A Revision Article of Gas Wells Performance Methods

I. Pseudo-Steady State (Laminar) Flow Condition

The exact solution to the differential form of Darcy's equation for compressible fluids under the pseudo-steady state flow condition is expressed mathematically as:

\[
Q_g = \frac{k h (\psi_r - \psi_{wf})}{1422 T \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]} = \frac{k h}{1422 T \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]} \int_{p_{ref}}^{p} \frac{2 \mu_g Z}{\psi} dp \quad (1a)
\]

Since \( B_g \) is given by:

\[
B_g = \frac{(14.7)Z T}{(5.6146)(520) \rho} = 0.005035 \frac{Z T}{\rho}
\]

Plugging into the above equation yields:

\[
Q_g = \frac{k h}{141.22 \times 10^3 \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]} \int_{p_{ref}}^{p} \frac{1}{\mu_g B_g} dp \quad (1b)
\]

Where:

- \( Q_g \) = gas flow rate, Mscf/day
- \( K \) = permeability, md
- \( h \) = reservoir thickness, ft
- \( T \) = reservoir temperature, °R
- \( r_e \) = drainage radius, ft
- \( r_w \) = wellbore radius, ft
- \( s \) = skin factor
- \( \mu_g \) = gas viscosity, cp
- \( Z \) = gas compressibility factor
- \( B_g \) = gas volume factor, bbl/scf
- \( \psi_r \) = average (static) reservoir real gas pseudo-pressure, psi²/cp
- \( \psi_{wf} \) = wellbore bottom-hole flowing real gas pseudo-pressure, psi²/cp
Equation (1) can be approximated since the pressure function exhibits three distinct pressure application regions as follows:

1. **Low Pressure Region**

   When both $P_r$ and $P_{wf}$ are less than 2000 psi, the pressure functions $\frac{2p}{\mu_g Z}$ and $\frac{1}{\mu_g B_g}$ exhibit a linear relationship with pressure; i.e. the product $\mu_g Z$ is essentially constant when evaluated at pressures below 2000 psi. Implementing this observation into the above integral gives the following approximation:

   $$Q_g = \left\{ \frac{kh(p_r^2 - p_{wf}^2)}{1422T(\mu_g Z)\left[ \frac{p_r^2 + p_{wf}^2}{2} \right]} \ln \left( \frac{r_r}{r_w} \right) - 0.75 + s \right\} = J(p_r^2 - p_{wf}^2)$$

   Where $J$ is the productivity index of the gas well which is analogous to that of an oil well; i.e.:

   $$J = \frac{kh(\psi_r - \psi_{wf})}{1422T(\mu_g Z)\left[ \frac{p_r^2 + p_{wf}^2}{2} \right] \ln \left( \frac{r_r}{r_w} \right) - 0.75 + s} = \frac{Q_g}{\psi_r - \psi_{wf}}$$

   Construct the IPR curve by assuming various values of $P_{wf}$ and calculating the corresponding $Q_g$. This method is commonly called the pressure-squared approximation method.

2. **Intermediate Pressure Region**

   When both $P_r$ and $P_{wf}$ are between 2000 and 3000 psi, the pressure functions $\frac{2p}{\mu_g Z}$ and $\frac{1}{\mu_g B_g}$ show distinct curvature. Construct the IPR curve by assuming various values $p_{wf}$ and calculating the corresponding $Q_g$ using the above equation:

   $$Q_g = \left\{ \frac{kh(\psi_r - \psi_{wf})}{1422T\ln \left( \frac{r_r}{r_w} \right) - 0.75 + s} \right\} = \left\{ \frac{kh}{1422T\ln \left( \frac{r_r}{r_w} \right) - 0.75 + s} \right\} \int_{p_{wf}}^{p_r} \frac{2p}{\mu_g Z} dp$$
\[
\begin{align*}
= & \left\{ \frac{kh}{141.22 \times 10^3 \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]} \right\} \int_{p_{wf}}^{p_r} \frac{2p}{\mu_B Z} dp \\
& \text{(3)}
\end{align*}
\]

3. High Pressure Region

When both \( P_r \) and \( P_{wf} \) are greater than 3000 psi, the pressure functions \( \frac{2p}{\mu_B Z} \) and \( \frac{1}{\mu_B B_g} \) are nearly constants. This suggests that the pressure term in the above equation is taken outside the integral to give the following approximation:

\[
Q_g = \left\{ \frac{kh(p_r - p_{wf})}{141.22 \times 10^3 (\mu_B B_g) \frac{1}{2} \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]} \right\} \text{(4)}
\]

Construct the IPR curve by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_g \). This method is commonly called the pressure-approximation method.
II. Semi-Steady State (Turbulent) Flow Condition

To account for the additional pressure drops due to the turbulent flow, the rate-dependent skin factor $DQ_g$ is included in the pseudo-steady state equations. The resulting set of equations is:

1. Pressure-Squared Approximation Form

$$Q_g = \frac{kh(p^2_r - p^2_{wf})}{1422T(\mu_g Z) \ln \left( \frac{r_e}{r_w} \right) - 0.75 + DQ_g}$$

Where:

$$D = \text{inertial or turbulent flow term} = \frac{Fkh}{1422T}$$

$$F = \text{non-Darcy flow coefficient} = 3.161 \times 10^{-12} \left[ \frac{\beta T \nu_g}{h^2 r_w} \right]$$

$$\beta = \text{turbulence parameter} = 1.88 \times 10^{10} k^{-1.47} \phi^{-0.53}$$

2. Pressure Approximation Form

$$Q_g = \frac{kh(p_r - p_{of})}{141.22 \times 10^3 (\mu_g B_g) \left( \frac{p_{of} + p_r}{2} \right) \ln \left( \frac{r_e}{r_w} \right) - 0.75 + DQ_g}$$

3. Real Gas Potential (Pseudo-Pressure) Form

$$Q_g = \frac{kh(\psi_r - \psi_{of})}{1422T \ln \left( \frac{r_e}{r_w} \right) - 0.75 + DQ_g}$$

The above equations do not represent explicit expressions for calculating the gas flow rate. There are two separate empirical treatments that can be used to represent the turbulent flow problem in
gas wells. Both treatments are directly derived from the above equations. The two treatments are called the simplified treatment approach and the laminar inertial turbulent approach.

1. The Simplified Treatment Approach

Based on the analysis of a large number of gas wells, Rawlins and Schellhardt (1936) postulated that the relationship between the gas flow rate and pressure can be expressed as:

\[ Q_g = C(p_r^2 - p_{wf}^2)^n \]

Where:
- \( Q_g \) = gas flow rate, Mscf/day
- \( P_r \) = average reservoir pressure, psi
- \( P_{wf} \) = wellbore flowing pressure, psi
- \( n \) = deliverability exponent
- \( C \) = performance coefficient, Mscf/day/psi

This equation is called Deliverability or Back-pressure equation. Taking the logarithm of both sides of the equation yields:

\[ \log(Q_g) = \log(C) + n \log(p_r^2 - p_{wf}^2) \]

This equation suggests that a plot of \( Q_g \) versus \( (p_r^2 - p_{wf}^2) \) on a log-log paper should yield a straight line having a slope of \( n \). In the industry, the plot is reversed by plotting \( (p_r^2 - p_{wf}^2) \) versus \( Q_g \) produce a straight line with a slope of \( (1/n) \). This plot is called Deliverability or Back-pressure plot. The deliverability exponent can be determined from any two points on the line. \( C \) is then calculated as follows:

\[ C = \frac{Q_g}{(p_r^2 - p_{wf}^2)^n} \]

Once \( n \) and \( C \) are determined, the gas flow rate \( Q_g \) at any pressure \( P_{wf} \) can be calculated and the IPR curve may be constructed. There are essentially three types of deliverability tests. These are:

- Conventional Deliverability (Backpressure) Test
- Isochronal Test
- Modified isochronal Test
2. The Laminar Inertia! Turbulent Approach

• Pressure-Squared Quadratic Approach (recommended at pressures below 2000 psi)

The pressure-squared equation:

\[
Q_g = \frac{kh(p_r^2 - p_{wf}^2)}{1422T(\mu_g Z)\left(\frac{p_{wf}^2 + p_r^2}{2}\right)} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + DQ_g\right]
\]

Can be written in a more simplified form as follows:

\[
p_r^2 - p_{wf}^2 = aQ_g^2 + bQ_g
\]

Where:

\[
b = \text{laminar flow coefficient, which is given by:}
\]

\[
b = \left[\frac{1422T(\mu_g Z)}{kh} \left(\frac{p_{wf}^2 + p_r^2}{2}\right)\right] \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]
\]

\[
a = \text{inertial-turbulent flow coefficient, which is given by:}
\]

\[
a = \left[\frac{1422T(\mu_g Z)}{kh} D\right]
\]

The above equation can be linearized by dividing both sides of the equation by \(Q_g\) to yield:

\[
\frac{p_r^2 - p_{wf}^2}{Q_g} = aQ_g + b
\]

The coefficients \(a\) and \(b\) are determined by plotting \(\frac{p_r^2 - p_{wf}^2}{Q_g}\) versus \(Q_g\) on a linear scale and should yield a straight line with a slope of \(a\) and an intercept of \(b\). Given the values of \(a\) and \(b\), the quadratic flow equation can be solved for \(Q_g\) at any \(P_{wf}\) from:

\[
Q_g = \frac{-b + \sqrt{b^2 + 4a(p_r^2 - p_{wf}^2)}}{2a}
\]

The IPR curve is constructed by assuming various values of \(P_{wf}\) and calculating the corresponding \(Q_g\).
• **Pressure Quadratic Approach** (recommended at pressures above 3000 psi)

The pressure equation:

\[
Q_g = \frac{kh(p_r - p_{wf})}{141.22 \times 10^3 (\mu_g B_g) \frac{p_r + p_{wf}}{2} \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s + DQ_g \right]}
\]

Can be written in a more simplified form as follows:

\[
p_r - p_{wf} = aQ_g^2 + bQ_g
\]

Where:

\[b = \text{laminar flow coefficient, which is given by:}\]

\[
b = \left[ \frac{141.22 \times 10^{-3} (\mu_g Z)}{kh} \left( \frac{p_r - p_{wf}}{2} \right) \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right]
\]

\[a = \text{inertial-turbulent flow coefficient, which is given by:}\]

\[
a = \left[ \frac{141.22 \times 10^{-3} (\mu_g Z)}{kh} \left( \frac{p_r - p_{wf}}{2} \right) D \right]
\]

The above equation can be linearized by dividing both sides of the equation by \( Q_g \) to yield:

\[
\frac{p_r - p_{wf}}{Q_g} = aQ_g + b
\]

The coefficients a and b are determined by plotting \( \frac{p_r - p_{wf}}{Q_g} \) versus \( Q_g \) on a linear scale should yield a straight line with a slope of a and an intercept of b. Given the values of a and b, the quadratic flow equation can be solved for \( Q_g \) at any \( P_{wf} \) from:

\[
Q_g = \frac{-b + \sqrt{b^2 + 4a(p_r - p_{wf})}}{2a}
\]

The IPR curve is constructed by assuming various values of \( P_{wf} \) and calculating the corresponding \( Q_g \).
• **Pseudo-pressure Quadratic Approach** (more rigorous and applicable to all ranges of pressure)

The pseudo-pressure equation:

\[
Q_g = \frac{kh(\psi_r - \psi_{wf})}{1422T \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s + DQ_g \right]}
\]

Can be written in a more simplified form as follows:

\[
\psi_r - \psi_{wf} = aQ_g^2 + bQ_g
\]

where:

\[b = \text{laminar flow coefficient, which is given by:}\]

\[
b = \left( \frac{1422}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) - 0.75 + s \right] \right)
\]

\[a = \text{inertial-turbulent flow coefficient, which is given by:}\]

\[a = \left( \frac{1422}{kh} \right) D
\]

The above equation can be linearized by dividing both sides of the equation by \(Q_g\) to yield:

\[
\frac{\psi_r - \psi_{wf}}{Q_g} = aQ_g + b
\]

The coefficients \(a\) and \(b\) are determined by plotting \(\frac{\psi_r - \psi_{wf}}{Q_g}\) versus \(Q_g\) on a linear scale and should yield a straight line with a slope of \(a\) and an intercept of \(b\). Given the values of \(a\) and \(b\), the quadratic flow equation can be solved for \(Q_g\) any \(\psi_{wf}\) from:

\[
Q_g = -b + \frac{\sqrt{b^2 + 4a(\psi_r - \psi_{wf})}}{2a}
\]

\[
= -b + \frac{\sqrt{b^2 + 4a \int \frac{2p}{\mu_g Z} dp}}{2a}
\]

The IPR curve is constructed by assuming various values of \(\psi_{wf}\) and calculating the corresponding \(Q_{vg}\).