



SOLVING MATERIAL BALANCE PROBLEMS FOR SINGLE UNITS WITHOUT REACTIONS



EXAMPLE 1: Extraction of Streptomycin from a Fermentation Broth.

Streptomycin is used as an antibiotic to fight bacterial diseases, and is produced by the fermentation of a bacterium in a biological reactor with a nutrient of glucose and amino acids. After the fermentation process, Streptomycin is recovered by contacting the fermentation broth with an organic solvent in an extraction process. The extraction process is able to recover the Streptomycin because Streptomycin has a greater affinity for dissolving in the organic solution than in the aqueous. Determine the mass fraction of Streptomycin in the exit organic solvent assuming that no water exits with the solvent and no solvent exits with the aqueous solution. Assume that the density of the aqueous solution is 1 g/cm^3 and the density of the organic solvent is 0.6 g/cm^3 .



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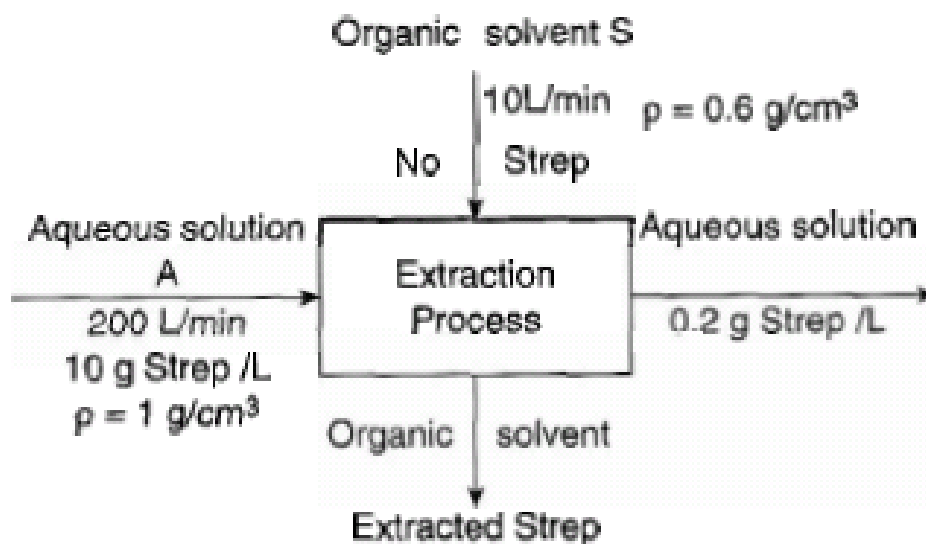


Solution:

Step 1

This is an open (flow), steady-state process without reaction. Assume because of the low concentration of Strep. in the aqueous and organic fluids that the flow rates of the entering fluids equal the flow rates of the exit fluids.

Steps 2,3, and 4





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Step 5

Basis: 1 min

Steps 6 and 7

The degree-of-freedom analysis is:

Number of variables (8): 4 flows (in L) plus 4 concentrations (in g/L)

Number of equations (8):

Basis: Feed = 200 L (flow of aqueous entering aqueous solution) 1

Specifications: Concentration of Strep in entering aqueous solution 1

Concentration of Strep in exit aqueous solution 1

Concentration of Strep in entering organic solvent 1

Flow of exiting aqueous solution (same as existing flow) 1

Flow of entering organic solution 1

Flow of exiting organic solution (same as existing flow) 1

Strep material balance 1

Total 8



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The degrees of freedom are 0.

Steps 8 and 9

The material balances are *in* = *out* in grams. Let *x* be the g of Strep per L of solvent S

Strep M.B:

$$\frac{200 \text{ L of A}}{\left| \frac{10 \text{ g Strep}}{1 \text{ L of A}} \right|} + \frac{10 \text{ L of S}}{\left| \frac{0 \text{ g Strep}}{1 \text{ L of S}} \right|} = \frac{200 \text{ L of A}}{\left| \frac{0.2 \text{ g Strep}}{1 \text{ L of A}} \right|} + \frac{10 \text{ L of S}}{\left| \frac{x \text{ g Strep}}{1 \text{ L of S}} \right|}$$

$$x = 196 \text{ g Strep/L of S}$$



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To get the g Strep /g solvent, use the density of the solvent:

$$\frac{196 \text{ g Strep}}{1 \text{ L of S}} \left| \frac{1 \text{ L of S}}{1000 \text{ cm}^3 \text{ of S}} \right| \frac{1 \text{ cm}^3 \text{ of S}}{0.6 \text{ g of S}} = 0.3267 \text{ g Strep/g of S}$$

$$\text{The mass fraction Strep} = \frac{0.3267}{1 + 0.3267} = 0.246$$

EXAMPLE 3.6 Continuous Distillation

A novice manufacturer of alcohol for gasohol is having a bit of difficulty with a distillation column. The operation is shown in Figure E3.6. Technicians think too much alcohol is lost in the bottoms (waste). Calculate the composition of the bottoms and the mass of the alcohol lost in the bottoms.

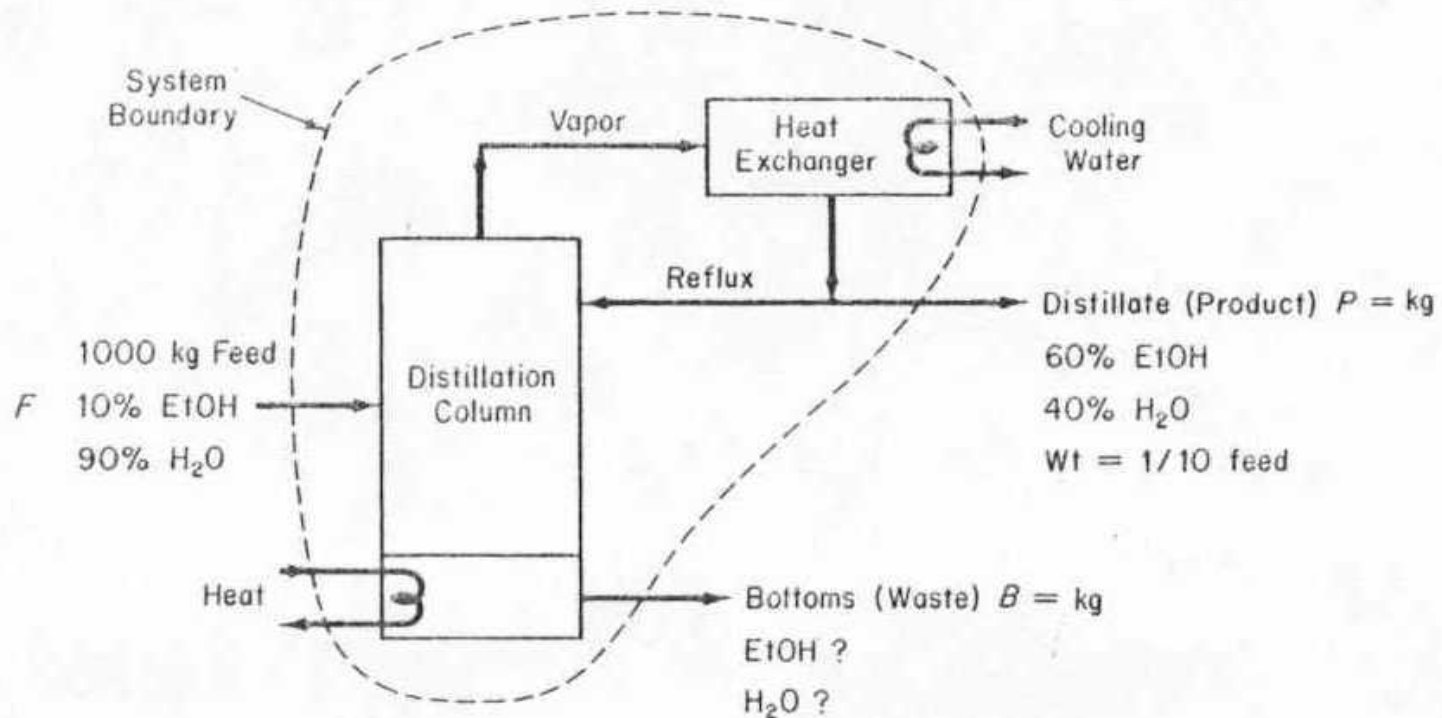


Figure E3.6

To be solved in class (in teams)



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Steps 1,2,and 3

All of the symbols and known data have been placed on the Figure.

Step 4

We are given that P is 1/10 of F, so that $P = 0.1(1000) = 100$ kg

Step 5

Basis: 1 hour so that $F = 1000$ kg of feed

Steps 6 and 7

Number of variables: 9

$$m_{\text{EtOH}}^F, m_{\text{H}_2\text{O}}^F, m_{\text{EtOH}}^P, m_{\text{H}_2\text{O}}^P, m_{\text{EtOH}}^B, m_{\text{H}_2\text{O}}^B, F, P, B$$



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Number of equations: 9

Basis: $F = 1000$ kg

Specifications: $m_{\text{EtOH}}^F = 1000(0.10) = 100$

$$m_{\text{H}_2\text{O}}^F = 1000(0.90) = 900$$

$$m_{\text{EtOH}}^P = 0.60P$$

$$m_{\text{H}_2\text{O}}^P = 0.40P$$

$$P = 0.1 F = 100 \text{ kg}$$

Material balances: EtOH and H₂O

Implicit equations: $\sum m_i^B = B$ or $\sum \omega_i^B = 1$

The problem has zero degrees of freedom.



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Steps 8 and 9

Let's substitute the total mass balance $F = P + B$ for one of the component mass balances and calculate B by direct subtraction

$$B = 1000 - 100 = 900 \text{ kg}$$

The solution for the composition of the bottoms can then be computed directly from the material balances:

	<i>kg feed in</i>	<i>kg distillate out</i>	<i>kg bottoms out</i>	<i>Mass fraction</i>
EtOH balance:	$0.10(1000)$	$- 0.60(100)$	$= 40$	0.044
H ₂ O balance:	$0.90(1000)$	$- 0.40(100)$	$= \underline{860}$	<u>0.956</u>
			900	1.000



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Step 10

As a check let's use the redundant equation:

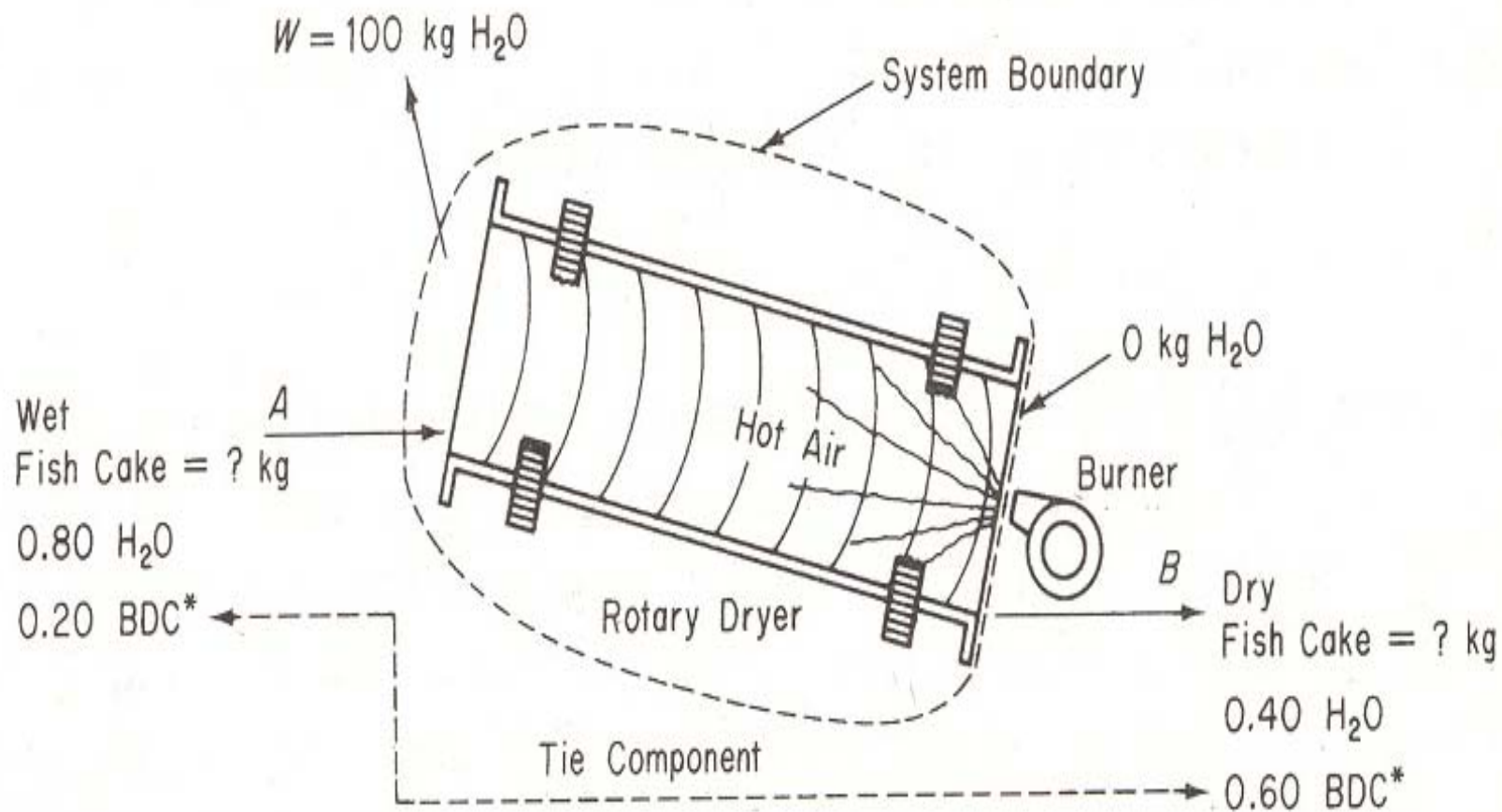
$$m_{\text{EtOH}}^B + m_{\text{H}_2\text{O}}^B = B \quad \text{or} \quad \omega_{\text{EtOH}}^B + \omega_{\text{H}_2\text{O}}^B = 1$$

$$40 + 860 = 900 \text{ kg}$$

$$0.044 + 0.956 = 1$$

EXAMPLE 3 Drying

Fish caught by human beings can be turned into fish meal, and the fish meal can be used as feed to produce meat for human beings or used directly as food. The direct use of fish meal significantly increases the efficiency of the food chain. However, fish-protein concentrate, primarily for aesthetic reasons, is used mainly as a supplementary protein food. As such, it competes with soy and other oilseed proteins. In the processing of the fish, after the oil is extracted, the fish cake is dried in ro-tary drum dryers, finely ground, and packed. The resulting product contains 65% protein. In a given batch of fish cake that contains 80% water (the remainder is dry cake), 100 kg of water is removed, and it is found that the fish cake is then 40% water. Calculate the weight of the fish cake originally put into the dryer.



*Bone Dry Cake

Figure E3.8



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Solution:

Steps 1,2,3, and 4

This is a steady-state process without reaction. The system is the dryer.

Step 5

Basis: 100 kg of water evaporated = W

Steps 6 and 7

There are four streams:

Two in (air and fish cake) and two out (air and fish cake),

The air is not shown in the Figure because it is not involved in the process. Only the water in the air is involved.

Two independent balances can be written.



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The degree-of-freedom analysis gives zero degrees of freedom

Step 8 and 9

The water balance

$$0.80A = 0.40B + 100$$

	<u>In</u>	<u>Out</u>	
Total balance:	A	$= B + W = B + 100$	} mass balances
BDC balance:	$0.20A$	$= 0.60B$	

The solution is

$$A = 150 \text{ kg initial cake and } B = (150)(0.20/0.60) = 50 \text{ kg}$$



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Step 10

Check via the water balance:

$$0.80(150) \stackrel{?}{=} 0.40(50) + 100$$

$$120 = 120$$



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EXAMPLE: Crystallization

A tank holds 10,000 kg of a saturated solution of Na_2CO_3 at 30°C . You want to crystallize from this solution 3000 kg of $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$ without any accompanying water. To what temperature must the solution be cooled?

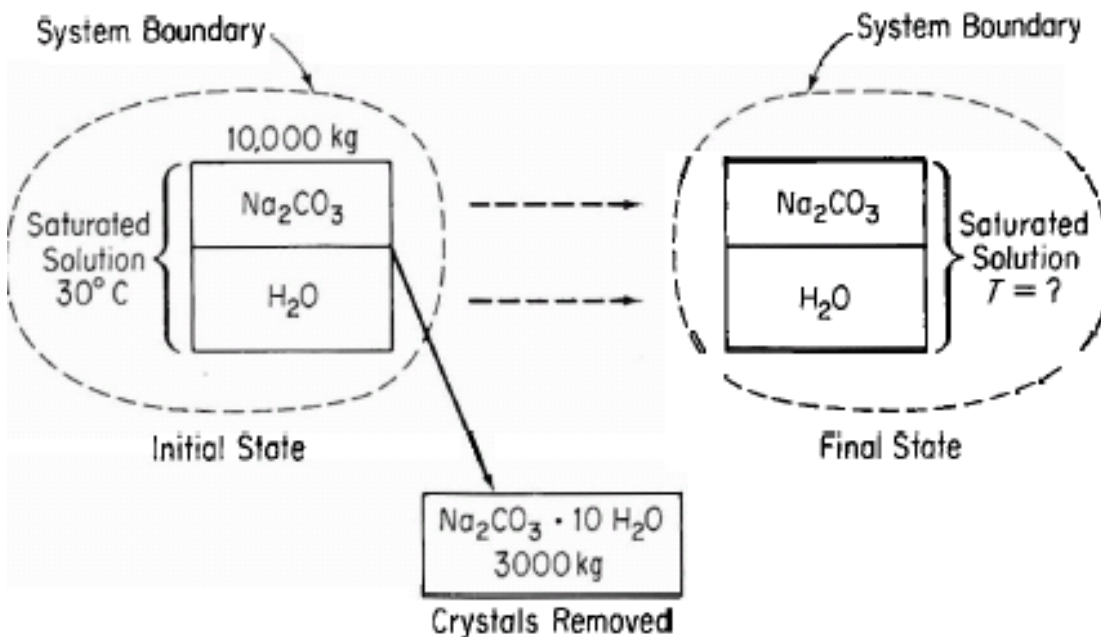
Solution

Steps 1,2, and 3

The next Figure is a diagram of the process



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Step 4

You definitely need solubility data for Na_2CO_3 as a function of the temperature:



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Temp.(°C)	Solubility (g Na ₂ CO ₃ /100 g H ₂ O)
0	7
10	12.5
20	21.5
30	38.8

Because the initial solution is saturated at 30°C, you can calculate the composition of the initial solution:

$$\frac{38.8 \text{ g Na}_2\text{CO}_3}{38.8 \text{ g Na}_2\text{CO}_3 + 100 \text{ g H}_2\text{O}} = 0.280 \text{ mass fraction Na}_2\text{CO}_3$$



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Next, you should calculate the composition of the crystals.

Basis: 1 g mol $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$

<u>Comp.</u>	<u>Mol</u>	<u>Mol wt.</u>	<u>Mass</u>	<u>Mass fr</u>
Na_2CO_3	1	106	106	0.371
H_2O	10	18	<u>180</u>	<u>0.629</u>
Total			286	1.00



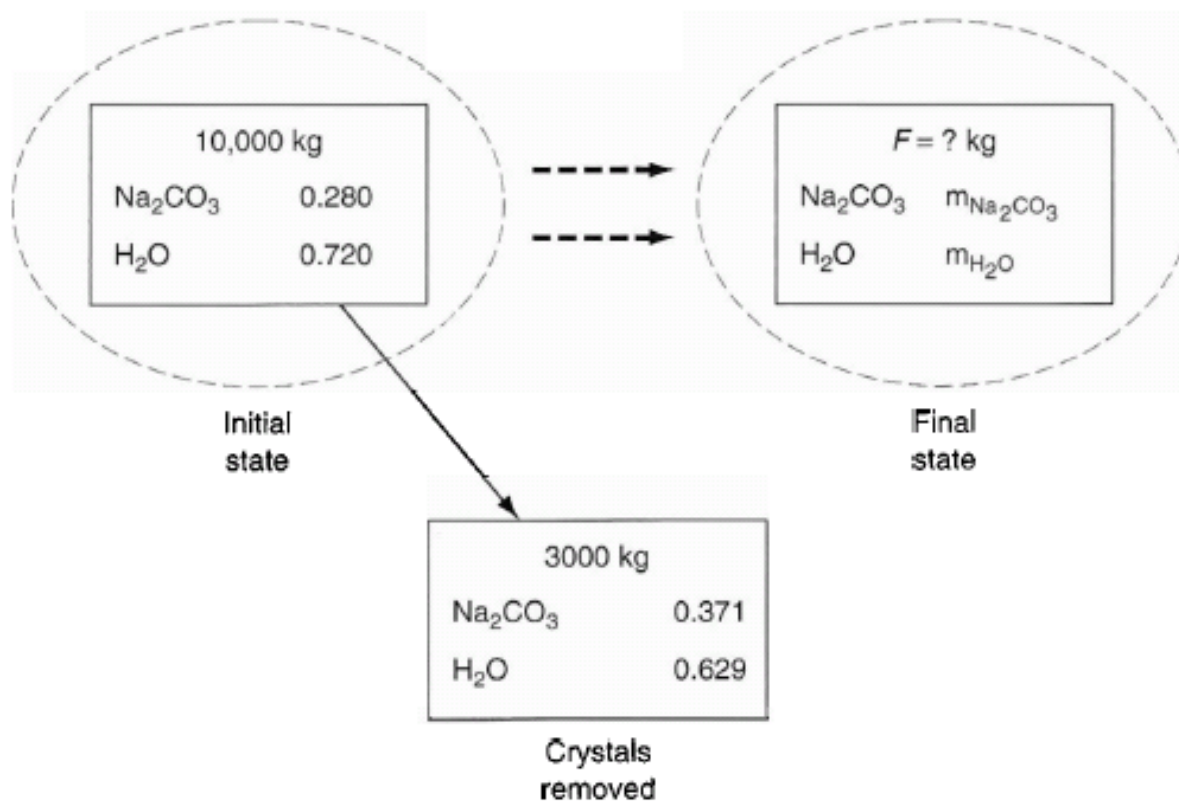
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Step 5

Basis: 10,000 kg of saturated solution at 30°C

Steps 2 and 3 (Repeated)





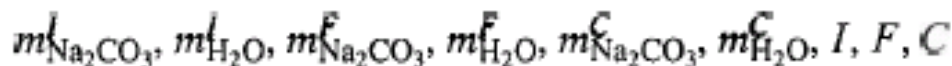
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Steps 6 and 7

The analysis of the degrees of freedom yields a value of zero (I stands for the initial, F the final state, and C the crystals).

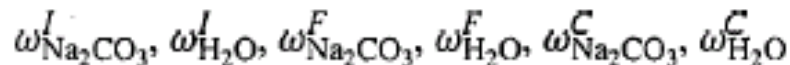
Number of variables: 9



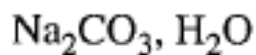
Number of equations: 9

Basis: $I = 10,000$ kg

Specifications:



Material balances:





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Steps 8 and 9

After substituting the specifications and basis into the material balances, (only two are independent) you get (in kg).

<i>Accumulation in Tank</i>					
	<i>Final</i>		<i>Initial</i>		<i>Transport out</i>
Na ₂ CO ₃	$m_{\text{Na}_2\text{CO}_3}^F$	–	10,000(0.280)	=	–3000(0.371)
H ₂ O	$m_{\text{H}_2\text{O}}^F$	–	10,000(0.720)	=	–3000(0.629)
Total	F	–	10,000	=	–3000



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The solution for the composition and amount of the final solution is

<i>Component</i>	<i>kg</i>
$m_{\text{Na}_2\text{CO}_3}^F$	1687
$m_{\text{H}_2\text{O}}^F$	<u>5313</u>
F (total)	7000

Step 10

Check using the total balance:

$$7,000 + 3,000 = 10,000$$



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To find the temperature of the final solution, calculate the composition of the final solution in terms of grams of Na_2CO_3 /100 grams of H_2O so that you can use the tabulated solubility data listed in Steps 2-4 above.

$$\frac{1,687 \text{ kg Na}_2\text{CO}_3}{5,313 \text{ kg H}_2\text{O}} = \frac{31.8 \text{ g Na}_2\text{CO}_3}{100 \text{ g H}_2\text{O}}$$

Thus, the temperature to which the solution must be cooled lies between 20°C and 30°C . By linear interpolation

$$30^\circ\text{C} - \frac{38.8 - 31.8}{38.8 - 21.5} (10.0^\circ\text{C}) = 26^\circ\text{C}$$