Math 241 (A02) - Midterm Exam
Winter 2005/2006
Part I (18 points)

Show all work to get full credit. No work may amount to no credit.

Name: PID#: 

1. (4 Points) Identify the following statements as true or false:
   a. Every homogenous LS with $m$ equations and $n$ unknowns has a nontrivial solution if $m < n$.
   b. The identity matrix is a diagonal matrix with diagonal entries equal to 1.
   c. An upper triangular matrix is a square matrix with all entries above the diagonal equal to zero.
   d. The transpose of a $10 \times 4$ matrix is a $10 \times 10$ matrix.
   e. The subspace spanned by the columns of a matrix $A$ is called the range of $A$.
   f. Any linearly independent set of $n$ vectors in $F^n$ is a basis of $F^n$.
   g. If a subspace $W$ has a basis of 5 vectors, then the dimension of $W$ is 6.
   h. For any two $n \times n$ matrices $A$ and $B$, $AB = BA$.

2. (5 Points)
   a. For $A = \begin{bmatrix} 1 & 2 & 3 \\
                         1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\
                         0 & 2 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 7 & -3 & 11 \\
                         5 & 0 & 9 \end{bmatrix}$
   verify that $(A + C)^T = A^T + C^T$ and $(AB)^T = B^T A^T$.
   b. Find the symmetric matrix $A$ such that

   $$X^T A X = 7x_1^2 + 9x_2^2 - 5x_3^2 + 2x_1 x_2 + 22x_1 x_3$$

   where $X = \begin{bmatrix} x_1 \\
                           x_2 \\
                           x_3 \end{bmatrix}$.
3. (3 points) Determine whether \( \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \end{bmatrix} \) and \( \begin{bmatrix} -1 \\ 8 \end{bmatrix} \) form a basis of \( \mathbb{R}^2 \). If not, find a subset that forms a basis.
4. (3 points) Let
\[ A = \begin{bmatrix} 3 & 4 & -7 \\ -1 & 3 & -2 \\ 1 & 2 & 1 \end{bmatrix}. \]

Does the vector \(\begin{bmatrix} 10 \\ 1 \\ 8 \end{bmatrix}\) belong to the subspace spanned by the columns of \(A\)? If yes, express it as a linear combination of the columns of \(A\).
5. (3 points) Reduce the following matrix into reduced row echelon form:

\[
\begin{bmatrix}
3 & 1 & 0 & 0 \\
1 & 0 & -1 & 0 \\
2 & 3 & 4 & 5
\end{bmatrix}.
\]