OP-AMPS in SIGNAL PROCESSING APPLICATIONS

Filters, Integrators, Differentiators, and Instrumentation Amplifier
Sensors: definition and principles
Sensors : taxonomies

• Measurand
  – physical sensor
  – chemical sensor
  – biological sensor (*cf* : biosensor)

• Invasiveness
  – invasive(contact) sensor
  – noninvasive(noncontact) sensor

• Usage type
  – multiple-use(continuous monitoring) sensor
  – disposable sensor

• Power requirement
  – passive sensor
  – active sensor
What is electronics engineering all about?

Physical variables
- F, P, d, v, t, f

Electrical variables
- V, I, t, f

Process

Sensors/ transducers

Filters

Amplifiers

Feedback

A/D converters

Computers

D/A converters

Actuators/transducers

New process
What is a Signal Processor?

- It selects useful parts of a signal
- It cleans signal from impurities
- It compares signals
- It converts signals into forms that can be easily recognized

The ECG wave in its raw form

Isoelectric line

PQRST
Examples of biomedical signals
Examples of Signal Processors

- Basic amplifiers – already discussed
- Instrumentation amplifier – an improved differential amplifier
- Active filters
- Integrators and differentiators
- Extra Reading:
  - Precision rectifiers
  - Logarithmic amplifiers
  - Negative capacitance amplifiers
Signal Conditioning

- Ideal operational amplifier
- Inverting, non-inverting and instrument amplifier
- Integrator, differentiator
- Filters
- Examples of signal conditioning module
Operational (OP) amplifier is a high-gain dc differential amplifier. It is made of an integrated circuit chip. It has two inputs, negative terminal $v_1$ and positive terminal $v_2$ and one output $v_0$. The effective input to the amplifier is the voltage difference of the two inputs $v_1 - v_2$. 
Properties of Ideal OP Amplifier

- Gain if infinity \((A=\infty)\).
- \(V_0 = 0\), when \(v_1 = v_2\)
- Input impedance is infinity.
- Output impedance is zero.
- Bandwidth is infinitely wide and has no phase shift.
Basic Rules for Ideal OP Amplifier

• When the OP amplifier output is in its linear range, the two input terminals are at the same voltage.

• No current flows into either input terminal of the OP amp.
Inverting Amplifier
Non-inverting Amplifier
Instrument Amplifier
Active Filters

- Low-pass
- High-pass
- Band-pass
- Band-stop (notch)
- All-pass (phase-shift)
Derive the equation for gain $G$

What are the important parameters?

$$G_L = \frac{R_f}{R_i} ; \tau_f = R_f C_f ; f_c = \frac{1}{2\pi \tau_f}$$

$$G = \frac{V_0(j\omega)}{V_i(j\omega)}$$

$$G = -\frac{Z_f}{Z_i}$$

$$= -\frac{R_f}{R_i} \frac{\left(\frac{1}{j\omega C_f}\right) + R_f}{\left(1 + j\omega R_f C_f\right) R_i} = -\frac{R_f}{R_i} \frac{1}{1 + j\omega \tau_f} = \frac{G_L}{1 + j\omega \tau_f}$$
Low-pass filter: characteristics

Gain \( G_L \)

\[
\begin{array}{c|cccc}
0.1 & 1 & 10 & 100 \\
\hline
0.1 & 0.01 & & \\
\hline
-0.01 & & & \\
\end{array}
\]

Slope = -1

Phase

\[
\begin{array}{c|cccc}
-\pi & 1 & & \\
\hline
-5\pi/4 & & & \\
-3\pi/2 & & & \\
\end{array}
\]

\[
G_L = \frac{R_f}{R_i} ; \quad \tau_f = R_f C_f ; \quad f_c = \frac{1}{2\pi \tau_f}
\]
Integrator – ideal: circuit

\[ i = \frac{V_i}{R_i} = C_f \frac{dV_0}{dt} \]

Virtual Ground

\[ V_0 = -\frac{1}{R_i C_f} \int_0^t V_i \, dt + V_{iC} \]

Express \( V_0 \) in terms of \( V_i \)

\[ G = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{1}{j\omega C_f} \]

Where \( \tau = R_i C_f \)

\[ G = -\frac{1}{j\omega R_i C_f} = -\frac{1}{j\omega \tau_f} \]
Integrator – ideal: characteristics

\[ \frac{G}{G_L} \] versus \( f/f_c \)

- Slope = -1

\[ \theta = -\frac{3\pi}{2} = -270^\circ \]

\[ G = \frac{1}{\omega \tau} = \frac{\omega_c}{\omega} = \frac{f_c}{f} \]

Draw \( \frac{G}{G_L} \) versus \( f/f_c \)

Draw phase versus \( f/f_c \)
The integrator with bias currents

Through which component the bias current flows?
**Integrator - practical**

- **R\(_f\)** provides a route for DC bias currents.

- **Integrator**
  
  ![Integrator Diagram]

- **Gain**
  \[
  G = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{R_f}{R_i} \frac{1}{1 + j\omega\tau_f} = \frac{G_L}{1 + j\omega\tau_f}
  \]

- **Phase**
  \[
  \tau_f = R_f C_f; f_c = \frac{1}{2\pi\tau_f}
  \]

- **Gain Plot**
  
  - Without **R\(_f\)**
  - With **R\(_f\)**
  - Slope = -1
  
  **Low-pass filter**

- **Frequency Response**
  
  - Phase
    - \(-\pi\)
    - \(-3\pi/2\)
    - \(-5\pi/4\)
  
  - Gain
    - 0.1
    - 0.01

- **Integ.**
  
  - Without **R\(_f\)**
  - With **R\(_f\)**
The piezoelectric sensor generates charge, which is transferred to the capacitor, $C$, by the charge amplifier. Feedback resistor $R$ causes the capacitor voltage to decay to zero.
Charge amplifier: circuit

\[ dq_s / dt = i_s = Kdx / dt \]

Virtual Ground

Piezoelectric sensor

\[ I_{SC} = I_{SR} = 0 \]

\[ V_0 = -V = - \frac{1}{C_f} \int_0^t \frac{Kdx}{dt} dt = - \frac{KX}{C_f} \]
The charge amplifier responds to a step input with an output that decays to zero with a time constant \( \tau = R_f C_f \).
High-pass filter: circuit

Derive the equation for gain \( G \)

\[
G = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i}
\]

What are the important parameters?

\[
G_H = -\frac{R_f}{R_i} ; \ \tau_i = R_iC_i ; \ f_c = \frac{1}{2\pi\tau_i}
\]

\[
G = -\frac{R_f}{R_i + \frac{1}{j\omega C_i}} = -\frac{j\omega R_f C_i}{1 + j\omega R_i C_i} = -\frac{R_f}{R_i} \frac{j\omega \tau_i}{1 + j\omega \tau_i} = \frac{j\omega \tau_i G_H}{1 + j\omega \tau_i}
\]
High-pass filter: characteristics

Gain/\(G_H\)

\[ \begin{array}{c|c|c|c}
  f/f_c & \text{slope } = +1 & \text{phase } \frac{-3\pi}{4} & \text{phase } \frac{-\pi}{2} \\
  0.1 & 0.01 & 0.1 & 0.1 \end{array} \]

Draw \(G/G_H\) versus \(f/f_c\)

Slope = +1

Draw phase versus \(f/f_c\)
Differentiator - ideal

\[ i = C_i \frac{dV_i}{dt} \]

\[ V_0 = -R_f C_i \frac{dV_i}{dt} \]

\[ f_c = \frac{1}{2\pi \tau} \]

Express \( V_0 \) in terms of \( V_i \)

\[ G = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R_f}{1/j\omega C_i} \]

\[ G = -j\omega R_f C_i = -j\omega \tau \]

Gain

Slope = +1

Phase shift = -90°

Write expression for gain \( G \)
Differentiator - practical

In an ideal differentiator, the high frequency response is limited by the open-loop gain of the op-amp yielding a very noisy output voltage.

\[ \tau_f = R_f C_f ; f_c = 1/2\pi\tau_f \]

\[ C_f \text{ is used to limit the hf gain, hence the hf noise.} \]

The gain at hf:

\[ G_H = -\frac{C_i}{C_f} \]

\[ \tau_f = R_f C_f ; f_c = 1/2\pi\tau_f \]

\[ G = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{R_f}{R_f + \frac{1}{j\omega C_f}} \]

\[ \frac{1}{j\omega C_i} = \frac{j\omega R_f C_i}{1 + j\omega R_f C_i} = \frac{C_i}{C_f 1 + j\omega \tau_f} \]
Differentiator characteristics

Differentiator

Gain/G_H

-\pi/2

Phase

With C_f

Slope = +1

-3\pi/4

-\pi

f/f_c

0.1 1 10 100

0.1

0.01

With C_f

Without C_f

High-pass filter
Measurement of integrator and differentiator characteristics

- Form your lab team and assign duties.
- Go to the lab bench.
- Built the circuit given: integrator for team 1 and differentiator for team 2
- Connect the power supply.
- Connect the CRO
- Do the experiment according to the procedures in the sheet - 6.
- Switch off the PS and signal generator
- Return your seat
Band-pass filter: circuit

Mid-band gain: \( G_{MB} = -\frac{R_f}{R_i} \)

Time-constants: \( \tau_i = R_iC_i \); \( \tau_f = R_fC_f \)

Critical frequencies: \( f_L = \frac{1}{2\pi\tau_i} \); \( f_H = \frac{1}{2\pi\tau_f} \)

\[
G = \frac{V_0(j\omega)}{V_i(j\omega)} = -\frac{Z_f}{Z_i} = -\frac{\left(\frac{R_f}{j\omega C_f}\right)}{R_i + \left(\frac{1}{j\omega C_i}\right)} = -\frac{j\omega C_f R_f}{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}
\]

\[
G = -\frac{R_f}{R_i} \frac{j\omega \tau_i}{(1 + j\omega \tau_i)(1 + j\omega \tau_f)} = \frac{G_{MB}j\omega \tau_i}{(1 + j\omega \tau_i)(1 + j\omega \tau_f)}
\]
Band-pass filter: characteristics

Gain/$G_H$

Slope = +1

Slope = -1
Band-pass filter (non-inverting)
Frequency response of bpf

Gain

Open-loop gain

Overall gain

Frequency (Hz)

Gain

100K

| \frac{V_o}{V+} |

100
60
1
0.8

25 159 15.9K 2.5M
BPF with bias compensation

\[ R_a = 10K \]
\[ R_1 = 820K \]
\[ C_1 = 0.1\mu F \]
\[ R_2 = 820K \]
\[ C_2 = 1nF \]
\[ R_f = 220K \]
Comparators: Basic Rules

- $V_0 = A(V_2 - V_1)$
- $V_0 = 0$ if $V_1 = V_2$
- $V_0 = +V_{SAT}$ if $V_1 < V_2$
- $V_0 = -V_{SAT}$ if $V_1 > V_2$
- No current flows into either input terminals of the op-amp.
**Simple comparator (inverting)**

\[ V_- = \frac{V_i R_2 + V_{ref} R_1}{R_1 + R_2} \]

Output stays at \(+V_{SAT}\) if \(V_- < 0\)

Output goes to \(-V_{SAT}\) if \(V_- > 0\)
Comparator with hysteresis

\[ V_- = \frac{V_i R_2 + V_{ref} R_1}{R_1 + R_2} \]

\[ V_+ = \frac{V_0 R_3}{R_3 + R_4} \]

Output stays at \(+V_{SAT}\) if \(V_- < V_+\)

Output goes to \(-V_{SAT}\) if \(V_- > V_+\)
Characteristics

\[ V_H = -V_{\text{ref}} \frac{R_1}{R_2} - V_{\text{SAT}} \frac{R_3}{(R_3 + R_4)} \]

\[ V_L = -V_{\text{ref}} \frac{R_1}{R_2} + V_{\text{SAT}} \frac{R_3}{(R_3 + R_4)} \]
Op-Amps in Reality

- Fabricated using integrated-circuit technologies
  - Inside an op-amp:
    1) Transistors
    2) Parasitic capacitors
    3) Internal resistors

- Op-amp chip configuration
  - Quad-channel (i.e. 4-in-1)
  - 14 pins in the op-amp chip
Instrumentation Amplifier
Overall Circuit Structure

- Overall differential gain is given by:

\[
G_D = G_\alpha G_\beta = \left(1 + \frac{2R_2}{R_1}\right) \left(\frac{R_4}{R_3}\right)
\]

\[
G_\alpha = 1 + \frac{2R_2}{R_1}
\]

\[
G_\beta = \frac{R_4}{R_3}
\]

Input Conditioner

Difference Amplifier
In-Amp: Input Conditioner

• Note that $i_{R1} = i_{R2a} = i_{R2b}$. Then the diff. output voltage is given by:

$$v_b' - v_a' = i_{R1} (R_1 + 2R_2)$$

• The diff. input voltage is equal to:

$$v_b - v_a = i_{R1} R_1$$

• The differential gain is thus equal to:

$$G_D = \frac{v_b' - v_a'}{v_b - v_a} = 1 + \frac{2R_2}{R_1}$$
In-Amp: Input Conditioner

• This input conditioner does not amplify common-mode signals!

• Brief proof of principle:
  – Since \( v_b - v_a = i_{R1} R_1 \):
    \[
    v_a = v_b = v_{cm} \quad \Rightarrow \quad i_{R1} = 0
    \]
  – The output voltages are thus equal to:
    \[
    v'_a = v'_b = v_{cm}
    \]
In-Amp: Difference Amplifier

1) From voltage divider principles:

\[ v_+ = v_b \left( \frac{R_2}{R_1 + R_2} \right) \]

2) Note that \( i_{R1a} = i_{R2a} \). Thus:

\[ \frac{v_o - v_-}{R_2} = \frac{v_- - v_a}{R_1} \]

\[ \Rightarrow \frac{v_o}{R_2} = \left( \frac{1}{R_2} + \frac{1}{R_1} \right) v_- - \frac{v_a}{R_1} \]
In-Amp: Difference Amplifier

3) Note that $v_\neq v_+$. So from the voltage divider expression:

$$v_- = v_b \left( \frac{R_2}{R_1 + R_2} \right)$$

4) Substituting the above into the output voltage expression, we get:

$$\frac{v_o}{R_2} = \left( \frac{R_2 + R_1}{R_2 R_1} \right) \left( \frac{R_2}{R_2 + R_1} \right) v_b - \frac{v_a}{R_1}$$

$$\Rightarrow v_o = \frac{R_2}{R_1} (v_b - v_a) \quad \Rightarrow \quad G_d = \frac{v_o}{v_b - v_a} = \frac{R_2}{R_1}$$

$$\begin{align*}
\end{align*}$$