بسم الله الرحمن الرحيم

King Abdulaziz University

Review for Trig and Trig Substitution Integral

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على عشرة إسئلة.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus II
Math202

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Version 1.0

Enter Name: ______

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Start Answer each of the following.

1. $\displaystyle\int \frac{dx}{(x^2+1)^{\frac{3}{2}}} =$

- ☐ $\dfrac{x}{\sqrt{x^2+1}} + C$
- ☐ $\dfrac{\sqrt{x^2+1}}{x} + C$
- ☐ $\dfrac{1}{\sqrt{x^2+1}} + C$
- ☐ $\dfrac{1}{x\sqrt{x^2+1}} + C$

2. $\int \frac{x^2}{\sqrt{1-x^2}}\,dx =$

- ☐ $\dfrac{\sin^{-1}(x) + \sqrt{1-x^2}}{2} + C$
- ☐ $\dfrac{\sin^{-1}(x) - \sqrt{1-x^2}}{2} + C$
- ☐ $\dfrac{\sin^{-1}(x) - x\sqrt{1-x^2}}{2} + C$
- ☐ $\dfrac{\sin^{-1}(x) + x\sqrt{1-x^2}}{2} + C$

3. $\displaystyle\int_{2\sqrt{2}}^{4} \frac{dx}{\sqrt{x^2-4}} =$

- ☐ $\frac{\pi}{4}$
- ☐ $\frac{\pi}{24}$
- ☐ $\frac{\pi}{12}$
- ☐ $\frac{\pi}{6}$

4. $\displaystyle\int \frac{dx}{\sqrt{9+4x-x^2}} =$

- ☐ $\frac{1}{5}\sec^{-1}\left(\frac{x-4}{5}\right)+C$
- ☐ $\cosh^{-1}\left(\frac{x-4}{5}\right)+C$
- ☐ $\sinh^{-1}\left(\frac{x-4}{5}\right)+C$
- ☐ $\sin^{-1}\left(\frac{x-4}{5}\right)+C$

5. $\int \frac{\sin^3 x}{\sqrt{\cos x}}\, dx =$

- ☐ $\frac{2}{5}\sqrt{\cos^5 x} + 2\sqrt{\cos x} + C$
- ☐ $\frac{2}{5}\sqrt{\cos^5 x} - 2\sqrt{\cos x} + C$
- ☐ $\frac{2}{5}\sqrt{\sin^5 x} + 2\sqrt{\sin x} + C$
- ☐ $\frac{2}{5}\sqrt{\sin^5 x} - 2\sqrt{\sin x} + C$

6. $\int \frac{\tan^3 x}{\sqrt{\sec x}}\, dx =$

- ☐ $\frac{2}{3}\sqrt{\tan^3 x} + 2\sqrt{\tan x} + C$
- ☐ $\frac{2}{3}\sqrt{\sec^3 x} - 2\sqrt{\sec x} + C$
- ☐ $\frac{2}{3}\sqrt{\sec^3 x} + \dfrac{2}{\sqrt{\sec x}} + C$
- ☐ $\frac{2}{3}\sqrt{\sec^3 x} - 2\sqrt{\sec x} + C$

7. $\int \sinh^3 x \cosh^8 x\, dx =$

- ☐ $\frac{1}{11}\cosh^{11} x - \frac{1}{9}\cosh^9 x + C$
- ☐ $\frac{1}{11}\sinh^{11} x + \frac{1}{9}\sinh^9 x + C$
- ☐ $\frac{1}{11}\cosh^{11} x + \frac{1}{9}\cosh^9 x + C$
- ☐ $\frac{1}{11}\sinh^{11} x - \frac{1}{9}\sinh^9 x + C$

8. $\int \csc^4 x \cot^4 x \, dx =$

- ☐ $\frac{1}{7}\cot^7 x + \frac{1}{5}\cot^5 x + C$
- ☐ $\frac{-1}{7}\csc^7 x - \frac{1}{5}\csc^5 x + C$
- ☐ $\frac{1}{7}\csc^7 x + \frac{1}{5}\csc^5 x + C$
- ☐ $\frac{-1}{7}\cot^7 x - \frac{1}{5}\cot^5 x + C$

9. $\int \tan^2 x \sec x \, dx =$

- ☐ $\frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$
- ☐ $\frac{1}{2}\sec x \tan x - \frac{1}{2}\ln|\sec x + \tan x| + C$
- ☐ $\frac{-1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x|$
- ☐ $\frac{1}{2}\sec x \tan x - \ln|\sec x + \tan x|$

10. The correct trigonometric substation for the integral $\int \frac{dx}{x^2\sqrt{4-9x^2}}\, dx$ is

- [ ] $x = \frac{2}{3}\tan\theta$
- [ ] $x = \frac{2}{3}\sec\theta$
- [ ] $x = \frac{2}{3}\sin\theta$
- [ ] $x = \frac{3}{2}\sec\theta$

End

Score: Correct

Answers:

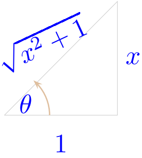
Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1. Let

$$x = \tan\theta,$$
$$dx = \sec^2\theta\, d\theta$$
$$\sqrt{x^2+1} = \sec\theta.$$



$$\int \frac{dx}{(\sqrt{x^2+1})^3} = \int \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$$
$$= \int \frac{d\theta}{\sec\theta}$$
$$= \int \cos\theta\, d\theta$$
$$= \int \cos\theta\, d\theta$$
$$= \sin\theta + C = \frac{x}{\sqrt{x^2+1}} + C.$$

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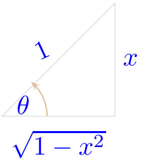
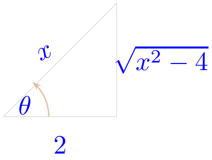
Solution to 2. Let

$$x = \sin\theta,$$
$$dx = \cos\theta\, d\theta$$
$$\sqrt{1-x^2} = \cos\theta.$$



$$\begin{aligned}
\int \frac{x^2\,dx}{\sqrt{1-x^2}} &= \int \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta \\
&= \int \sin^2\theta\,d\theta \\
&= \int \left[\frac{1-\cos(2\theta)}{2}\right] d\theta \\
&= \frac{1}{2}[\theta - \frac{1}{2}\sin(2\theta)] + C \\
&= \frac{1}{2}[\theta - \sin\theta\cos\theta] + C = \frac{1}{2}[\sin^{-1}(x) - x\sqrt{1-x^2}] + C.
\end{aligned}$$

Solution to 3. Let $x = 2\sec\theta$,
$dx = 2\sec\theta\tan\theta\, d\theta$
$\sqrt{x^2-4} = 2\tan\theta$.
Now,
$\theta = \sec^{-1}\left(\frac{x}{2}\right)$
hence when $x = 2\sqrt{2}$ then
$\theta = \sec^{-1}\left(\sqrt{2}\right) = \frac{\pi}{3}$.
Also when $x = 4$ then
$\theta = \sec^{-1}(2) = \frac{\pi}{4}$



$$\int_{2\sqrt{2}}^{4} \frac{dx}{\sqrt{x^2-4}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2\sec\theta 2\tan\theta}\, 2\sec\theta\tan\theta\, d\theta$$
$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2}\, d\theta$$
$$= \frac{1}{2}\left[\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$
$$= \frac{1}{2}\left[\frac{\pi}{3} - \frac{\pi}{4}\right]$$
$$= \frac{\pi}{24}.$$

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Solution to 4.

$$\begin{aligned} 9 + 8x - x^2 &= -x^2 + 8x + 9, \\ &= -(x^2 - 8x + 4^2 - 4^2) + 9, \\ &= -(x^2 - 8x + 4^2) + 16 + 9, \\ &= 25 - (x-4)^2. \end{aligned}$$

$$\begin{aligned} \int \frac{dx}{\sqrt{9 + 4x - x^2}} &= \int \frac{1}{\sqrt{5^2 - (x-4)^2}}\, dx \\ &= \sin^{-1}\left(\frac{x-4}{5}\right) + C. \end{aligned}$$

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Solution to 5.

$$\begin{aligned}
\int \frac{\sin^3 x}{\sqrt{\cos x}}\,dx &= \int \frac{\sin^2 x}{\sqrt{\cos x}}\,\sin x\,dx \\
&= \int \frac{1-\cos^2 x}{\sqrt{\cos x}}\,\sin x\,dx \\
&= -\int \frac{1-\cos^2 x}{\sqrt{\cos x}} - \sin x\,dx \\
&= \int \frac{\cos^2 x - 1}{\sqrt{\cos x}}\,d(\cos x) \\
&= \int (\cos^{\frac{3}{2}} x - \cos^{\frac{-1}{2}} x)\,d(\cos x) \\
&= \frac{2}{5}\cos^{\frac{5}{2}} x + 2\cos^{\frac{1}{2}} x + C \\
&= \frac{2}{5}\sqrt{\cos^5 x} + 2\sqrt{\cos x} + C
\end{aligned}$$

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Solution to 6.

$$\begin{aligned}
\int \frac{\tan^3 x}{\sqrt{\sec x}}\,dx &= \int \frac{\tan^2 x}{\sec x\sqrt{\sec x}}\,\sec x\tan x\,dx \\
&= \int \frac{\sec^2 x - 1}{\sec^{\frac{3}{2}}}\,\sec x\tan x\,dx \\
&= \int (\sec^{\frac{1}{2}} x - \sec^{\frac{-3}{2}} x)\,d(\sec x) \\
&= \frac{2}{3}\sec^{\frac{3}{2}} x + 2\sec^{\frac{-1}{2}} x + C \\
&= \frac{2}{3}\sqrt{\sec^3 x} + \frac{2}{\sqrt{\sec x}} + C
\end{aligned}$$

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Solution to 7.

$$\begin{aligned}\int \sinh^3 x \cosh^8 x\,dx &= \int \sinh^2 x \cosh^8 x\ \sinh x\,dx\\ &= \int (\cosh^2 x - 1)\cosh^8 x\ \sinh x\,dx\\ &= \int (\cosh^{10} x - \cosh^8 x)\,d(\cosh x)\\ &= \frac{1}{11}\cosh^{11} x - \frac{1}{9}\cosh^9 x + C.\end{aligned}$$

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Solution to 8.

$$\int \csc^4 x \cot^4 x \, dx = = \int \csc^2 x \cot^4 x \ \csc^2 x \, dx$$

$$= - \int (1 + \cot^2 x) \cot^4 x - \csc^2 x \, dx$$

$$= - \int (\cot^4 x + \cot^6 x) \, d(\cot x)$$

$$= \frac{-1}{7} \cot^7 x - \frac{1}{5} \cot^5 x + C.$$

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Solution to 9.

$$\begin{aligned}
\int \tan^2 x \sec x \, dx = &= \int (\sec^2 x - 1) \sec x \, dx \\
&= \int \sec^3 x \, dx - \int \sec x \, dx \\
&= \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x \, dx - \int \sec x \, dx \\
&= \frac{1}{2} \sec x \tan x - \frac{1}{2} \int \sec x \, dx \\
&= \frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C.
\end{aligned}$$

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Solution to 10. $\displaystyle\int \frac{dx}{x^2\sqrt{4-9x^2}}\,dx = \int \frac{dx}{x^2\sqrt{2^2-(3x)^2}}\,dx$

Hence $3x = 2\sin\theta$. Therefore $x = \frac{2}{3}\sin\theta$. ■