Lecture 7: Galactic rotation

Astronomers attribute this difference to dark matter.
Differential Rotation of the Galaxy

The Sun orbits at 230 km/s or about 500,000 mph
The sun’s speed and distance from the galactic center

- Relative to nearby stars, the sun travels at a speed of about 20 km/s.
- We use indirect approaches to find out how fast the sun moves around the Galaxy. One method utilizes the motion of globular clusters. Such an analysis shows that the sun revolves at about 220 km/s.
- How far is the sun from the Galaxy’s center? That’s a tough question because we cannot see the center optically. However, we look above and below the galactic plane, where the obscuration is less, to observe objects thought to be symmetrical about the galactic center.
- We can use RR Lyrae variables found in globular clusters to measure their distances. By this technique, the sun’s distance from the galactic center is approximately 28,000 ly, within a range of 24,000 to 33,000 ly.
We investigate the “Rotation Curve” of our Galaxy by plotting the velocity as a function of radial distance $d$ from the center.
Rotation curve

The relation between rotational speed of objects in a galaxy and their distance from its center
• The rotation curve tells us the overall distribution of matter in the Galaxy, because gravity controls those orbital motions.

• Imagine that the Galaxy’s mass is mostly concentrated in the nucleus. The stars in the galactic disk should revolve around the nucleus of the Galaxy much as the planets revolve around the sun. The stellar motion should follow Kepler’s laws, and the orbital speeds of the stars should decrease with increasing distance from the Galaxy’s center.
In fact, the orbital speeds do not follow the expected Keplerian decline. That exposes an important fact about the Galaxy: the major part of the mass is not concentrated at the center! From close to the center out to 1000 ly, the curve rises steeply, then drops, bottoming out at about 10,000 ly. It then rises slowly out to the position of the sun, then drops again. In the outer parts of the Galaxy, the curve rises beyond the sun’s orbit. It then appears to flatten out at a distance of 50,000 ly from the galactic center. If the motions followed Kepler’s laws, we would expect the rotation curve to decline beyond the sun’s orbit, and it does not.

What does this rotation curve say? Much of the Galaxy’s material must lie out beyond the sun’s orbit. From the rotation curve out to 60,000 ly, the Galaxy mass is $3.4 \times 10^{11}$ solar masses. Other estimates give close to $10^{12}$ solar masses. At least as much mass lies exterior to the sun as interior to it.
Most of the matter in the Galaxy has not yet been identified

- According to Kepler’s Third Law, the farther a star is from the center, the slower it should orbit.
- Observations show that speed actually increases with distance from the center.
- This could be due to gravity from extra mass we cannot see - called DARK MATTER.
Spiral galaxies all tend to have flat rotation curves indicating large amounts of dark matter.
Astronomers attribute this difference to dark matter.

Expected

Observed

Distance from center of galaxy

Rotation velocity

Montanned

A

B

منحنى دوران المجرة هو رسم بياني لسرعة الدوران المدارية للنجوم والغازات التي تتحرك حولها بدلالة أبعادها عن مركز المجرة. منحنى الدوران يجب أن يبدأ بسرعات ضئيلة قرب مركز المجرة، ثم تزداد السرعة عند المسافات المتوسطة، ومن ثم تعود لتهبط إلى سرعات أقل بعيد عن مركز المجرة. إلا أن الفلكيين عند قياسهم لمنحنيات الدوران للمجرات لا يوجد سلوكا كهذا أبدا، فمنحنيات الدوران تزداد قريبا من المركز، لكنها تستوي بعد ذلك فلا تهبط إلى سرعات منخفضة أبدا. فلا بد من وجود كتل كبيرة غير مرفعة خارج حدود المجرة تؤثر بجاذبيتها في النجوم البعيدة عن مركز المجرة فتجعلها تحافظ على سرعتها المدارية.

يعزو علماء الفلك هذا الفرق إلى وجود المادة الخفية.
-The nature of dark matter?

This is one of the big unsolved problems of astronomy. The dark matter in the Galaxy and other galaxies may be:

- lots of very low mass and low luminosity stars, such as red dwarfs, brown dwarfs or white dwarfs
- numerous black holes
- some form of matter, possibly as subatomic particles which are so far unknown to science
Galactic rotation from halo stars:
• Mean $V_R$ of 70 globular clusters is about 200 km/s towards $l = 270^\circ$
• This is evidence that the Sun and other disk stars are moving at about 200 km/s towards $l = 90^\circ$, as consequence of galactic disk rotation

The so-called “asymmetric drift” showing the distribution of stellar radial velocities for stars in the solar neighbourhood. The Population II stars have high velocities preferentially in the $l = 270^\circ$ direction, a result of galactic rotation
Galactic rotation parameters

Circular velocity: $\Theta_o = 220$ km/s
Radius of solar orbit $R_o = 8.5$ kpc
Angular velocity $\omega_o = \Theta_o / R_o = 26$ km/s/kpc
Orbital period $P_o = 2\pi / \omega_o = 240 \times 10^6$ yr

Note that $\omega = \omega(R) \rightarrow$ differential rotation.
$\omega = \text{constant}$ would imply solid-body rotation, but this is not observed.
The radial velocity, $V_R$, relative to the Sun, of a disk star in a circular orbit about the galactic centre depends on its distance $d$ from the Sun. For a given line of sight the radial velocity is a maximum when the angle $\alpha$ is zero, $d = R_o \cos l$. 

![Diagram showing radial velocities in the differentially rotating Galaxy](image)
From disk stars
Radial velocity of star relative to Sun is

\[ V_r = \Theta \cos \alpha - \Theta_0 \sin l \]

Sine rule

\[ \frac{\sin l}{R} = \frac{\sin(90^\circ + \alpha)}{R_0} = \frac{\cos \alpha}{R_0} \]

Now

\[ \omega = \Theta / R ; \omega_0 = \Theta_0 / R_0 \]

Hence

\[ \therefore V_r = \omega R \cos \alpha - \omega_0 R_0 \sin l = \omega R_0 \sin l - \omega_0 R_0 \sin l \]
Approximations:

\[ \omega - \omega_o \approx \left( \frac{d\omega}{dR} \right)_o (R - R_o) \]

\[ - (R - R_o) \approx d \cos l \]

Therefore

\[ V_r = \left( \frac{d\omega}{dR} \right)_o (R - R_o).R_o \sin l \]

\[ = - \left( \frac{d\omega}{dR} \right)_o d \cos l.R_o \sin l \]

\[ = - \frac{1}{2} Rd \left( \frac{d\omega}{dR} \right)_o \sin 2l \]
Define Oort’s constant for galactic rotation as:

\[
A = -\frac{1}{2} R_0 \left( \frac{d\omega}{dR} \right)_0
\]

Hence

\[
V_R = Ad \sin 2l
\]

\( A \) is a measure of the amount of differential rotation in the Galaxy.

The best value for Oort’s constant \( A \) comes from distant B-type stars and Cepheids in galactic plane and is \( A = 15 \text{ km/s/kpc} \).
For stars in the disk of a given distance $d$, the radial velocities (measured by the Doppler effect in stellar spectra) show a double sign wave as a function of galactic longitude, $l$. 

Fig. 3.45a,b. Radial velocity after discounting the solar motion $v_{rL}$ [km s$^{-1}$] as a function of the galactic longitude: (a) for classical Cepheids with mean distance $\bar{r} = 2.3$ kpc (from Joy, 1939) and (b) for early type B-stars with $\bar{r} = 2.0$ kpc (from Feast and Thackeray, 1958).
Radial velocities of HII regions are plotted as a function of galactic longitude. The plot shows a double sine wave, like for disk stars, but with some deviations.
Galactic rotation from HI and CO clouds

For disk objects in a differentially rotating galaxy,

\[ V_R = \Theta \cos \alpha - \Theta_o \sin l \approx (\omega - \omega_o) R_o \sin l \]

The radial velocity, \( V_R \), is a maximum at point P along a given line of sight, when \( \alpha = 0 \) and \( R_{\text{min}} = R_0 \sin l \).

Fig. 1: Radial velocities in the differentially rotating Galaxy
At point P one obtains

\[ V_{\text{(max)}} = \Theta_{R=R_{\text{min}}} - \Theta_{O} \sin l \]

\[ \therefore \Theta_{R=R_{\text{min}}} = V_{\text{(max)}} + \Theta_{O} \sin l \]

\[ \therefore \Theta(R = R_{O} \sin l) = V_{R_{\text{max}}} + \Theta_{O} \sin l \]

Procedure:
For a number of longitudes \( l \), find \( V_{R_{\text{max}}} \) for 21-cm radiation from HI clouds along that line of sight, and hence obtain \( \Theta(R) \). This works for \( R < R_{0} \) only.
HI cloud radial velocities along a given line of sight
HI radial velocities are a maximum in this direction for gas at position A, where the velocity is about 70 km/s. The cloud at A has a galactic orbital radius of $R_0 \sin l$. 

**FIGURE 20–2** Line profiles and Doppler shifts. (A) Line profiles from a number of H I clouds at longitude 48°. (B) The line-of-sight geometry for the profiles in A.
• For $R>R_0$ (directions $90^\circ < l < 270^\circ$) there is no maximum in radial velocity.
• But for $R>R_0$ we can use CO in dense molecular clouds. These are often associated with star-forming regions, and there are ways to estimate distances to stars and hence to the CO.
• Then obtain the outer parts of the galactic rotation curve from

$$\Theta(R) = \left( V_R + \Theta_0 \sin l \right) / \cos \alpha$$

• Note that when distance $d$ to a cloud is known, then both angle $\alpha$ and orbital radius $R$ are also known.
Rotation curve for the Galaxy

• The best rotation curve $\Theta(R)$ for the Galaxy comes from combining radial velocity data for stars, HI clouds and CO in dense molecular clouds
• Result: $\Theta$ varies little with radius, though there are dips at around $R = 3$ kpc and 10 kpc
• Mean velocity of galactic orbits is $\Theta \sim 220$ km/s
• This “flat” rotation curve is a major surprise
• Expected result, if galactic mass is from stars, is $\Theta \propto R^{-1/2}$
Galactic rotation curve $\Theta(R)$ based mainly on CO. The “flat” (i.e. nearly constant) curve is evidence for extra mass in the form of dark matter in the Galaxy.
Rotation curve for the Galaxy, based mainly on HI clouds

Fig. 18.19. Rotation curve of the Milky Way based on motions of hydrogen clouds. Each point represents one cloud. The thick line represents the rotation curve determined by Maarten Schmidt in 1965. If all mass were concentrated within the radius 20 kpc, this curve would continue according to Kepler's third law (broken line). The rotation curve determined by Leo Blitz on the basis of more recent observations begins to rise again at 12 kpc.
## The galactic rotation curve

<table>
<thead>
<tr>
<th>$R$ (kpc)</th>
<th>$\Theta$ (km/s)</th>
<th>$P$ ($10^6$ yr)</th>
<th>$\omega$ (rad/Myr)</th>
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<tr>
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<tr>
<td>8.5 ($R_0$)</td>
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<tr>
<td>16</td>
<td>235</td>
<td>420</td>
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</table>
- The Galaxy’s mass

• Knowing the sun’s velocity and distance, we apply Newton’s version of Kepler’s third law to deduce the mass of the Galaxy.

\[ M_G + M_\odot = \frac{R_\odot^3}{P_\odot^2} \]

where \( R_\odot \) must be in AU and \( P_\odot \) in years. The mass then comes out in solar masses. At \( 220 \) km/s it takes the sun about \( 2.8 \times 10^8 \) years to complete one cycle. Now \( R_\odot \) is roughly \( 30,000 \) ly and \( 1 \) ly = \( 6.32 \times 10^4 \) AU; so \( 30,000 \) ly becomes \( 1.9 \times 10^9 \) AU. Then

\[ M_G + M_\odot = \frac{(1.9 \times 10^9)^3}{(2.8 \times 10^8)^2} \]

\[ M_G + M_\odot = 0.9 \times 10^{11} M_\odot \]

• The mass of the sun is so small compared with that of the Galaxy that we ignore it, so the Galaxy’s mass interior to the sun’s orbit is about \( 10^{11} \) solar masses.
Another method for determining the Galaxy’s mass

• At the sun’s position in the Galaxy, we may approximate the solar motion as a circular Keplerian orbit about a massive central body of mass $M_G$. Since the centripetal acceleration maintaining this circular orbit is produced by the gravitational attraction between the core ($M_G$) and the Sun ($M_\odot$), we have

\[
F_{\text{cent}} = F_{\text{grav}} \quad \Rightarrow \quad M_\odot v_\odot^2 / R_\odot = G (M_\odot M_G / R_\odot^2)
\]

\[
v_\odot^2 / R_\odot = G M_G / R_\odot^2
\]

where $v_\odot$ (220 km/s) is the sun’s circular speed, $R_\odot$ is the distance to the galactic center (8.5 kpc) and $G$ (6.67x10^{-11} N m^2/kg^2) is the constant of the universal gravitation. The previous equation gives the mass of our Galaxy within the solar orbit:

\[
M_G = v_\odot^2 R_\odot / G \\
= (2.2x10^5 \text{ m/s})^2 (8.5 \times 10^3 \times 3.086 \times 10^{16} \text{ m}) / (6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2)
\]

\[
= 1.9 \times 10^{41} \text{ kg} \approx 10^{11} M_\odot
\]

This only the mass interior to the sun’s orbit

** (1 N = 1 kg m/s^2, 1 $M_\odot = 1.989 \times 10^{30}$ kg, 1 pc = 3.0856 $\times 10^{16}$ m )