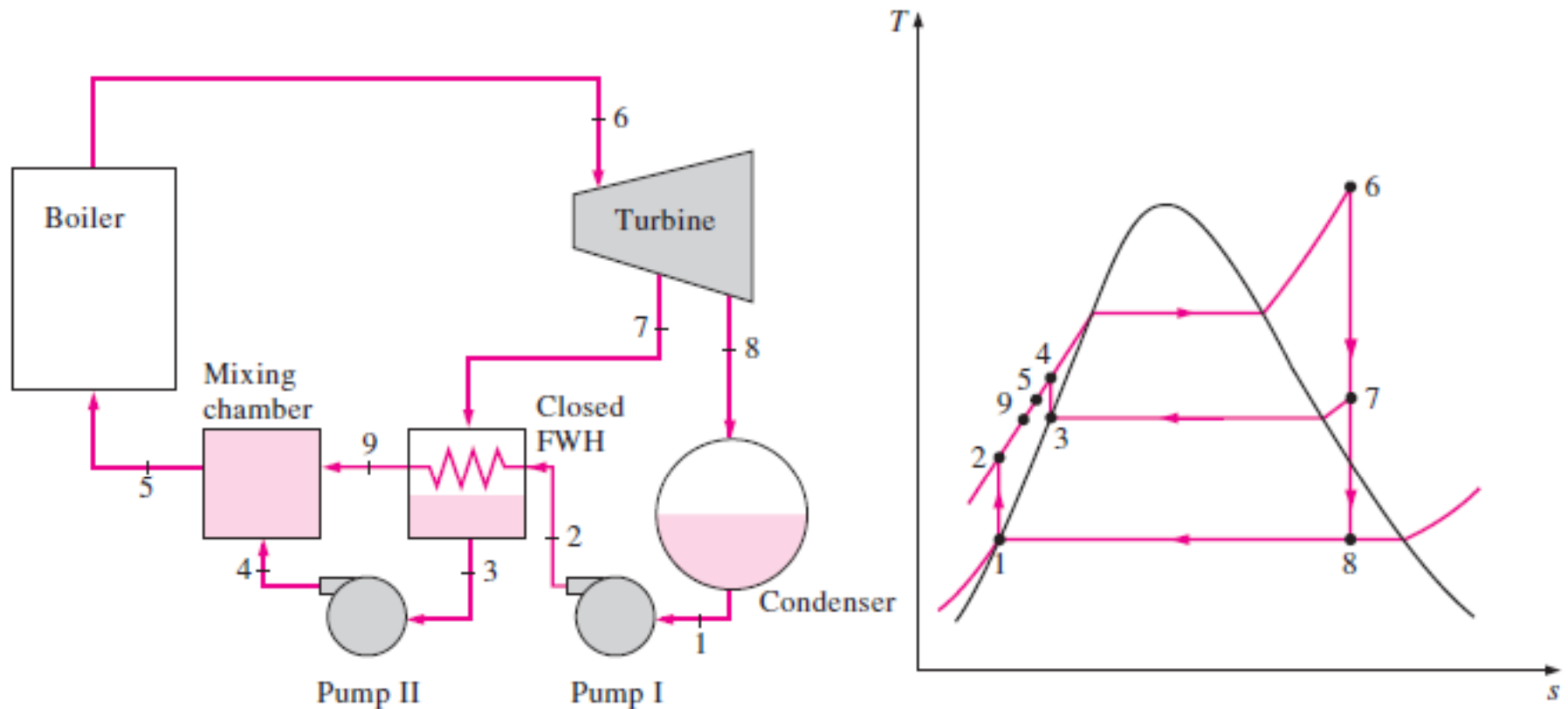
$$W_{pump\ I,in} = V_1 (P_2 - P_1)$$

$$W_{pump\ II,in} = V_3 (P_4 - P_3)$$

The thermal efficiency of the Rankine cycle increases as a result of regeneration.

This is because regeneration raises the average temperature at which heat is transferred to the steam in the boiler by raising the temperature of the water before it enters the boiler.

## *Closed feedwater heaters*



**FIGURE 10–16**

The ideal regenerative Rankine cycle with a closed feedwater heater.



Another type of feedwater heater frequently used in steam power plants is the **closed feedwater heater, in which heat is transferred from the extracted** steam to the feedwater without any mixing taking place.

The two streams now can be at different pressures, since they do not mix.

### *EXAMPLE 10–5 The Ideal Regenerative Rankine Cycle*

Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feedwater heater.

Determine the fraction of steam extracted from the turbine and the thermal efficiency of the cycle.

***Solution*** : A steam power plant operates on the ideal regenerative Rankine cycle with one open feedwater heater. The fraction of steam extracted from the turbine and the thermal efficiency are to be determined.

***Assumptions:*** *1 Steady operating conditions exist.*  
*2 Kinetic and potential*  
energy changes are negligible.

***Analysis:*** *The schematic of the power plant and the T-s diagram of the cycle* are shown in Fig. 10–18.

We note that the power plant operates on the ideal regenerative Rankine cycle. Therefore, the pumps and the turbines are isentropic; there are no pressure drops in the boiler, condenser, and feedwater heater; and steam leaves the condenser and the feedwater heater as saturated liquid. First, we determine the enthalpies at various states:

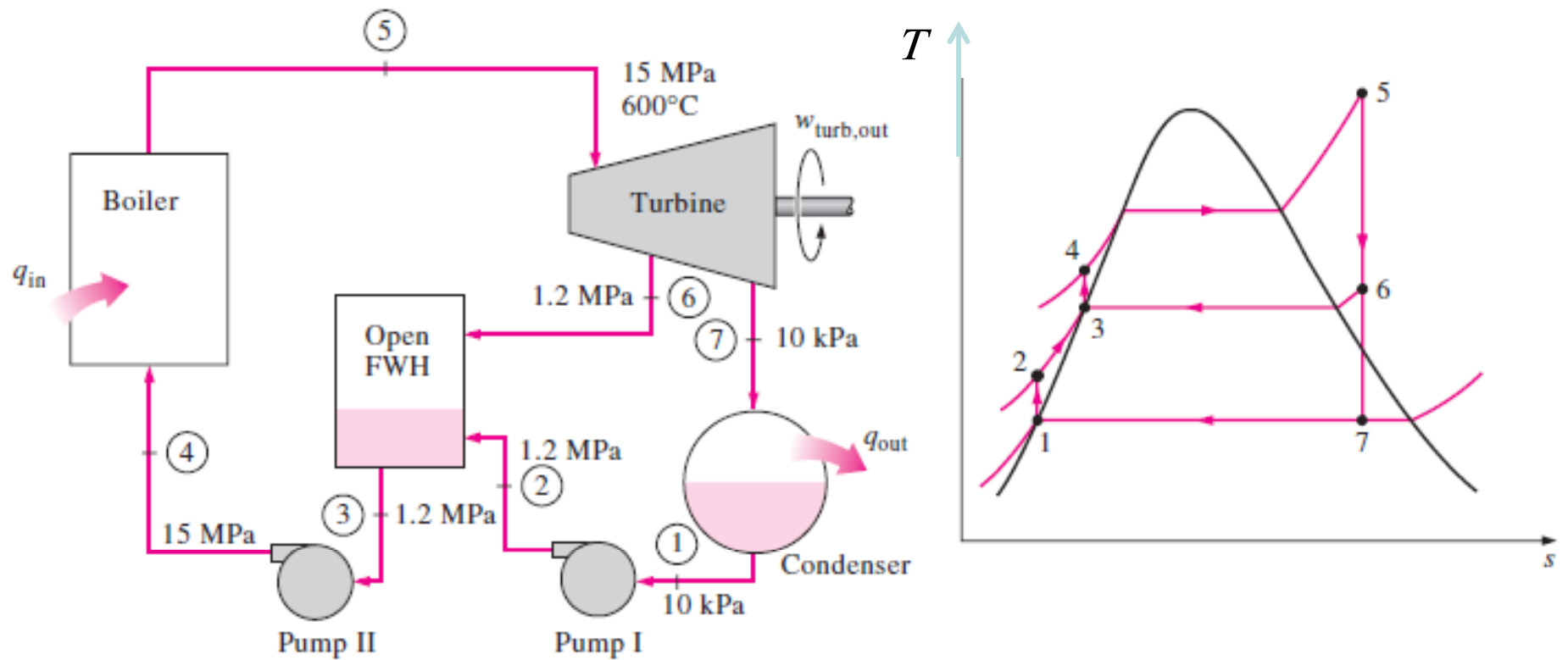
$$\left. \begin{array}{l} \text{State 1:} \quad P_1 = 10 \text{ kPa} \\ \quad \quad \text{Sat. liquid} \end{array} \right\} \begin{array}{l} h_1 = h_f @ 10 \text{ kPa} = 191.81 \text{ kJ/kg} \\ v_1 = v_f @ 10 \text{ kPa} = 0.00101 \text{ m}^3/\text{kg} \end{array}$$

$$\text{State 2: } P_2 = 1.2 \text{ MPa}$$

$$s_2 = s_1$$

$$\begin{aligned} w_{\text{pump l,in}} &= v_1(P_2 - P_1) = (0.00101 \text{ m}^3/\text{kg})[(1200 - 10) \text{ kPa}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= 1.20 \text{ kJ/kg} \end{aligned}$$

$$h_2 = h_1 + w_{\text{pump l,in}} = (191.81 + 1.20) \text{ kJ/kg} = 193.01 \text{ kJ/kg}$$



**FIGURE 10–18**

Schematic and  $T-s$  diagram for Example 10–5.

$$\text{State 3: } \left. \begin{array}{l} P_3 = 1.2 \text{ MPa} \\ \text{Sat. liquid} \end{array} \right\} \begin{array}{l} v_3 = v_f @ 1.2 \text{ MPa} = 0.001138 \text{ m}^3/\text{kg} \\ h_3 = h_f @ 1.2 \text{ MPa} = 798.33 \text{ kJ/kg} \end{array}$$

$$\text{State 4: } P_4 = 15 \text{ MPa}$$

$$s_4 = s_3$$

$$w_{\text{pump II, in}} = v_3(P_4 - P_3)$$

$$= (0.001138 \text{ m}^3/\text{kg})[(15,000 - 1200) \text{ kPa}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right)$$

$$= 15.70 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{\text{pump II, in}} = (798.33 + 15.70) \text{ kJ/kg} = 814.03 \text{ kJ/kg}$$

$$\text{State 5: } \left. \begin{array}{l} P_5 = 15 \text{ MPa} \\ T_5 = 600^\circ\text{C} \end{array} \right\} \begin{array}{l} h_5 = 3583.1 \text{ kJ/kg} \\ s_5 = 6.6796 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{State 6: } \left. \begin{array}{l} P_6 = 1.2 \text{ MPa} \\ s_6 = s_5 \end{array} \right\} \begin{array}{l} h_6 = 2860.2 \text{ kJ/kg} \\ (T_6 = 218.4^\circ\text{C}) \end{array}$$

$$\text{State 7: } P_7 = 10 \text{ kPa}$$

$$s_7 = s_5 \quad x_7 = \frac{s_7 - s_f}{s_{fg}} = \frac{6.6796 - 0.6492}{7.4996} = 0.8041$$

$$h_7 = h_f + x_7 h_{fg} = 191.81 + 0.8041(2392.1) = 2115.3 \text{ kJ/kg}$$



The energy analysis of open feedwater heaters is identical to the energy analysis of mixing chambers. The feedwater heaters are generally well insulated ( $\dot{Q} = 0$ ), and they do not involve any work interactions ( $\dot{W} = 0$ ). By neglecting the kinetic and potential energies of the streams, the energy balance reduces for a feedwater heater to

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \rightarrow \sum_{\text{in}} \dot{m}h = \sum_{\text{out}} \dot{m}h$$

or

$$yh_6 + (1 - y)h_2 = 1(h_3)$$

where  $y$  is the fraction of steam extracted from the turbine ( $= \dot{m}_6 / \dot{m}_5$ ). Solving for  $y$  and substituting the enthalpy values, we find

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{798.33 - 193.01}{2860.2 - 193.01} = 0.2270$$

Thus,

$$q_{\text{in}} = h_5 - h_4 = (3583.1 - 814.03) \text{ kJ/kg} = 2769.1 \text{ kJ/kg}$$

$$\begin{aligned} q_{\text{out}} &= (1 - y)(h_7 - h_1) = (1 - 0.2270)(2115.3 - 191.81) \text{ kJ/kg} \\ &= 1486.9 \text{ kJ/kg} \end{aligned}$$

and

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1486.9 \text{ kJ/kg}}{2769.1 \text{ kJ/kg}} = 0.463 \text{ or } 46.3\%$$

*Discussion* This problem was worked out in *Example 10–3c* for the same pressure and temperature limits but without the regeneration process. A comparison of the two results reveals that the thermal efficiency of the cycle has increased from 43.0 to 46.3 percent as a result of regeneration. The net work output decreased by 171 kJ/kg, but the heat input decreased by 607 kJ/kg, which results in a net increase in the thermal efficiency.

# Homework

10-2C , 10-3C , 10-4C , 10-5C , 10-6C , 10-7C , 10-8C , 10-9C , 10-12C , 10-13C , 10-14 , 10-16, 10-20, 10-21 , 10-22, 10-23, 10-24 , 10-25, , 10-26C , 10-34, , 10-36, 10-37, 10-38 , 10-42C , 10-43C , 10-47 , 10-48 , 10-51 , 10-52 , 10-56 , 10-58.