DISPERersive SWITCHING IN BISTABLE MODELS

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Received 18 August 2008

Novel switching processes in optical bistable models of homogeneously or inhomogeneously broadened 2-level atoms placed in ring cavity with atoms in contact with normal or squeezed vacuum reservoirs are investigated. This is done through computational examination of the relevant input-output relationship by simultaneously varying the atomic and cavity detuning parameters for fixed values of the input laser field.

Keywords: Optical bistability and switching.

1. Introduction

In optics, bistable behavior of a physical system is mainly due to a device (driven optical cavity) filled with a nonlinear medium together with the action of a feedback process (provided by the cavity mirrors). For proper choice of the system parameters, the input-output field relationship exhibits bistable (or multistable) behavior. Such a bistable device with certain nonlinear materials has many potential applications\(^1\)–\(^4\) among which is the switching operation (switch-on and -off) between the two stable states of the output field.

The basic optical bistable (OB) absorptive model of homogeneously 2-level atoms placed in a ring cavity at exact atomic and cavity resonances (i.e., all three frequencies of atomic transition, input laser field and the single cavity mode are equal) has been analyzed by Bonifacio and Lugiato.\(^5\) This was subsequently generalized\(^7,8\) to include the dispersive effects (i.e., both atomic transition and cavity mode frequencies are detuned from the input field frequency) — as well as the inhomogeneous broadening of the atomic transition. Regions of bistability were identified analytically for nonzero values of the cavity and atomic detuning parameters.

In the case where the 2-level atoms is in interaction with a broadband squeezed vacuum (SV) field, the authors of Refs. 9 and 10 has investigated the “phase switching” effect where for fixed values of the input field and by varying the phase of the SV field the output field exhibits either one-way or 2-way switching effect. Also, for
a mesoscopic (nondissipative) multistable system,\textsuperscript{11,12} it has been shown that for a fixed value of the input field and by varying the atomic cooperative parameter (C) (which comprises the atomic density) the output field exhibits multiple “cooperative switching”. Similar cooperative switching process for OB model of dissipative 2-level atoms in the normal vacuum (NV) and SV fields has been analyzed in Ref. 13.

In our paper, we further investigate the switching effect, namely, the “dispersive switching” in the output field at fixed value of the input field by simultaneously varying both atomic and cavity detunings with the same amount (same or opposite sign). We examine the two main OB models of homogeneous 2-level atoms in NV and SV fields. The case of inhomogeneous broadening is also examined.

2. Homogeneously Broadened Atoms

2.1. Normal vacuum case

For the OB model of homogeneously broadened 2-level atoms in a ring cavity (or Fabry-cavity) within the spatial mean field approximation and for nonzero atomic and cavity detunings, the characteristic input-output field relation is of the normalized form,\textsuperscript{7,8}

\[
Y = \frac{X}{(1 + \delta^2 + X)^2}[(1 + \delta^2 + X + 2C)^2 + (\theta(1 + \delta^2 + X) - 2C\delta)^2].
\] (1)

The notations are: \(X, Y\) are the dimensionless output and input field intensities respectively, \(C\) is the cooperative parameter, \(\theta\) and \(\delta\) are the normalized cavity and atomic detuning parameters respectively.

Now, we observe that for \(\theta = \lambda \delta\), where \(\lambda\) is a +ve or -ve number, Eq. (1) is a cubic equation in \(\delta\) and hence for fixed values of \(Y\) and certain range of \(\delta\), the output field \(X\) will show a bistable behavior and hence switching effect. Since both \(X\) and \(\delta\) in Eq. (1) (for \(\theta = \lambda \delta\)) occur as cubics, it is not easy to analyze mathematically the range in which switching effect may occur. Hence, our investigation is computational just as in the SV case.\textsuperscript{9,10}

The 3D plot of \((Y, X, \delta)\) for \(C = 5, \theta = \delta\) is given in Fig. 1(a). The contour plot of \(X\) with \(\delta\) to be referred to as dispersive switching diagram, at fixed input field \(Y = 35\) is given in Fig. 1(b); as inverted omega or mushroom shape. In this case, we have two bistable regions in the interval \(0.28 < |\delta| < 0.56\). If we plot the input-output field \((X, Y)\) at fixed \(\theta = \delta = 0.5\) (Fig. 1(c)), then at \(Y = 35\), \(X\) has two stable (lower and upper) values corresponding to points \(P_1, P_2\) (the point \(P_2\) is unstable). Figure 1(b) shows how these points move as \(\delta\) varies. As \(\delta\) increases, \(P_1\) moves until it reaches the unstable point \(A_1\) and the system then switches on to the stable right upper branch and staying on that branch with further increase in \(\delta\).

Note that if at the point \(P_1\) in Fig. 1(b), \(\delta\) decreases then the system similarly switches-on at \(A_2\) to the stable left upper branch. The movement of point \(P_3\) on the upper branch in Fig. 1(b) shows that the system will switch-down at \(A_3\) to the
Fig. 1. For \( \theta = \delta \) and \( C = 5 \), (a) the three dimensional \((\delta - X - Y)\)-plot, (b) the \((\delta - X)\)-contour plot at \( Y = 35 \) and (c) the optical bistable behavior for \( \delta = 0.5 \).

stable lower branch, but the further decrease in \( \delta \) will cause the system to switch-up at \( A_2 \) to the upper left stable branch. Hence, we have two-way dispersive switching effect.

For increasing value of \( C = 10, \theta = \delta \), Fig. 2 shows the corresponding behavior, where the 3D plot shows a pronounced peak, and for fixed \( Y = 90 \), the switching diagram shows only one-way switching (switch-up): The point \( P_1 \) on the isle in
Fig. 2. For $\theta = \delta$ and $C = 10$, (a) the three dimensional ($\delta - X - Y$)-plot, (b) the ($\delta - X$)-contour plot at $Y = 90$ and (c) the optical bistable behavior for $\delta = 0.5$.

Fig. 2(b) moves to the unstable point $A_1(A_2)$ by increasing (decreasing) the value of $\delta$ where the system switches up to the single stable upper branch. The movement of the point $P_3$ on the upper branch will only cause a change in $X$ with peak value at $\delta = 0$, but no jumping effect occurs. For $\theta = -\delta$, the corresponding dispersive switching diagram for $C = 5, 10$, Figs. 3(a), 3(b), are essentially the inverse of Figs. 1(b), 2(b) with qualitative difference for the fixed values of $Y$ and the stable lower branch in Fig. 2(b) is almost flat.
2.2. Squeezed vacuum case

In the case where the 2-level atoms are in contact with SV field, the characteristic relation (1) is generalized to

\[ Y = \frac{X}{(1 + \delta^2 + b_1 X)^2} \left[ (1 + \delta^2 + b_1 (X + 2C))^2 + (\theta (1 + \delta^2 + b_1 X) - 2Cb_2)^2 \right] \]  \hspace{1cm} (2)

where \( b_1 = 1 - 2|M|/1 + 2N \cos(\phi) \), \( b_2 = \delta + 2|M| \sin(\phi)/1 + 2N \), \( \phi = \phi_s - 2\phi_f \) is the relative phase of the SV field, with respect to the input field. The SV parameters \( N \) (average photon number) and \( M = |M|e^{i\phi_s} \) (degree of squeezing) are related by \( |M|^2 = N(N + 1) \) for ideal squeezing.

In Fig. 4(a), we show the dispersive switching diagram for \( C = 10, N = 0.1, \phi = 0, \theta = \delta \) at fixed \( Y = 70 \), with \( (X - Y) \) relation at \( \theta = \delta = 0 \) shown in Fig. 4(b). The movement of \( P_1 \) as \( \delta \) increases will lead to switch-up at the unstable point \( A_1 \) to the right upper stable branch where further increase in \( \delta \) results in decrease of \( X \) but decreasing \( \delta \) will lead to switch-down at \( A_2 \) to the stable lower branch. As for point \( P_3 \) on the isolated isle, the increase or decrease in \( \delta \) will result in switching-down process to the stable lower branch where further increase (or decrease) in \( \delta \) will lead again to switch-up at point \( A_1 \) (or \( A_3 \)). For larger value of \( Y = 72 \), the switching diagram in Fig. 4(c) shows the merger of the isolated isle (Fig. 4(a)) which results in multiple switch-up and -down processes. For \( Y = 80 \), Fig. 4(d), shows two isolated isles underneath a continuous stable upper branch with possible switch-up processes only. In all these cases, the switching diagram is symmetric with respect to \( \delta \). Changing the SV phase to \( \phi = \pi/2 \) makes the switching diagram asymmetric in \( \delta \) (Figs. 5). The case of \( \theta = -\delta \) with \( \phi = \pi/2 \) and \( Y = 80 \) has the effect to isolate an asymmetric island around \( \delta = 0 \) (Fig. 6(a)) to induce one-way
Fig. 4. For $\theta = \delta, \phi = 0, N = 0.1$ and $C = 10$, the $(\delta - X)$-contour plot at (a) $Y = 70$ and (b) the optical bistable behavior for $\delta = 0$, the contours at (c) $Y = 72$ and (d) $Y = 80$.

switching-down effect. Increasing $Y$ to 100 causes the merge of this island with the lower stable branch and results in an asymmetric switching diagram of two-way (up and down) effect (Fig. 6(b)).

3. Inhomogeneously Broadened Atoms

In the case of inhomogeneously broadened 2-level atoms with Lorentzian distribution of peak (central) frequency $\omega_a$, the input-output field relation in the general case of SV field is of the form:

$$Y = X \left[ 1 + 2C b_1 \left( \frac{\sigma' + g_1}{g_1 g_2} \right)^2 + \left( \theta - 2C \frac{\sigma' b_3 + b_2 g_1}{g_1 g_2} \right)^2 \right]$$  (3)
where \( g_1 = g_1(X) = \sqrt{1 + b_1 X} \), \( g_2 = g_2(X) = \delta^2 + (\sigma' + g_1(X)) \), \( b_{1,2} \) are defined below Eq. (2), \( b_3 = 2|M|\sin(\phi)/1 + 2N \), \( \sigma' \) is the normalized Lorentzian width; and \( \delta \) is now the atomic detuning (i.e., the difference between the (atomic) peak frequency \( \omega_a \) and \( \omega_f \) the (laser) input field frequency).

In the case of normal vacuum \( (b_1 = 1, b_2 = \delta, b_3 = 0) \), we have obtained a qualitative dispersive switching diagram similar to that obtained in the homogeneous
Fig. 6. For $\theta = -\delta$, $\phi = \pi/2$, $N = 0.1$ and $C = 10$, the $(\delta - X)$-contour plot at (a) $Y = 80$ and (b) $Y = 100$.

Fig. 7. A contour at $Y = 490$, $\sigma' = 1$, $N = 0.1$, $C = 40$, $\theta = -\delta$, $\phi = \pi/2$ (inset zooming view for $-1.6 \leq \delta \leq -1.5$).
case. The search in the SV case for \( \sigma' = 1, \theta = -\delta, \phi = \pi/2, N = 0.1 \) shows that the dispersive switching diagram has asymmetric double OB shape, Fig. 7.

4. Summary
We have computationally explored the possibility of a new switching effect in OB models of 2-level (homogeneously/inhomogeneously broadened) atoms in the presence of normal or squeezed vacuum (SV) fields. This is done for certain input field values by simultaneously varying atomic and cavity detuning (\( \delta, \theta \)) such that \( \theta = \lambda \delta \), with \( \lambda \) constant. The switching diagram (output field versus dispersive detuning) exhibits a richness of possible one-, two-, multiple-switching process and even double bistable behavior in the inhomogeneously broadened case with SV field.

Thus, a single dispersive control (as a result of linearly and simultaneously changing both atomic and cavity detunings) offers interesting manipulation of switching effects in optical bistable devices.

Acknowledgment
The author wishes to thank Prof. S. S. Hassan (University of Bahrain) for his fruitful comments.

References