بسم الله الرحمن الرحيم

King Abdulaziz University

Review for The First Exam

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على عشرون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus II
Math202

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Version 1.0

Enter Name:

I.D. Number:

Start Answer each of the following.

1. $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) =$

- 0
- $2x$
- e
- 1

2. $\operatorname{sech}^{-1}\left(\frac{2}{3}\right) =$

- ☐ $\ln\left(\dfrac{3-\sqrt{5}}{2}\right)$
- ☐ $\ln\left(\dfrac{3-\sqrt{13}}{2}\right)$
- ☐ $\ln\left(\dfrac{3+\sqrt{5}}{2}\right)$
- ☐ $\ln\left(\dfrac{3+\sqrt{13}}{2}\right)$

3. If $y = \operatorname{sech}^3\left(x^2 + e^x\right)$, then

- ☐ $y' = 3(2x + e^x)\operatorname{sech}^3\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$
- ☐ $y' = -3(2x + e^x)\operatorname{sech}^3\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$
- ☐ $y' = (2x + e^x)\operatorname{sech}^3\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$
- ☐ $y' = 3(2x + e^x)\operatorname{sech}^2\left(x^2 + e^x\right)\tanh\left(x^2 + e^x\right)$

4. $\int \frac{dx}{x\sqrt{4-9x^2}} =$

- ☐ $\frac{1}{2}\ln\left|\frac{2+\sqrt{4-9x^2}}{3x}\right| + C$
- ☐ $\frac{-1}{2}\ln\left|\frac{2-\sqrt{4-9x^2}}{3x}\right| + C$
- ☐ $\frac{1}{2}\ln\left|\frac{2-\sqrt{4-9x^2}}{3x}\right| + C$
- ☐ $\frac{-1}{2}\ln\left|\frac{2+\sqrt{4-9x^2}}{3x}\right| + C$

5. $\int_{0}^{\frac{\pi}{2}} \sin^3 x \cos^2 x \, dx =$

- ☐ $\frac{2}{15}$
- ☐ $\frac{-2}{15}$
- ☐ $\frac{1}{15}$
- ☐ $\frac{-1}{15}$

6. $\int \frac{\tan^3 x}{\sqrt[4]{\sec x}}\, dx =$

- ☐ $\frac{4}{7}\sqrt[4]{\tan^7 x} + 4\sqrt[4]{\tan x} + C$
- ☐ $\frac{4}{7}\sqrt[4]{\sec^7 x} - 4\sqrt[4]{\sec x} + C$
- ☐ $\frac{4}{7}\sqrt[4]{\sec^7 x} + \frac{4}{\sqrt[4]{\sec x}} + C$
- ☐ $\frac{4}{7}\sqrt[4]{\sec^7 x} - 4\sqrt[4]{\sec x} + C$

7. $\int x\,\mathrm{sech}^2\, x\, dx =$

- ☐ $x \tanh x - \ln(\cosh x) + C$
- ☐ $x \tanh x + \ln(\cosh x) + C$
- ☐ $x \tanh x - \ln(\sinh x) + C$
- ☐ $x \tanh x + \ln(\sinh x) + C$

8. $\int \frac{1}{(1+x^2)^2}\,dx =$

- ☐ $\frac{1}{2}\tan^{-1} x - \frac{x}{x^2+1} + C$
- ☐ $\frac{1}{2}\tan^{-1} x + \frac{x}{x^2+1} + C$
- ☐ $\frac{1}{2}\tan^{-1} x - \frac{x}{2x^2+2} + C$
- ☐ $\frac{1}{2}\tan^{-1} x + \frac{x}{2x^2+2} + C$

9. $\int \frac{1}{x^3\sqrt{x^2-9}}\,dx =$

- ☐ $\frac{1}{54}\sec^{-1}\left(\frac{x}{3}\right) - \frac{\sqrt{x^2-9}}{27x^2} + C$
- ☐ $\frac{1}{54}\sec^{-1}\left(\frac{x}{3}\right) + \frac{\sqrt{x^2-9}}{27x^2} + C$
- ☐ $\frac{1}{54}\sec^{-1}\left(\frac{x}{3}\right) - \frac{\sqrt{x^2-9}}{54x^2} + C$
- ☐ $\frac{1}{54}\sec^{-1}\left(\frac{x}{3}\right) + \frac{\sqrt{x^2-9}}{54x^2} + C$

10. The correct trigonometric substation for the integral $\displaystyle\int \frac{x\,dx}{\sqrt{9x^2-36x+11}}\,dx$ is

- ☐ $x=\frac{5}{3}\sin\theta$
- ☐ $x=\frac{5}{3}\sec\theta$
- ☐ $x=\frac{5}{3}\sin\theta+2$
- ☐ $x=\frac{5}{3}\sec\theta+2$

11. A trigonometric substation converts the integral $\int \frac{x\,dx}{\sqrt{9x^2-36x+11}}\,dx$ to

- ☐ $\int \frac{5\sin\theta}{9}\,d\theta$
- ☐ $\int \frac{5\sec^2\theta+6\sec\theta}{9}\,d\theta$
- ☐ $\int \frac{5\sec^2\theta}{9}\,d\theta$
- ☐ $\int \frac{5\sin\theta+6}{9}\,d\theta$

12. $\lim_{x\to 0} \frac{\sinh x - x}{x^2} =$

- ☐ 1
- ☐ 0
- ☐ -1
- ☐ e

13. $\lim_{x\to 0^+}\left(\dfrac{\cos(3x)}{\cos(2x)}\right)^{\frac{1}{x^2}} =$

- ☐ 1
- ☐ $\frac{-5}{2}$
- ☐ $\frac{1}{\sqrt{e^5}}$
- ☐ $\sqrt{e^5}$

14. $\lim_{x\to\infty}\left[\ln x - \sinh^{-1} x\right] =$

- ☐ 1
- ☐ $-\ln 2$
- ☐ 0
- ☐ $\ln 2$

15. $\displaystyle\int \frac{1}{\sqrt{x^2+2x+5}}\,dx =$

- ☐ $\ln\left(x+1-\sqrt{x^2+2x+5}\right)+C$
- ☐ $\ln\left(x-1-\sqrt{x^2+2x+5}\right)+C$
- ☐ $\ln\left(x-1+\sqrt{x^2+2x+5}\right)+C$
- ☐ $\ln\left(x+1+\sqrt{x^2+2x+5}\right)+C$

16. $\displaystyle\int \frac{x+1}{x^2-7x+12}\,dx =$

- ☐ $\ln\left(\dfrac{|x+4|^5}{(x-3)^4}\right)+C$
- ☐ $\ln\left(\dfrac{|x-4|^5}{(x-3)^4}\right)+C$
- ☐ $\ln\left(\dfrac{|x-4|^5}{(x+3)^4}\right)+C$
- ☐ $\ln\left(\dfrac{|x+4|^5}{(x+3)^4}\right)+C$

17. $\int \frac{x^3 + x}{x^2 - 1}\,dx =$

- ☐ $\frac{1}{2}x^2 + \ln\left|x^2 - 1\right| + C$
- ☐ $\frac{1}{2}x^2 - \ln\left|x^2 - 1\right| + C$
- ☐ $x^2 + \frac{1}{2}\ln\left|x^2 - 1\right| + C$
- ☐ $\frac{1}{2}x^2 - \frac{1}{2}\ln\left|x^2 - 1\right| + C$

18. The partial decomposition of $\dfrac{4x+6}{(x^2+1)(x-1)^2}$ is

- ☐ $\dfrac{3x-2}{x^2+1}+\dfrac{3}{x-1}$
- ☐ $\dfrac{3x-2}{x^2+1}+\dfrac{5}{(x-1)^2}$
- ☐ $\dfrac{3x-2}{x^2+1}+\dfrac{-3}{x-1}+\dfrac{5}{(x-1)^2}$
- ☐ $\dfrac{-3x-2}{x^2+1}+\dfrac{3}{x-1}+\dfrac{5}{(x-1)^2}$

19. The integral $\int_0^\infty \frac{1}{e^x + e^{-x}}\, dx$

- ☐ Converges to $\frac{\pi}{4}$
- ☐ Converges to $\frac{\pi}{2}$
- ☐ Converges to $\frac{3\pi}{4}$
- ☐ Diverges

20. The integral $\int_0^1 x \ln x \, dx$

- ☐ Diverges
- ☐ Converges to 0
- ☐ Converges to $\frac{1}{4}$
- ☐ Converges to $\frac{-1}{4}$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$\begin{aligned}\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) &= \ln(e^x) + \ln(e^{-x}) \\ &= x + (-x) \\ &= 0.\end{aligned}$$

Another solution:

$$\begin{aligned}\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x) &= \\ &\ln((\cosh x + \sinh x)(\cosh x - \sinh x)) \\ &= \ln(\cosh^2 x - \sinh^2 x) \\ &= \ln 1 = 0.\end{aligned}$$

■

Solution to 2.

$$\text{Since } \operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) 0 < x \le 1$$

$$\text{then } \operatorname{sech}^{-1}\left(\frac{2}{3}\right) = \ln\left(\frac{1+\sqrt{1-(\frac{2}{3})^2}}{\frac{2}{3}}\right)$$

$$= \ln\left(\frac{3(1+\sqrt{1-(\frac{4}{9})}}{2}\right)$$

$$= \ln\left(\frac{3(1+\frac{\sqrt{5}}{3})}{2}\right)$$

$$= \ln\left(\frac{3+\sqrt{5}}{2}\right).$$

■

Solution to 3.

$$
\begin{aligned}
y &= \left(\operatorname{sech}\left(x^2+e^x\right)\right)^3 \\
y' &= 3\left(\operatorname{sech}\left(x^2+e^x\right)\right)^2(-\operatorname{sech}\left(x^2+e^x\right)\tanh\left(x^2+e^x\right))(2x+e^x) \\
&= 3\operatorname{sech}^2\left(x^2+e^x\right)(-\operatorname{sech}\left(x^2+e^x\right)\tanh\left(x^2+e^x\right))(2x+e^x) \\
&= -3(2x+e^x)\operatorname{sech}^3\left(x^2+e^x\right)\tanh\left(x^2+e^x\right).
\end{aligned}
$$

■

Solution to 4. Remember that

$$\int \frac{du}{u\sqrt{a^2 - u^2}} = \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$\int \frac{dx}{x\sqrt{4 - 9x^2}} = \int \frac{1}{3x\sqrt{2^2 - (3x)^2}} \, 3dx$$

$$= \frac{-1}{2} \ln \left| \frac{2 + \sqrt{4 - 9x^2}}{3x} \right| + C.$$

■

Solution to 5.

$$\int_0^{\frac{\pi}{2}} \sin^3 x \cos^2 x\, dx = \int_0^{\frac{\pi}{2}} \sin^2 x \cos^2 x \ \sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \cos^2 x \ \sin x\, dx$$

$$= - \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \cos^2 x\, (- \sin x)\, dx$$

$$= \int_0^{\frac{\pi}{2}} (\cos^4 x - \cos^2 x)\, d(\cos x)$$

$$= \left[\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x\right]_0^{\frac{\pi}{2}} = \left[0 - (\frac{1}{5} - \frac{1}{3})\right] = \frac{2}{15}.$$

■

Solution to 6.

$$\begin{aligned}
\int \frac{\tan^3 x}{\sqrt[4]{\sec x}}\,dx &= \int \frac{\tan^2 x}{\sec x\sqrt[4]{\sec x}}\,\sec x\tan x\,dx \\
&= \int \frac{\sec^2 x - 1}{\sec^{\frac{5}{4}} x}\,\sec x\tan x\,dx \\
&= \int (\sec^{\frac{3}{4}} x - \sec^{\frac{-5}{4}} x)\,d(\sec x) \\
&= \frac{4}{7}\sec^{\frac{7}{4}} x + 4\sec^{\frac{-1}{4}} x + C \\
&= \frac{4}{7}\sqrt[4]{\sec^7 x} + \frac{4}{\sqrt[4]{\sec x}} + C
\end{aligned}$$

■

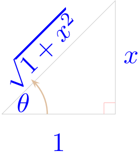
Solution to 7.

$$\begin{aligned}
\int x \operatorname{sech}^2 x \, dx &= \int x \, d(\tanh x) & u = x \quad dv = \operatorname{sech}^2 x \, dx \\
&= x \tanh x - \int \tanh x \, dx & du = dx \quad v = \tanh x \\
&= x \tanh x - \int \frac{\sinh x}{\cosh x} \, dx \\
&= x \tanh x - \ln(\cosh x) + C.
\end{aligned}$$

■

Solution to 8. Let

$$x = \tan\theta,$$
$$dx = \sec^2\theta\, d\theta$$
$$(1+x^2)^2 = \sec^4\theta.$$

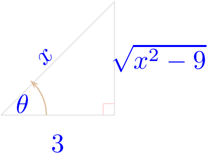


$$\begin{aligned}
\int \frac{dx}{(1+x^2)^2} &= \int \frac{\sec^2\theta}{\sec^4\theta}\, d\theta \\
&= \int \cos^2\theta\, d\theta \\
&= \int \left[\frac{1+\cos(2\theta)}{2}\right] d\theta = \frac{1}{2}[\theta + \frac{1}{2}\sin(2\theta)] + C \\
&= \frac{1}{2}[\theta + \sin\theta\cos\theta] + C = \frac{1}{2}[\tan^{-1}(x) + \frac{x}{\sqrt{1+x^2}}\frac{1}{\sqrt{1+x^2}}] + C \\
&= \frac{1}{2}[\tan^{-1}(x) + \frac{x}{1+x^2}] + C.
\end{aligned}$$

■

Solution to 9. Let

$$x = 3\sec\theta,$$
$$dx = 3\sec\theta\tan\theta\, d\theta$$
$$\sqrt{x^2-9} = 3\tan\theta$$
$$x^3 = 27\sec^3\theta.$$



$$\int \frac{dx}{x^3\sqrt{x^2-9}} = \int \frac{3\sec\theta\tan\theta\, d\theta}{(27\sec^3\theta)(3\tan\theta)} = \frac{1}{27}\int \cos^2\theta\, d\theta$$
$$= \frac{1}{27}\int \left[\frac{1+\cos(2\theta)}{2}\right] d\theta = \frac{1}{54}[\theta + \frac{1}{2}\sin(2\theta)] + C$$
$$= \frac{1}{54}[\theta + \sin\theta\cos\theta] + C = \frac{1}{54}\left[\sec^{-1}\left(\frac{x}{3}\right) + \frac{3}{x}\frac{\sqrt{x^2-9}}{x}\right] + C$$
$$= \frac{1}{54}\sec^{-1}\left(\frac{x}{3}\right) + \frac{\sqrt{x^2-9}}{27x^2} + C.$$

■

Solution to 10.

$$\begin{aligned} 9x^2 - 36x + 11 &= 9(x^2 - 4x) + 11, \\ &= 9(x^2 - 4x + 4 - 4) + 11 \\ &= 9[(x-2)^2 - 4] + 11 \\ &= 9(x-2)^2 - 36 + 11 = (3(x-2))^2 - 5^2. \end{aligned}$$

$$\int \frac{x\,dx}{\sqrt{9x^2 - 36x + 11}}\,dx = \int \frac{x\,dx}{\sqrt{(3(x-2))^2 - 5^2}}\,dx$$

Hence $3(x-2) = 5\sec\theta$.
Therefore $x - 2 = \frac{5}{3}\sec\theta$.
Thus $x = \frac{5}{3}\sec\theta + 2$. ■

Solution to 11.

$$\begin{aligned}9x^2-36x+11 &= 9(x^2-4x)+11,\\ &= 9(x^2-4x+4-4)+11\\ &= 9[(x-2)^2-4]+11\\ &= 9(x-2)^2-36+11 = (3(x-2))^2-5^2.\end{aligned}$$

$$\int \frac{x\,dx}{\sqrt{9x^2-36x+11}}\,dx = \int \frac{x\,dx}{\sqrt{(3(x-2))^2-5^2}}\,dx$$

Hence $3(x-2)=5\sec\theta$. Therefore $x-2=\frac{5}{3}\sec\theta$.
Thus $x=\frac{5}{3}\sec\theta+2$, and $dx=\frac{5}{3}\sec\theta\tan\theta\,d\theta$.
Also, $\sqrt{(3(x-2))^2-5^2}=5\tan\theta$.

$$\begin{aligned}\int \frac{x\,dx}{\sqrt{9x^2-36x+11}}\,dx &= \int \frac{x\,dx}{\sqrt{(3(x-2))^2-5^2}}\,dx\\ &= \int \frac{\left(\frac{5}{3}\sec\theta+2\right)\ \frac{5}{3}\sec\theta\tan\theta d\theta}{5\tan\theta}\\ &= \int \left(\frac{5}{3}\sec\theta+2\right)\frac{1}{3}\sec\theta d\theta = \int \frac{5\sec^2\theta+6\sec\theta}{9}\,d\theta.\end{aligned}$$

■

Solution to 12.

$$\lim_{x\to 0}\frac{\sinh x - x}{x^2} \overset{L.H.}{=} \lim_{x\to 0}\frac{\cosh x - 1}{2x}$$

$$\overset{L.H.}{=} \lim_{x\to 0}\frac{\sinh x}{2}$$

$$= \frac{0}{2} = 0.$$

■

Solution to 13.

Let $y=\left(\frac{\cos(3x)}{\cos(2x)}\right)^{\frac{1}{x^2}}$ $\ln y=\frac{\ln\left(\frac{\cos(3x)}{\cos(2x)}\right)}{x^2}=\frac{\ln(\cos(3x))-\ln(\cos(2x))}{x^2}$

$$\lim_{x\to0^+}\ln y=\lim_{x\to0^+}\frac{\ln(\cos(3x))-\ln(\cos(2x))}{x^2}\quad\left(\frac{0}{0}\right)\text{ I.F.}$$

$$\overset{L.H.}{=}\lim_{x\to0^+}\frac{\frac{-3\sin(3x)}{\cos(3x)}-\frac{-2\sin(2x)}{\cos(2x)}}{2x}$$

$$=\lim_{x\to0^+}\frac{-3\tan(3x)+2\tan(2x)}{2x}\quad\left(\frac{0}{0}\right)\text{ I.F.}$$

$$\overset{L.H.}{=}\lim_{x\to0^+}\frac{-9\sec^2(3x)+4\sec^2(2x)}{2}=\frac{-9+4}{2}=\frac{-5}{2}.$$

Hence

$$\lim_{x\to0^+}\left(\frac{\cos(3x)}{\cos(2x)}\right)^{\frac{1}{x^2}}=e^{\frac{-5}{2}}=\frac{1}{\sqrt{e^5}}.$$

■

Solution to 14.

$$\lim_{x\to\infty}\left[\ln x - \sinh^{-1} x\right] = \lim_{x\to\infty}\left[\ln x - \ln\left(x+\sqrt{1+x^2}\right)\right] \quad (\infty-\infty) \text{ I.F.}$$

$$= \lim_{x\to\infty} \ln\left(\frac{x}{x+\sqrt{1+x^2}}\right)$$

$$= \lim_{x\to\infty} \ln\left(\frac{x}{x+\sqrt{x^2\left(\frac{1}{x^2}+1\right)}}\right)$$

$$= \lim_{x\to\infty} \ln\left(\frac{x}{x\left(1+\sqrt{\frac{1}{x^2}+1}\right)}\right)$$

$$= \lim_{x\to\infty} \ln\left(\frac{1}{1+\sqrt{\frac{1}{x^2}+1}}\right)$$

$$= \ln\left(\frac{1}{2}\right) = -\ln 2.$$

■

Solution to 15.

$$\begin{aligned} x^2+2x+5 &= (x^2+2x)+5, \\ &= (x^2+2x+1-1)+5 \\ &= (x+1)^2-1+5 \\ &= (x+1)^2+4 = (x+1)^2+2^2. \end{aligned}$$

$$\begin{aligned} \int \frac{1}{\sqrt{x^2+2x+5}}\,dx &= \int \frac{1}{\sqrt{(x+1)^2+2^2}}\,dx \\ &= \ln\left(x+1+\sqrt{(x+1)^2+2^2}\right)+C \\ &= \ln\left(x+1+\sqrt{x^2+2x+5}\right)+C. \end{aligned}$$

■

Solution to 16.

$$
\begin{aligned}
x^2 - 7x + 12 &= (x-4)(x-3), \\
\frac{x+1}{(x-4)(x-3)} &= \frac{A}{x-4} + \frac{B}{x-3} \\
x+1 &= A(x-3) + B(x-4) \\
x = 3: \qquad 4 &= -B \Rightarrow B = -4 \\
x = 4: \qquad 5 &= A \\
\frac{x+1}{(x-4)(x-3)} &= \frac{4}{x-4} + \frac{-5}{x-3}
\end{aligned}
$$

$$
\begin{aligned}
\int \frac{x+1}{\sqrt{x^2-7x+12}}\,dx &= \int \left(\frac{5}{x-4} + \frac{-4}{x-3} \right) dx \\
&= 5\ln|x-4| - 4\ln|x-3| + C \\
&= \ln|x-4|^4 - \ln(x-3)^4 + C \\
&= \ln\left(\frac{|x-4|^5}{(x-3)^4} \right) + C.
\end{aligned}
$$

■

Solution to 17.

$$
\begin{array}{r|l}
 & x \\
\hline
x^2 - 1 & x^3 + x \\
 & -x^3 \overset{+}{-} x \\
\hline
 & 2x
\end{array}
$$

$$
\begin{aligned}
\frac{x^3 + x}{x^2 - 1} &= x + \frac{2x}{x^2 - 1} \\
\int \frac{x^3 + x}{x^2 - 1}\, dx &= \int \left(x + \frac{2x}{x^2 - 1} \right) dx \\
&= \frac{1}{2}x^2 + \ln\left| x^2 - 1 \right| + C.
\end{aligned}
$$

■

Solution to 18.

$$\frac{4x+6}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$4x+6 = (Ax+B)(x-1)^2 + c(x-1)(x^2+1) + D(x^2+1)$$

$$4x+6 = (Ax+B)(x^2-2x+1) + C(x^3-x^2+x-1) + D(x^2+1)$$

$$4x+6 = (A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B+C)x + (B-C+D)$$

$$x=1: \qquad 10 = 2D \Rightarrow D = 5.$$

Also, we have

$$\begin{aligned} \text{coff.}x^3: &\qquad 0 = A + C && (1)\\ \text{coff.}x^2: &\qquad 0 = -2A + B - C + D && (2)\\ \text{coff.}x: &\qquad 4 = A - 2B + C && (3)\\ \text{constant}: &\qquad 6 = B - C + D && (4)\\ & && (5) \end{aligned}$$

Using (4) and (2) we get that $0 = -2A + 6 \Rightarrow A = 3$.
Using (1) we get $C = -A = -3$.

Using (3) we get $4 = 3 - 2B - 3 \Rightarrow 4 = -2B \Rightarrow B = -2$. Therefore

$$\frac{4x+6}{(x^2+1)(x-1)^2} = \frac{3x-2}{x^2+1} + \frac{-3}{x-1} + \frac{5}{(x-1)^2}$$

■

Solution to 19.

$$\int_0^\infty \frac{1}{e^x+e^{-x}}\,dx = \lim_{t\to\infty}\int_0^t \frac{1}{e^x+e^{-x}}\,dx$$

$$= \lim_{t\to\infty}\int_0^t \frac{1}{e^x+e^{-x}}\frac{e^x}{e^x}\,dx$$

$$= \lim_{t\to\infty}\int_0^t \frac{1}{e^{2x}+1}\,e^x\,dx$$

$$= \lim_{t\to\infty}\int_0^t \frac{1}{(e^x)^2+1}\,e^x\,dx$$

$$= \lim_{t\to\infty}\left[\tan(e^x)\right]_0^t$$

$$= \lim_{t\to\infty}\left[\tan(e^t) - \tan 1\right] = \left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{\pi}{2}.$$

■

Solution to 20. Note that

$$\lim_{t\to 0^+} t^2 \ln t \quad (0.(-\infty)) \text{ I.F.}$$

$$\begin{aligned}\lim_{t\to 0^+} t^2 \ln t &= \lim_{t\to 0^+} \frac{\ln t}{\frac{1}{t^2}} \quad \left(\frac{-\infty}{\infty}\right) \text{ I.F.}\\ &\stackrel{L.H.}{=} \lim_{t\to 0^+} \frac{\frac{1}{t}}{\frac{-2}{t^3}}\\ &= \lim_{t\to 0^+} \frac{t^2}{-2}\\ &= 0.\end{aligned}$$

$$\begin{aligned}
\int_0^1 x\ln x\,dx &= \lim_{t\to 0^+}\int_t^1 x\ln x\,dx \\
&= \lim_{t\to 0^+}\int_t^1 \ln x\, d\left(\frac{1}{2}x^2\right) && u=\ln x \quad dv = x\,dx \\
&= \lim_{t\to 0^+}\left[\frac{1}{2}x^2\ln x|_t^1 - \int_t^1 \frac{1}{2}x^2\frac{1}{x}\,dx\right] && du=\frac{dx}{x} \quad v=\frac{1}{2}x^2 \\
&= \lim_{t\to 0^+}\left[0-\frac{1}{2}t^2\ln t - [\frac{1}{4}x^2]_t^1\right] \\
&= \lim_{t\to 0^+}\left[-\frac{1}{2}t^2\ln t - \frac{1}{4} + \frac{1}{4}t\right] \\
&= \left[0-\frac{1}{4}\right] = \frac{-1}{4}.
\end{aligned}$$

■