I. Material Balance Calculations For Gas Reservoirs

For a single-phase gas reservoir, the MB equation takes the form:

\[
G(B_g - B_w) + GB_g \left( \frac{C_i + C_w S_{wi}}{1 - S_{wi}} \right) \Delta p_I + W_I B_{sw} + G_I B_{ig} = G_p B_g + W_p B_w
\]

The following cases will be considered:

I.1 Closed (Volumetric) Gas Reservoirs with Negligible Water and Rock Compressibilities

I.2 Gas Reservoirs with Water Influx and Negligible Water and Rock Compressibilities

I.3 Gas Reservoirs with Water Influx and Water and Rock Compressibilities

I.4 Abnormally High-Pressure Gas Reservoirs

I.5 Abnormally High-Pressure Gas Reservoirs with Dissolved Gas in Water (Fetkovitch Method)

I.6 Abnormally High-Pressure Gas Reservoirs with Dissolved Gas in Water (Kazemi's Method)

I.7 Graphical Technique for the Abnormally High-Pressure Gas Reservoirs (A Non-p/z Form of the Gas Material Balance)

I.8 Wet Gas Reservoirs
I.1 Closed (Volumetric) Gas Reservoir with Negligible Water and Rock Compressibilities

\[ G(B_g - B_{g*}) = G_p B_g \]

Since

\[ B_g = \frac{V_g bbl}{V_g SCF} \] and \[ V_g = \frac{z n R T}{p} \]

Then

\[ B_g = \frac{z_i n R T_f}{p_f} \frac{p_i}{z_i n R T_s} = \frac{P_s}{p_f} \frac{z_f}{P_f} \frac{C}{C} \frac{z}{p} \]

Plugging into the above equation, we obtain:

\[ G\left(\frac{z}{p} - \frac{z_i}{p_i}\right) = \frac{z}{p} G_p \frac{z}{p} (G - G_p) = \frac{z_i}{p_i} G \frac{z}{p} \left(1 - \frac{G_p}{G}\right) = \frac{z_i}{p_i} \]

Which can arranged in a straight line form as follows:

\[ \frac{p}{z} \frac{z}{z_i G} G_p \]

Which suggests plotting \( G_p \) on the x-axis versus \( p/z \) on the y-axis and drawing the best fit line through the set of points. The slope of the best fit line would be \(-p_i/(z_i G)\) and the y-intercept would be \( p_i/z_i \).
Given the following data:

Initial pressure = 3250 psia
Reservoir temperature = 213 \degree F
Standard pressure = 15.025 psia
Standard temperature = 60 \degree F
Cumulative production = 1 \times 10^9 SCF
Average reservoir pressure = 2864 psia
Gas deviation factor at 3250 psia = 0.910
Gas deviation factor at 2864 psia = 0.888
Gas deviation factor at 500 psia = 0.951

A) Calculate the initial gas in place for a closed gas reservoir with negligible water and rock compressibilities.

B) If abandonment pressure is 500 psia, calculate the initial gas reserve.

C) Based on a 500 psia abandonment pressure, calculate the remaining gas.
Solution:

A) Since:

\[
\frac{P}{Z} = \frac{P_i}{Z_i} - \frac{P_i}{Z_i} \frac{G_p}{G}
\]

Rearranging and solving for G yields:

\[
\frac{P_i}{Z_i} - \frac{P}{Z} = \frac{P_i}{Z_i} \frac{G_p}{G} \quad \Rightarrow \quad \frac{P_i - P}{Z} = \frac{P_i}{Z_i} \frac{G_p}{G} = \left(\frac{P_i}{Z_i} - \frac{P}{Z}\right) G_p
\]

Which yields:

\[
G = \frac{3250/0.910}{3250/0.910 - 2864/0.888} \times 10^9 = 10.316 \text{ MMM SCF}
\]

B) Rearranging the above equation for \(G_p\), we obtain:

\[
G_p = \left(\frac{P_i}{Z_i} - \frac{P}{Z}\right) G
\]

Which yields:

\[
G_p = \frac{3250/0.910 - 500/0.951}{3250/0.910} \times 10.316 \times 10^9 = 8.797 \text{ MMM SCF}
\]

C) The remaining gas is \(G - G_p = (10.316 - 8.797) \times 10^9 = 1.519 \text{ MMM SCF}\)
Problem #1: Given the following data:

Initial reservoir pressure = 4200 psia  
Reservoir temperature = 180 °F  
Gas deviation factor at 2000 psia = 0.8

<table>
<thead>
<tr>
<th>p/z</th>
<th>Gp, MMM SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>4600</td>
<td>0</td>
</tr>
<tr>
<td>3700</td>
<td>1</td>
</tr>
<tr>
<td>2800</td>
<td>2</td>
</tr>
</tbody>
</table>

A) What will be the cumulative gas produced when the average reservoir pressure has dropped to 2000 psia?

B) Assuming the reservoir rock has a porosity of 12%, water saturation of 30%, and reservoir thickness is 15 ft. How many acres does the reservoir cover?

Problem #2: Given the following data:

<table>
<thead>
<tr>
<th>Reservoir Pressure, p (psia)</th>
<th>Gas Deviation Factor, z (dimensionless)</th>
<th>Cumulative Gp (MMM SCF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2080</td>
<td>0.759</td>
<td>0.000</td>
</tr>
<tr>
<td>1885</td>
<td>0.767</td>
<td>6.873</td>
</tr>
<tr>
<td>1620</td>
<td>0.787</td>
<td>14.002</td>
</tr>
<tr>
<td>1205</td>
<td>0.828</td>
<td>23.687</td>
</tr>
<tr>
<td>888</td>
<td>0.866</td>
<td>31.009</td>
</tr>
<tr>
<td>645</td>
<td>0.900</td>
<td>36.207</td>
</tr>
</tbody>
</table>

A) Estimate the amount of gas initially in place.

B) Discard the first point and estimate the amount of initial gas in place.

C) Discuss your results.
Solution of Problem #1:

A) Preparing the p/z plot and drawing the best-fit line through the data points, we get:

Equation for the best-fit is: \( p/z = -900(G_p) + 4600 \); i.e \( G_p = (4600 - p/z)/900 \).

Thus at \( p/z = 2000/0.8 = 2500 \), \( G_p = 2.33 \) MMM SCF.

B) Setting \( p/z = 0 \) in the best-fit equation yields the value of \( G \) to be 5.11 MMM SCF.

Since \( G = 43,560 \times A \times h \times \phi \times (1 - S_{wi})/B_{gi} \) and \( B_{gi} = (14.7 \times (180+460))/ (1 \times 520 \times 4600) = 0.00393311 \)

Thus \( A = GB_{gi}/(43,560 \times h \times \phi \times (1 - S_{wi})) = (5.11 \times 10^9 \times 0.00393311)/(43,560 \times 15 \times 0.12 \times (1 - 0.30)) = 366 \) acres.
Solution of Problem #2:

A) Preparing the p/z plot and drawing the best-fit line through the data points, we get:

Equation for the first best-fit (solid line) is: $p/z = -57.1317(G_p) + 2806.51$; i.e $G_p = (2806.51 - p/z)/57.1317$. Thus setting $p/z = 0$ in the best-fit equation yields the value of $G$ to be 49.12 MMM SCF.

B) Discarding the first point and drawing the best-fit through the rest of points (dashed line in Fig. 2) yields the following best-fit equation: $p/z = -59.7042(G_p) + 2877.32$; i.e $G_p = (2877.32 - p/z)/59.7042$. Therefore setting $p/z = 0$ in the best-fit equation yields the value of $G$ to be 48.20 MMM SCF.

Fig. 2: p/z versus $G_p$
I.2 Gas Reservoirs with Water Influx and Negligible Water and Rock Compressibilities

\[ G(B_g - B_{g*}) + W_e = G_p B_g + W_p B_w \]

Rearranging and dividing all through by \( B_g \) yields:

\[ G \left( 1 - \frac{B_g}{B_{g*}} \right) = G_p - \frac{W_e - W_p B_w}{B_g} \]

Substituting for \( B_g = C \frac{z}{p} \) on the LHS and dividing by \( G \) yields:

\[ \left( 1 - \frac{p/z}{p_i/z_i} \right) = \frac{1}{G} \left( G_p - \frac{W_e - W_p B_w}{B_g} \right) \]

Solving for \( p/z \) yields:

\[ \frac{p}{z} = \frac{p_i}{z_i} - \frac{p_i}{z_i} \frac{G_p - \frac{W_e - W_p B_w}{B_g}}{G} \]

Which suggests plotting \( (G_p - (W_e - W_p B_w)/B_g) \) on the x-axis versus \( p/z \) on the y-axis and drawing the best fit line through the set of points. The slope of the best fit line would be \(-p/(z_i G)\) and the y-intercept would be \( p_i/z_i \).
Given the following data:

- Initial bulk reservoir volume = 415.3 MM ft³
- Average porosity = 0.172
- Average connate water saturation = 0.25
- Initial pressure = 3200 psia
- Initial gas FVF, \( B_{gi} = 0.005262 \) ft³/SCF
- Final pressure = 2925 psia
- Gas FVF at final pressure = 0.0057 ft³/SCF
- Cumulative water production = 15,200 STB
- Water FVF, \( B_w = 1.03 \) bbl/STB
- Cumulative gas produced, \( G_p = 935.4 \) MM SCF
- Bulk volume invaded by water at final pressure = 13.04 MM ft³

A) Calculate initial gas in place.

B) Calculate water influx, \( W_e \).

C) Calculate residual gas saturation, \( S_{gr} \).
Solution:

A) \[ G = 415.3 \times 10^6 \times 0.172 \times (1 - 0.25) / 0.005262 = 10.18 \text{ MMM SCF} \]

B) Since:

\[ G(B_g - B_{gr}) + W_e = G_p B_g + W_p B_w \]

Rearranging and solving for \( W_e \), we get:

\[ W_e = G_p B_g + W_p B_w - G(B_g - B_{gr}) \]

which yields:

\[ W_e = 935.4 \times 10^6 \times 0.0057 + 15,200 \times 5.6146 \times 1.03 - 10.18 \times 10^9 \times (0.0057 - 0.005262) \]

\[ = 960,400 \text{ ft}^3 \]

C) \( S_w = \) water volume/pore volume = \( V_w/V_p = \)

(connate water + water influx - water produced)/pore volume =

\( (13.04 \times 10^6 \times 0.172 \times 0.25 + 960,400 - 15,200 \times 5.6146 \times 1.03)/\)

\( 13.04 \times 10^6 \times 0.172 = 0.64 \)

Therefore, \( S_{gr} = 1 - 0.64 = 0.36 \) or 36%.
 EPS-441: Petroleum Development Geology

*Material Balance Calculations for a Gas Reservoir with Water Influx*
*Neglecting Water and Rock Compressibilities: p/z plot*

Given the following reservoir and rock properties:

Areal extent, \( A = 8870 \) acres
Formation thickness, \( h = 32.5 \) ft
Porosity, \( \phi = 30.8 \% \)
Initial water saturation, \( S_{wi} = 42.5 \% \)
Gas deviation factor at standard conditions, \( z_b = 1.0 \)

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>( G_p ) (MMM SCF)</th>
<th>( p ) (psia)</th>
<th>( z )</th>
<th>( B_g ) (bb/SCF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2333</td>
<td>0.882</td>
<td>0.001,172</td>
</tr>
<tr>
<td>2</td>
<td>2.305</td>
<td>2321</td>
<td>0.883</td>
<td>0.001,180</td>
</tr>
<tr>
<td>4</td>
<td>20.257</td>
<td>2203</td>
<td>0.884</td>
<td>0.001,244</td>
</tr>
<tr>
<td>6</td>
<td>49.719</td>
<td>2028</td>
<td>0.888</td>
<td>0.001,358</td>
</tr>
<tr>
<td>8</td>
<td>80.134</td>
<td>1854</td>
<td>0.894</td>
<td>0.001,496</td>
</tr>
<tr>
<td>10</td>
<td>105.930</td>
<td>1711</td>
<td>0.899</td>
<td>0.001,630</td>
</tr>
<tr>
<td>12</td>
<td>135.350</td>
<td>1531</td>
<td>0.907</td>
<td>0.001,820</td>
</tr>
<tr>
<td>14</td>
<td>157.110</td>
<td>1418</td>
<td>0.912</td>
<td>0.001,995</td>
</tr>
<tr>
<td>16</td>
<td>178.300</td>
<td>1306</td>
<td>0.921</td>
<td>0.002,187</td>
</tr>
<tr>
<td>18</td>
<td>192.089</td>
<td>1227</td>
<td>0.922</td>
<td>0.002,330</td>
</tr>
<tr>
<td>20</td>
<td>205.744</td>
<td>1153</td>
<td>0.928</td>
<td>0.002,495</td>
</tr>
</tbody>
</table>

1) Estimate the initial gas in place by:

   A) Volumetric calculation

   B) MB calculation neglecting water influx

   C) MB calculation with water influx

2) Discuss the results
Solution:

A) The volumetric calculation of the initial gas in place yields:

\[ G = \frac{43,560 \times 8870 \times 32.50 \times 0.308 \times (1-0.425)}{(0.001172 \times 5.6146)} \]
\[ = 338 \text{ MMM SCF} \]

B) Prepare a p/z plot versus \( G_p \) assuming no water influx:

The best-fit equation is: \( p/z = -6.82595(G_p) + 2632.1 \); i.e \( G_p = (2632.1 - p/z)/6.82595 \).

Thus setting \( p/z = 0 \) in the best-fit equation yields the value of \( G \) to be 385.6 MMM SCF.

This value is much higher than the volumetric calculated value.

A close look to the plot indicates an upward curvature of the line connecting the data points indicating water encroachment. Thus, the p/z plot versus \( G_p \) can not be used to estimate gas in place.
C) Including water influx, \( W_e \), in the gas material balance equation yields:

\[
G(B_g - B_{gi}) + W_e = G_p B_g
\]

and preparing the following table:

<table>
<thead>
<tr>
<th>( \frac{p}{z} )</th>
<th>( W_e, \text{ bbl} )</th>
<th>( G_p - \frac{W_e}{B_g} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2645.12</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>2628.54</td>
<td>1.62E+04</td>
<td>2.29E+09</td>
</tr>
<tr>
<td>2492.08</td>
<td>8.66E+05</td>
<td>1.96E+10</td>
</tr>
<tr>
<td>2283.78</td>
<td>4.66E+06</td>
<td>4.63E+10</td>
</tr>
<tr>
<td>2073.83</td>
<td>1.04E+07</td>
<td>7.32E+10</td>
</tr>
<tr>
<td>1903.23</td>
<td>1.79E+07</td>
<td>9.50E+10</td>
</tr>
<tr>
<td>1687.98</td>
<td>2.73E+07</td>
<td>1.20E+11</td>
</tr>
<tr>
<td>1554.82</td>
<td>3.53E+07</td>
<td>1.39E+11</td>
</tr>
<tr>
<td>1418.02</td>
<td>4.69E+07</td>
<td>1.57E+11</td>
</tr>
<tr>
<td>1330.80</td>
<td>5.62E+07</td>
<td>1.68E+11</td>
</tr>
<tr>
<td>1242.46</td>
<td>6.62E+07</td>
<td>1.79E+11</td>
</tr>
</tbody>
</table>

Plot \( (G_p - \frac{W_e}{B_g}) \) on the x-axis versus \( \frac{p}{z} \) on the y-axis and draw the best-fit line through the set of points, see Fig. 4. The best-fit equation is: \( \frac{p}{z} = -7.84554 \times 10^{-9} (G_p - \frac{W_e}{B_g}) + 2645.58 \); i.e \( G_p - \frac{W_e}{B_g} = (2645.58 - \frac{p}{z})/7.84554 \). Thus setting \( \frac{p}{z} = 0 \) in the best-fit equation yields the value of \( G \) to be 337.2 MMM SCF.

D) Discussion of results:

The amount of water influx was calculated on the assumption that the volumetric calculation of the gas in place is valid. However, a more elaborate technique for water influx calculation should be used. Such techniques will be covered in the "Water Influx" chapter.
Fig. 4: \( p/z \) plot versus \( G_p \cdot \frac{We}{B_g} \)