Chapter 8: Conservation of Energy

This chapter actually completes the argument established in the previous chapter and outlines the standing concepts of energy and conservative rules of total energy. I will divide the chapter into two main parts: conservative and nonconservative forces and conservation of energy.

A- Conservative and Nonconservative Forces

There are two basic properties of the conservative forces that are related to the work done by this force. These are:

- *If the work done by the force is independent of the path, the force is said to be a conservative force. Otherwise, it is nonconservative.*
- *The work done by a conservative force in any closed path is zero. The closed path means a zero displacement.*

As examples of conservative forces, the *gravitational* and *spring* forces do. On the other side, *friction* force gives an example of nonconservative forces.

B- Conservation Law of Mechanical Energy

The law of conservation of energy states that: *the total mechanical energy of a system remains unchanged (constant) in any isolated system of objects that interact only through conservative forces.* In other words, it states that: *the energy may neither be created nor destroyed, but transferred from a form of energy to another.*

The total mechanical energy, \( E \), is defined as:

\[
E = K + U
\]

Where \( K \) is the total kinetic energy and \( U \) is the total potential energy of the system.
The conservation law of energy is, therefore,

\[ E_i = E_f \]

Where \( E_i \) and \( E_f \) are the initial and final energies, respectively. That implies to

\[ K_i + U_i = K_f + U_f \]

The potential energy is defined (from chapter 7) as:

\[ U = W_g = F \cdot d = mg \cdot d = mg \cdot d \cos \theta \]

Whereas the kinetic energy is defined as:

\[ K = \frac{1}{2} m v^2 \]

In the following diagram, kinetic and potential energies are calculated for a car starting its motion from rest at the top of a cliff. The total energy of the car is conserved at each point till it reaches the bottom of the cliff.
Examples:

1. Using the work-energy theorem, show that $W = -\Delta U$.

Solution

The work-energy theorem states that

$$W = \Delta K$$

However, the law of conservation of energy is:

$$K_i + U_i = K_f + U_f$$

It can be rewritten as

$$K_f - K_i = U_i - U_f = -(U_f - U_i)$$

Or

$$\Delta K = -\Delta U$$

Therefore,

$$W = -\Delta U$$

2. Find the height from which you would have to drop a ball so that it would have a speed of 9.0 m/s just before it hits the ground.

Solution

The initial energy of the system is

$$E_i = m \, gh$$

The final energy of the system is

$$E_f = \frac{1}{2} \, mv^2$$

Therefore,

$$\frac{1}{2} \, mv^2 = m \, gh$$
Hence,

\[ h = \frac{v^2}{2g} = \frac{9.0^2}{19.6} = 4.1 \text{ m} \]

3. A ball is thrown vertically upwards and reaches a maximum height of 19.6 m. Calculate its initial speed.

**Solution**

The initial energy of the system is

\[ E_i = \frac{1}{2} m v^2 \]

The final energy of the system is

\[ E_f = m \, gh \]

Therefore,

\[ \frac{1}{2} m v^2 = m \, gh \]

\[ v = [2 \, gh]^{0.5} = [2 \times 9.8 \times 19.6]^{0.5} = 19.6 \text{ m/s} \]

3. A block having a mass of 4.0 kg is given an initial velocity \( v_A = 2.0 \text{ m/s} \) to the right and collides with a spring of force constant \( k = 64.0 \text{ N/m} \). Calculate the maximum compression of the spring after the collision, assuming the surface is frictionless.

**Solution**

The initial energy, \( E_i \), is

\[ E_i = K_i + U_i \]
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There is only kinetic energy, therefore

\[ E_i = \frac{1}{2} mv^2 \]

The final energy, \( E_f \), is

\[ E_f = K_f + U_f \]

While we have asked about the maximum compression, that means the body will momentarily stop. Therefore, at this point, there is only potential energy due to the spring. Hence,

\[ E_f = \frac{1}{2} kx^2 \]

From the conservation law, we have

\[ E_i = E_f \]

Therefore,

\[ \frac{1}{2} mv^2 = \frac{1}{2} kx^2 \]

That gives

\[ x = (mv^2/k)^{0.5} = (4 \times 2^2/64)^{0.5} = 0.5 \text{ m} \]

4. A 4.00 kg particle is freely released from point (A) and slides on the frictionless. Determine (a) the particle’s speed at points (B) and (C) and (b) the total work done by the force of gravity in moving the particle from (A) to (C).

Solution
(a) The initial energy at point (A), $E_{iA}$, is

$$E_{iA} = K_{iA} + U_{iA}$$

Because the body starts from rest, there is only potential energy

$$E_i = U_{iA}$$

The final energy at point (B), $E_{fB}$, is

$$E_{fB} = K_{fB} + U_{fB}$$

At this point we have both kinetic and potential energies. From the conservation law, we find

$$E_{iA} = E_{fB}$$

Or

$$U_{iA} = K_{fB} + U_{fB}$$

That means

$$K_{fB} = U_{iA} - U_{fB}$$

Therefore,

$$\frac{1}{2} m v_B^2 = mg (d_A - d_B)$$

$$v_B = [2 g (d_A - d_B)]^{0.5} = [2 \times 9.8 \times (5.0 - 3.2)]^{0.5} = 5.95 \text{ m/s}$$

The final energy at point (C), $E_{fC}$, is

$$E_{fC} = K_{fC} + U_{fC}$$

However,

$$E_{iA} = E_{fC}$$

Or

$$U_{iA} = K_{fC} + U_{fC}$$

That means

$$K_{fC} = U_{iA} - U_{fC}$$
Therefore,
\[ \frac{1}{2} m v_C^2 = mg (d_A - d_C) \]
\[ v_C = [2 g (d_A - d_C)]^{0.5} = [2 \times 9.8 \times (5.0 - 2.0)]^{0.5} = 7.68 \text{ m/s} \]

(b) The work done by the gravity between points A and C is
\[ W_{AC} = - \Delta U_{AC} = -(U_C - U_A) \]
Therefore
\[ W_{AC} = - mg (d_C - d_A) = - 2 \times 9.8 \times (5.0 - 2.0) = -58.8 \text{ J} \]

5. Two masses are connected by a light string, which is passing over a light frictionless pulley. The mass \( m_1 \) is released from rest. Using the law of conservation of energy, determine the speed of \( m_2 \) just as \( m_1 \) hits the ground.

Solution

The two masses will move under the influence of a similar acceleration. Therefore their velocities will be the same when the mass \( m_1 \) hits the ground. Hence analysing each of them, separately, gives the followings:
For the mass, $m_1$, the initial kinetic and potential energies are
\[ K_{i1} = 0 \]
And
\[ U_{i1} = -m_1 gh \]
Therefore,
\[ E_{i1} = -m_1 gh \]
The final kinetic and potential energies (when it hits the ground) are
\[ K_{f1} = \frac{1}{2} m_1 v^2 \]
And
\[ U_{f1} = 0 \]
Therefore,
\[ E_{f1} = \frac{1}{2} m_1 v^2 - m_1 gh \]
For the mass, $m_2$, the initial kinetic and potential energies are
\[ K_{i2} = U_{i2} = 0 \]
Therefore,
\[ E_{i2} = 0 \]
The final kinetic and potential energies are
\[ K_{f2} = \frac{1}{2} m_2 v^2 \]
And
\[ U_{f2} = -m_2 gh \]
Therefore,
\[ E_{f2} = \frac{1}{2} m_2 v^2 - m_2 gh \]
The initial total energy of the system is
\[ E_i = E_{i1} + E_{i2} = -m_1 gh \]
The final total energy of the system is

\[ E_f = E_{f1} + E_{f2} = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - m_2 g h \]

It is known from the conservation law of energy that

\[ E_i = E_f \]

That implies to

\[ \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - m_2 g h = -m_1 g h \]

Therefore,

\[ \frac{1}{2} (m_1 + m_2) v^2 = (m_2 - m_1) g h \]

That gives

\[ v = \sqrt{\frac{2(m_2 - m_1)}{(m_1 + m_2)} g h} \]

That gives

\[ v = \sqrt{\frac{2 \times 9.8 \times 4.0 \times (5.0 - 3.0)}{(5.0 + 3.0)} g h} = 4.43 \text{ m/s} \]

6. Two bodies are connected by a cord, which passes through a small pulley. The coefficient of friction between the 3.00-kg block and the surface is 0.4. Estimate the speed of the 5.00-kg ball when it has fallen 1.50 m?

**Solution**

For the mass, \( m_i = 5.00 \text{ kg} \), the initial energy

\[ K_{ii} = U_{ii} = 0 \]
Therefore, 

\[ E_{ij} = 0 \]

The final kinetic and potential energies are

\[ K_{ij} = \frac{1}{2} m_i v^2 \]
\[ U_{ij} = -m_i gh \]

Therefore,

\[ E_{ij} = \frac{1}{2} m_i v^2 - m_i gh \]

For the mass, \( m_2 = 3.00 \text{ kg} \), we know that the total work is equivalent to the change in the kinetic energy

\[ W = \Delta K = - \Delta U \]

The work done by the friction force only, therefore

\[ W = -\mu_k m_2 gd \]

Hence

\[ \Delta K = -\mu_k m_2 gd \]

From the conservation law of energy we get

\[ \frac{1}{2} m_i v^2 - m_i gh = -\mu_k m_2 gd \]

Or

\[ v = \left[ 2 \left( m_1 - \mu_k m_2 \right) / \left( m_1 + m_2 \right) \right]^{0.5} \]

\[ v = \left[ 2 \left( 5.0 - 0.4 \times 3.0 \right) / \left( 5.0 + 3.0 \right) \times 9.8 \times 1.5 \right]^{0.5} = 3.73 \text{ m/s} \]
7. A motorcyclist is trying to jump across the valley by driving horizontally off the cliff at a speed of 38.0 m/s. Find the speed with which the cycle strikes the ground on the other side, as shown in the figure.

**Solution**

The initial energy at the top, $E_i$, is

$$E_i = K_i + U_i = \frac{1}{2} m v_0^2 - mg h_0$$

The final energy at the bottom, $E_f$, is

$$E_f = K_f + U_f = \frac{1}{2} m v_f^2 - mg h_f$$

From the conservation law, we find

$$\frac{1}{2} m v_0^2 - mg h_0 = \frac{1}{2} m v_f^2 - mg h_f$$

Then we get

$$v_f = \sqrt{v_0^2 - 2 g (h_0 - h_f)}$$

$$v_f = \sqrt{38.0^2 - 2 \times 9.8 \times (70.0 - 35.0)} = 27.53 \text{ m/s}$$
8. A person is sitting on a sledge at the top of a 23.7 m tall hill. Determine their speed when they reach the bottom of the hill.

**Solution**

The initial energy at the top, $E_i$, is

$$E_i = K_i + U_i = 0 + mg h$$

The final energy at the bottom, $E_f$, is

$$E_f = K_f + U_f = \frac{1}{2} mv^2 + 0$$

From the conservation law, we find

$$\frac{1}{2} mv^2 = mg h$$

Then we get

$$v = \sqrt{2 g h}$$

$$v = \sqrt{2 \times 9.8 \times 23.7} = 21.6 \text{ m/s}$$