The productivity index of a well, denoted by $J$, is a measure of the ability of the well to produce. It is given by:

$$J = \frac{Q_o}{P_r - P_{wf}}$$

Where:

- $J$ = Wellbore productivity index, STB/day/psig
- $P_r$ = Average (static) reservoir pressure, psig
- $Q_o$ = Wellbore stabilized oil flow rate, STB/day
- $P_{wf}$ = Wellbore stabilized bottom-hole flowing pressure, psig

The productivity index is generally measured during a production test on the well, where the well is shut-in until the static reservoir pressure $P_r$ is reached. The well is then allowed to flow at a constant flow rate of $Q_o$ and a stabilized bottom-hole flowing pressure of $P_{wf}$. It is important to note that the productivity index is a valid measure only if the well is flowing at pseudo-steady state conditions. Therefore, in order to accurately measure the productivity index of a well, it is essential that the well is allowed to flow at a constant rate for a sufficient amount of time to reach the pseudo-steady state condition.

A plot of the bottom hole pressure, $P_{wf}$, versus the oil flow rate, $Q_o$, of an oil well is called the Inflow Performance Relationship and is referred to as IPR. The IPR curve is constructed either for the present reservoir pressure or for the future reservoir pressure.

The present IPR curve of a well can be generated via many methods. The methods are grouped into two main categories: The first category, that uses reservoir parameters, includes the integral and Fetkovitch methods. The second category includes a set of Vogel-type empirical correlations. To construct the IPR curve of a well using well performance, perform the following steps:

- Use the static reservoir pressure $P_r$ and the stabilized wellbore rate and pressure $(Q_o & P_{wf})$ to calculate $J$ as follows:

$$J = \frac{Q_o}{P_r - P_{wf}}$$

- Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ as follows:

$$Q_o = J(P_r - P_{wf})$$

A Revision Article of Oil Wells Performance Methods
I. The Integral Form

Darcy’s equation can be expressed mathematically as:

\[
Q_o = \left( \frac{kh}{141.22 \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s} \right) \int_{P_r}^{P_w} \frac{k_{ro}}{\mu_o B_o} dp
\]

The IPR curve is constructed by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \).

To construct the IPR curve of a well using well performance along with the integral method, perform the following steps:

- Use the static reservoir pressure and the stabilized wellbore rate and pressure to calculate \( J \) as follows:

\[
J = \frac{Q_o}{\int_{P_{wf}}^{P_w} \frac{1}{\mu_o B_o} dp}
\]

- Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \) as follows:

\[
Q_o = J \int_{P_{wf}}^{P_w} \frac{1}{\mu_o B_o} dp
\]

II. Fetkovitch Method

Fetkovitch (1973) started with Darcy’s equation:

\[
Q_o = \left( \frac{kh}{141.22 \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s} \right) \int_{P_{wf}}^{P_w} \frac{k_{ro}}{\mu_o B_o} dp
\]

He considered two cases:
1. Saturated Oil Reservoir $P_r \leq P_b$

When the reservoir pressure $P_r$ and the bottom-hole flowing pressure $P_{wf}$ are both below the bubble-point pressure $P_b$, the oil flow rate can be written as:

$$Q_o = \frac{kh}{141.22(\mu_o B_o)Pr \left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)} \left(\frac{1}{2P_b}\right)(p_r^2 - p_{wf}^2) = \frac{J}{2P_b}(p_r^2 - p_{wf}^2) = C(p_r^2 - p_{wf}^2)$$

The constant $C$ is referred to as the performance coefficient. The IPR curve is constructed by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ using the above equation.

2. Undersaturated Oil Reservoir $P_r > P_b$

Two cases are considered:

**Case 1: $P_{wf} \geq P_b$**

$$Q_o = \frac{kh}{141.22(\mu_o B_o)Pr \left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)} \left(\frac{1}{2P_b}\right)(p_r^2 - P_b^2) = J(p_r^2 - P_b^2)$$

**Case 2: $P_{wf} < P_b$**

$$Q_o = \frac{kh}{141.22(\mu_o B_o)Pr \left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)} \left(\frac{1}{2P_b}\right)\left(p_b^2 - P_{wf}^2\right) + \left(\frac{1}{2P_b}\right)(p_r^2 - P_b^2)$$

$$= J \left(p_r^2 - P_b^2\right) + \left(\frac{1}{2P_b}\right)(p_b^2 - P_{wf}^2)$$

III. Constant Productivity Index Method

The IPR curve of a well can be generated using the constant productivity index approach. The productivity index of a well is defined as a measure of the well to produce. It is given by:
\[ J = \frac{Q_o}{P_r - P_{wf}} = \frac{Q_o}{\Delta P} \]

Where:

\( J \) = Productivity index, STB/day/psi  
\( Q_o \) = Oil flow rate, STB/day  
\( P_r \) = Volumetric average drainage area pressure (static pressure), psi  
\( P_{wf} \) = Bottom hole flowing pressure, psi

The productivity index is generally measured during a production test on the well. The well is shut-in until the static reservoir pressure is reached. The well is then allowed to flow at a constant flow rate of \( Q_o \) and a stabilized bottom-hole flowing pressure of \( P_{wf} \). It is important to note that the productivity index is a valid measure only if the well is flowing at pseudo-steady state conditions. Therefore, in order to accurately measure the productivity index of a well, it is essential that the well is allowed to flow at a constant rate for a sufficient amount of time to reach the pseudo-steady state condition. The method is summarized as follows:

- Use the stabilized wellbore rate and pressure \((Q_o \& P_{wf})\) to calculate \( J \) as follows:

\[ J = \frac{Q_o}{P_r - P_{wf}} \]

- Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \) as follows:

\[ Q_o = J(P_r - P_{wf}) \]

IV. Vogel Method

Vogel (1968) used a computer program to generate the IPRs for several hypothetical saturated oil reservoirs producing under a wide range of conditions. When applying his method, the only data required are: the average reservoir pressure \( P_r \), the oil bubble-point pressure \( P_b \), and the stabilized wellbore rate and pressure \((Q_o \& P_{wf})\). Vogel’s methodology can be used to predict the IPR curve for the following two types of reservoirs:

1. Saturated Oil Reservoirs \( P_r \leq P_b \)

- Use the stabilized wellbore rate and pressure \((Q_o \& P_{wf})\) to calculate \((Q_o)_{\text{max}}\) as follows:

\[ (Q_o)_{\text{max}} = \frac{Q_o}{\left[ 1 - 0.2\left( \frac{P_{wf}}{P_r} \right) - 0.8\left( \frac{P_{wf}}{P_r} \right)^2 \right]} \]
• Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ as follows:

$$Q_o = (Q_o)_{max} \left[ 1 - 0.2 \left( \frac{P_{wf}}{P_r} \right) - 0.8 \left( \frac{P_{wf}}{P_r} \right)^2 \right]$$

2. Undersaturated Oil Reservoirs $P_r > P_b$

• Use the stabilized wellbore rate and pressure $(Q_o & P_{wf})$ to calculate the productivity index $J$ as follows:

$$J = \left\{ \begin{array}{ll}
\frac{Q_o}{P_r - P_{wf}} & P_{wf} \geq P_b \\
\frac{Q_o}{(P_r - P_b) + \frac{P_b}{1.8}} \left[ 1 - 0.2 \left( \frac{P_{wf}}{P_b} \right) - 0.8 \left( \frac{P_{wf}}{P_b} \right)^2 \right] & P_{wf} < P_b 
\end{array} \right.$$

• Calculate the oil flow rate at the bubble-point pressure:

$$Q_{ob} = J (P_r - P_b)$$

• Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ as follows:

$$Q_o = \left\{ \begin{array}{ll}
J(P_r - P_{wf}) & P_{wf} \geq P_b \\
Q_{ob} + \frac{JP_b}{1.8} \left[ 1 - 0.2 \left( \frac{P_{wf}}{P_b} \right) - 0.8 \left( \frac{P_{wf}}{P_b} \right)^2 \right] & P_{wf} < P_b
\end{array} \right.$$

V. Wiggins (1993) Method

Wiggins (1993) used four sets of relative permeability and fluid property data as the basic input for a computer program to generate the IPRs for several hypothetical saturated oil reservoirs producing under a wide range of conditions. The only data required are: the average
reservoir pressure $P_r$, the oil bubble-point pressure $P_b$, and the stabilized wellbore rate and pressure ($Q_o$ & $P_{wf}$). This method is considered for the following two types of reservoirs:

1. **Saturated Oil Reservoirs** $P_r \leq P_b$
   - Use the stabilized wellbore rate and pressure ($Q_o$ & $P_{wf}$) to calculate ($Q_o$)$_{\text{max}}$ as follows:
     
     $$(Q_o)_{\text{max}} = \frac{Q_o}{1 - 0.52 \left(\frac{P_{wf}}{P_r}\right) - 0.48 \left(\frac{P_{wf}}{P_r}\right)^2}$$
   
     - Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ as follows:
     
     $$Q_o = (Q_o)_{\text{max}} \left[1 - 0.52 \left(\frac{P_{wf}}{P_r}\right) - 0.48 \left(\frac{P_{wf}}{P_r}\right)^2\right]$$

2. **Undersaturated Oil Reservoirs** $P_r > P_b$
   - Use the stabilized wellbore rate and pressure ($Q_o$ & $P_{wf}$) to calculate the productivity index $J$ as follows:
     
     $$J = \begin{cases} 
     \frac{Q_o}{(P_r - P_{wf})} & \text{if } P_{wf} \geq P_b \\
     \frac{Q_o}{(P_r - P_b) + \frac{P_r}{1.8}\left[1 - 0.52 \left(\frac{P_{wf}}{P_b}\right) - 0.48 \left(\frac{P_{wf}}{P_b}\right)^2\right]} & \text{if } P_{wf} < P_b 
     \end{cases}$$
   
     - Calculate the oil flow rate at the bubble-point pressure:
     
     $$Q_{ab} = J(P_r - P_b)$$
   
     - Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ as follows:
VI. Wiggins (1996) Method

In 1996, Wiggins derived an equation for the prediction of oil well performance. His equation can be used as follows:

1. Saturated Oil Reservoirs $P_r \leq P_b$

- Use the stabilized wellbore rate and pressure $(Q_o & P_{wf})$ to calculate $(Q_o)_{max}$ as follows:

\[
(Q_o)_{max} = \frac{Q_o}{1 - 0.0933 \left(\frac{P_{wf}}{P_r}\right) - 1.6183 \left(\frac{P_{wf}}{P_r}\right)^2 + 1.5579 \left(\frac{P_{wf}}{P_r}\right)^3 - 0.8464 \left(\frac{P_{wf}}{P_r}\right)^4}
\]

- Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_o$ as follows:

\[
Q_o = (Q_o)_{max} \left[1 - 0.0933 \left(\frac{P_{wf}}{P_r}\right) - 1.6183 \left(\frac{P_{wf}}{P_r}\right)^2 + 1.5579 \left(\frac{P_{wf}}{P_r}\right)^3 - 0.8464 \left(\frac{P_{wf}}{P_r}\right)^4\right]
\]

2. Undersaturated Oil Reservoirs $P_r > P_b$

- Use the stabilized wellbore rate and pressure $(Q_o & P_{wf})$ to calculate the productivity index $J$ as follows:

\[
J = \begin{cases} 
\frac{Q_o}{(P_r - P_{wf})} & P_{wf} \geq P_b \\
\frac{Q_o}{(P_r - P_b) + \frac{P_b}{1.8} \left[1 - 0.0933 \left(\frac{P_{wf}}{P_b}\right) - 1.6183 \left(\frac{P_{wf}}{P_b}\right)^2 + 1.5579 \left(\frac{P_{wf}}{P_b}\right)^3 - 0.8464 \left(\frac{P_{wf}}{P_b}\right)^4\right]} & P_{wf} < P_b
\end{cases}
\]

- Calculate the oil flow rate at the bubble-point pressure:
\[ Q_{ob} = J(P_r - P_b) \]

- Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \) as follows:

\[
Q = \begin{cases} 
J(P_r - P_{wf}) & P_{wf} \geq P_b \\
Q_{ob} + \frac{JP_b}{1.8} \left[ 1 - 0.0933 \left( \frac{P_{wf}}{P_b} \right) - 1.6183 \left( \frac{P_{wf}}{P_b} \right)^2 + 1.5579 \left( \frac{P_{wf}}{P_b} \right)^3 - 0.8464 \left( \frac{P_{wf}}{P_b} \right)^4 \right] & P_{wf} < P_b 
\end{cases}
\]

### VII. The Klins-Clark Method

Klins and Clark (1993) proposed an inflow expression similar in form to that of Vogel’s and can be used to estimate future IPR data. This method is considered for the following two types of reservoirs:

1. **Saturated Oil Reservoirs** \( P_r \leq P_b \)
   - Use the stabilized wellbore rate and pressure \( (Q_o & P_{wf}) \) to calculate \( (Q_o)_\text{max} \) as follows:
   
   \[
   (Q_o)_{\text{max}} = \frac{Q_o}{1 - 0.295 \left( \frac{P_{wf}}{P_r} \right) - 0.705 \left( \frac{P_{wf}}{P_r} \right)^d}
   \]
   
   Where:
   
   \[
   d = 0.28 + 0.72 \left( \frac{P_r}{P_b} \right) (1.24 + 0.001P_b)
   \]
   
   - Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \) as follows:
   
   \[
   Q_o = (Q_o)_{\text{max}} \left[ 1 - 0.295 \left( \frac{P_{wf}}{P_r} \right) - 0.705 \left( \frac{P_{wf}}{P_r} \right)^d \right]
   \]

2. **Undersaturated Oil Reservoirs** \( P_r > P_b \)
   - Use the stabilized wellbore rate and pressure \( (Q_o & P_{wf}) \) to calculate the productivity index \( J \) as follows:
Calculate the oil flow rate at the bubble-point pressure:

\[ Q_{ob} = J(P_r - P_b) \]

Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \) as follows:

\[
Q_o = \begin{cases} 
J(P_r - P_{wf}) & \text{if } P_{wf} \geq P_b \\
Q_{ob} + \frac{JP_b}{1.8} \left[ 1 - 0.295 \left( \frac{P_{wf}}{P_b} \right) - 0.705 \left( \frac{P_{wf}}{P_b} \right)^d \right] & \text{if } P_{wf} < P_b
\end{cases}
\]

VIII. FPP’s Method

The FPP solution method is given by:

\[ y = a_0 + a_1 x^2 + a_2 x^2 y \]

Using the least square method, we obtain a system of three equations in three unknowns \( a_0, a_1, a_2 \):

\[
\begin{align*}
& a_0 N + a_1 \sum_N x^2 + a_2 \sum_N x^2 y = \sum_N y \\
& a_0 \sum_N x^2 + a_1 \sum_N x^2 + a_2 \sum_N x^3 = \sum_N x^2 y \\
& a_0 \sum_N x^2 + a_1 \sum_N x^3 + a_2 \sum_N x^4 = \sum_N x^2 y
\end{align*}
\]

\( N \) is the number of data points. In matrix form, this can be written as:
The power $\gamma$ is solved for to minimize the residual root mean square (RRMS):

$$RRMS = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (z_{obs} - z_{cal})^2}$$

Once the set of equations has been solved, the following equations are solved:

$Q_{o,\text{max}} = a_0$

$a_1 = Q_{o,\text{max}} a_i$

$a_2 = Q_{o,\text{max}} a_2$

$x = \frac{P_{\text{wff}}}{P_r}$

$y = Q_o$

This methodology can be used to predict the IPR curve for the following two types of reservoirs:

1. **Saturated Oil Reservoirs** $P_r \leq P_b$

   - Construct the IPR curve by assuming various values of $P_{\text{wff}}$ and calculating the corresponding $Q_o$ as follows:

   $$Q_o = Q_{o,\text{max}} \left[ 1 - a_1 \left( \frac{P_{\text{wff}}}{P_r} \right)^\gamma - a_2 \left( \frac{P_{\text{wff}}}{P_r} \right)^{2\gamma} \right]$$

2. **Undersaturated Oil Reservoirs** $P_r > P_b$

   - Use the stabilized wellbore rate and pressure $(Q_o, P_{\text{wff}})$ to calculate the productivity index $J$ as follows:
Calculate the oil flow rate at the bubble-point pressure:

\[ Q_{ob} = J(P_r - P_b) \]

Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_o \) as follows:

\[
Q_o = \begin{cases} 
J(P_r - P_{wf}) & P_{wf} \geq P_b \\
Q_{ob} + \frac{JP_{h}}{1.8} \left[ 1 - a_1 \left( \frac{P_{wf}}{P_b} \right)^{2y} - a_2 \left( \frac{P_{wf}}{P_b} \right)^{2y} \right] & P_{wf} < P_b 
\end{cases}
\]
Predicting Future IPRs

As the average reservoir pressure $P_r$ declines, the IPR curve is shifted. There are several methods that are designed to address the problem of how the IPR might shift. Four simple approximation methods are presented.

1. Vogel’s Method

Vogel’s method provides a rough approximation of the future $(Q_o)_{\text{max}, f}$ at the specified future $P_{r, f}$ as follows:

$$(Q_o)_{\text{max}, f} = (Q_o)_{\text{max}, p} \left( \frac{P_{r, f}}{P_{r, p}} \right) \left[ 0.2 + 0.8 \left( \frac{P_{r, f}}{P_{r, p}} \right) \right]\n
$$

Where the subscripts $f$ and $p$ refer to the future and previous respectively. The calculated $(Q_o)_{\text{max}, f}$ can be used to predict the future IPR at $P_{r, f}$.

2. Fetkovitch

This method, proposed by Fetkovitch (1973), provides a simple approximation of the future $(Q_o)_{\text{max}, f}$ at the specified future $P_{r, f}$ as follows:

$$(Q_o)_{\text{max}, f} = (Q_o)_{\text{max}, p} \left( \frac{P_{r, f}}{P_{r, p}} \right)^3\n
$$

Where the subscripts $f$ and $p$ refer to future and present respectively. The calculated $(Q_o)_{\text{max}, f}$ can be used to predict the future IPR at $P_{r, f}$.

3. Wiggins’ Method

Wiggins (1993) proposed the following relationship:

$$(Q_o)_{\text{max}, f} = (Q_o)_{\text{max}, p} \left( \frac{P_{r, f}}{P_{r, p}} \right) \left[ 0.15 + 0.84 \left( \frac{P_{r, f}}{P_{r, p}} \right) \right]\n
$$

4. Standing’s Method

Standing (1970) extended the application of Vogel’s to predict future IPR of a well as a function of reservoir pressure. He noted that Vogel’s equation can be arranged as:
Standing introduced the productivity index $J$ as follows:

$$J = \frac{(Q_o)_{\text{max}}}{P_r} \left[ 1 + 0.8 \left( \frac{P_{wf}}{P_r} \right) \right]$$  \hspace{1cm} (2)

He then defined the present (current) zero drawdown productivity index as:

$$J_p^* = 1.8 \left[ \frac{(Q_o)_{\text{max}}}{P_r} \right]$$  \hspace{1cm} (3)

Where $J_p^*$ is Standing’s zero-drawdown productivity index. The $J_p^*$ is related to the productivity index $J$ by:

$$\frac{J}{J_p^*} = \frac{1}{1.8} \left[ 1 + 0.8 \left( \frac{P_{wf}}{P_r} \right) \right]$$  \hspace{1cm} (4)

Combining equations (1) and (3) to eliminate $(Q_o)_{\text{max}}$ yields:

$$Q_o = \left[ \frac{J_p^* (P_r)_f}{1.8} \right] \left[ 1 - \frac{P_{wf}}{(P_r)_f} \right] \left[ 1 + 0.8 \left( \frac{P_{wf}}{(P_r)_f} \right) \right]$$  \hspace{1cm} (5)

Where the subscripts $f$ refers to future condition. Standing suggested that $J_f^*$ is estimated from the present value of $J_p^*$ by the following expression:

$$J_f^* = J_p^* \left( \frac{k_{ro}}{u_o B_o} \right)_f$$  \hspace{1cm} (6)

If relative permeability data is not available, $J_f^*$ can be estimated from:

$$J_f^* = J_p^* \left( \frac{(P_r)_f}{(P_r)_p} \right)^2$$  \hspace{1cm} (7)

Standing’s methodology for predicting a future IPR is summarized in the following steps:

- Use the stabilized wellbore rate and pressure $(Q_o \ &\ P_{wf})$ to calculate $(Q_o)_{\text{max}}$ as follows:
\[(Q_o)_{\text{max}} = \frac{Q_o}{\left(1 - \frac{P_{wf}}{P_r}\right)\left[1 + 0.8\left(\frac{P_{wf}}{P_r}\right)\right]}\]

- Calculate \(J_p^*\) using:

\[J_p^* = 1.8 \left(\frac{(Q_o)_{\text{max}}}{P_r}\right)\]

- Calculate \(J_f^*\) using:

\[J_f^* = J_p^* \left(\frac{k_{ro}}{u_o B_o}\right)_{f} \left(\frac{k_{ro}}{u_o B_o}\right)_{p}\]

if relative permeability data is not available, \(J_f^*\) can be estimated from:

\[J_f^* = J_p^* \left(\frac{(P_r)_{f}}{(P_r)_{p}}\right)^2\]

- Construct the IPR curve by assuming various values of \(P_{wf}\) and calculating the corresponding \(Q_o\) as follows:

\[Q_o = \left[\frac{J_f^*(P_r)_{f}}{1.8}\right]\left[1 - \frac{P_{wf}}{(P_r)_{f}}\right]\left[1 + 0.8\left(\frac{P_{wf}}{(P_r)_{f}}\right)\right]\]